ICSE
Mathematics
Sample Paper 1

Time: 2 hr 30 min **Total Marks: 80**

General Instructions:

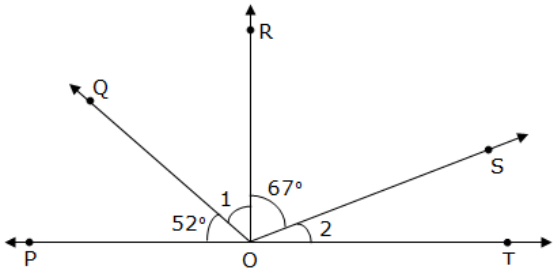
1. Answers to this paper must be written on the paper provided separately.
2. You will not be allowed to write during the first **15 minutes**.
3. This time is to be spent in reading the question paper.
4. The time given at the head of this paper is the time allowed for writing the answers.
5. Attempt **all** questions from **Section A.** Solve any **four** questions from **Section B.**
6. **All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.**
7. **Omission of essential working will result in loss of marks.**
8. The intended marks for questions or parts of questions are given in brackets [ ].

Section A (40 marks)

Question 1

(a) Do the ratios 30 cm to 4 m and 20 sec to 6 minutes form a proportion? [3]

(b) If RO is perpendicular to PT, find the measure of angles 1 and 2 in the figure below: [3]



(c) Simplify: $\frac{7\sqrt{3}}{\sqrt{6}-\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{8}+\sqrt{2}}$ [4]

Question 2

(a) The sum of two numbers is 55 and their H.C.F. and L.C.M. are 5 and 120 respectively, then, find the sum of the reciprocals of the numbers. [3]

(b) If the product of two positive consecutive even integers is 168, find the integers. [3]

(c) A's income is 60% more than that of B. By what percent is B's income less than A's? [4]

Question 3

(a) In a parallelogram ABCD, if its area is 20 cm^2, find the area of ΔABC and the distance between the sides AB and CD, if AB = 5 cm. [3]

(b) Given: A = {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, find (A × B) ∩ (B × C). [3]

(c) Simplify: $\frac{\left(2x^2y^3\right)^5 \times \left(2x^2y^2\right)^3}{\left(5x^4y\right)^6}$ [4]

Question 4

(a) Find the square root of $5\frac{19}{25}$. [3]

(b) Find the fraction which becomes ½ when its numerator is increased by 6 and is equal to 1/3 when its denominator is increased by 7.Find the fraction. [3]

(c) The table below classifies the days of the months of June, July and August according to the rainfall received in a locality. [4]

Rain (mm)	Days
10 – 20	8
20 – 30	10
30 – 40	14
40 – 50	20
50 – 60	15
60 – 70	8
70 – 80	7
80 – 90	6
90 – 100	4

Draw a histogram for this data.

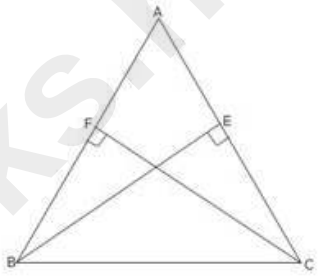
Section B (40 Marks)

Question 5

(a) Evaluate: $5 + 6 - 3 \times (2 + 70) - \frac{50}{2} + (3 + 7 \times 2 - 9)$ [3]

(b) Draw a circle of radius 2.5 cm. Show and define minor and major segments. [3]

(c) In ΔABC, BE and CF are altitudes on the sides AC and AB respectively such that BE = CF. Prove that AB = AC. [4]



Question 6

(a) Raj covered a certain distance in 6 hours. He covered some part of the journey by bus at 30 km/ h and the remaining part of the journey by train at 50 km/h. Find the distance covered for the entire journey. [3]

(b) Simplify: $\frac{x^2 - 3x - 10}{x^2 - x - 20} \times \frac{x^2 - 2x + 4}{x^3 + 8}$ [3]

(c) Draw triangle according to the following measures: [4]

ΔDEF: l(DE) = l(DF) = 6 cm, m∠D = 40°

Question 7

(a) How much compound interest is earned on Rs. 18,000 at 7% interest rate for 1 year? [3]

(b) Make d as the subject of the formula: $S = \frac{n}{2}\{2a + (n-1)d\}$ [3]

(c) The surface area of a cuboidal wooden box is 470 cm^2. If its length and breadth are 15 cm and 8 cm respectively, find its height. [4]

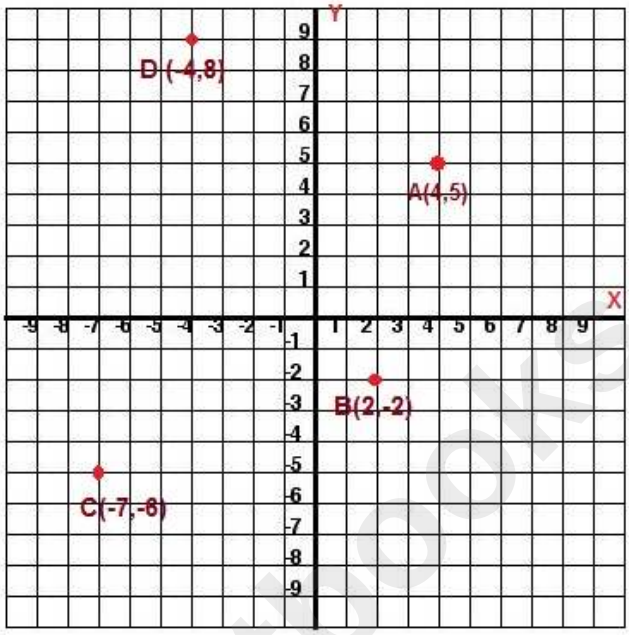
Question 8

(a) Simplify: $20x - [15x^3 + 5x^2 - \{8x^2 - (4 - 2x - x^3) - 5x^3\} - 2x]$ [3]

(b) Write down the co-ordinates of the images for the points plotted in the graph.

i. Point A and Point D reflected in the x-axis. [3]

ii. Point B and Point C reflected in the y-axis.



(c) The marks obtained by the students in a class test are given below: [4]

31, 12, 28, 45, 32, 16, 49, 12, 18, 26, 34, 39, 29, 28, 25, 46, 32, 13, 14, 26, 25, 34, 23, 23, 25, 45, 33, 22, 18, 37, 26, 19, 20, 30, 28, 38, 42, 21, 36, 19, 20, 40, 48, 15, 46, 26, 23, 33, 47, 40.

Taking class intervals 10-15, 15-20, …… 45-50; construct a frequency table.

Question 9

(a) Find the sum of the interior angles of a polygon of: [3]

i. 6 sides

ii. 8 sides

iii. 13 sides

(b) The area of a trapezium is 105 cm^2 and its height is 7 cm. If one of the parallel sides is longer than the other by 6 cm. Find the two parallel sides. [3]

(c) A and B are two sets such that $n(A - B) = 32 + x$, $n(B - A) = 5x$ and $n(A \cap B) = x$. Illustrate the information by means of a Venn-diagram. [4]

Given that $n(A) = n(B)$, calculate

i. the value of x.

ii. $n(A \cup B)$

Solution

Section A (40 marks)

Question 1

(a) Ratio of 30 cm to 4 m = 30 : 4 × 100 (1 m = 100 cm)

= 30 : 400

= 3 : 40

Ratio of 20 sec to 6 minutes = 20 : 6 × 60 (1 min = 60 sec)

= 20 : 360

= 1 : 18

Since, 3 : 40 ≠ 1 : 18, the given ratio does not form a proportion.

(b) ∠POR and ∠ROT are right angles.

Thus, ∠POQ + ∠1 = 90°

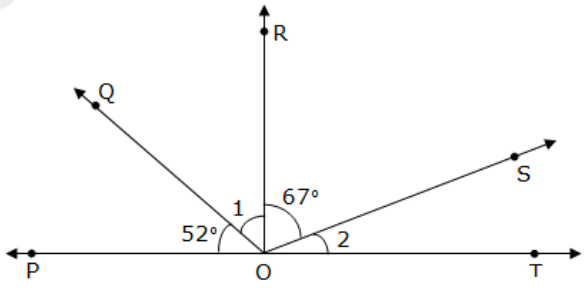
∴52° + ∠1 = 90°

∴∠1 = 90° – 52° = 38°

Also, ∠ROS + ∠2 = 90°

∴67° + ∠2 = 90°

∴∠2 = 90° - 67° = 23°



(c) $\frac{7\sqrt{3}}{\sqrt{6}-\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{8}+\sqrt{2}}$

R.F. of $\left(\sqrt{6}-\sqrt{3}\right)$ is $\left(\sqrt{6}+\sqrt{3}\right)$ and R.F. of $\left(\sqrt{8}+\sqrt{2}\right)$ is $\left(\sqrt{8}-\sqrt{2}\right)$.

$$\therefore \frac{7\sqrt{3}}{\sqrt{6}-\sqrt{3}}\times\frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{8}+\sqrt{2}}\times\frac{\sqrt{8}-\sqrt{2}}{\sqrt{8}-\sqrt{2}}$$

$$=\frac{7\sqrt{18}+7\times 3}{\left(\sqrt{6}\right)^2-\left(\sqrt{3}\right)^2}-\frac{2\sqrt{40}-2\sqrt{10}}{\left(\sqrt{8}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{7\sqrt{9\times 2}+21}{6-3}-\frac{2\sqrt{4\times 10}-2\sqrt{10}}{8-2}$$

$$=\frac{21\sqrt{2}+21}{3}-\frac{4\sqrt{10}-2\sqrt{10}}{6}$$

$$=\frac{21\left(\sqrt{2}+1\right)}{3}-\frac{2\sqrt{10}(2-1)}{6}$$

$$=\frac{21\left(\sqrt{2}+1\right)}{3}-\frac{2\sqrt{10}}{6}$$

$$=\frac{21\left(\sqrt{2}+1\right)}{3}-\frac{\sqrt{10}}{3}$$

$$=\frac{21\left(\sqrt{2}+1\right)-\sqrt{10}}{3}$$

Question 2

(a) Let the two numbers be 'a' and 'b'.

Then, a + b = 55

And, H.C.F. × L.C.M. = a × b = 5 × 120 = 600

$$\therefore \text{Required sum}=\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}=\frac{55}{600}=\frac{11}{120}$$

(b) Let x be the first positive even integer.

Then, the other positive even integer will be x + 2.

According to given information, we have

$x(x + 2) = 168$

$\therefore x^2 + 2x = 168$

$\therefore x^2 + 2x - 168 = 0$

$\therefore x^2 + 14x - 12x - 168 = 0$

$\therefore x(x + 14) - 12(x + 14) = 0$

$\therefore (x + 14)(x - 12) = 0$

$\therefore x + 14 = 0$ or $x - 12 = 0$

$\therefore x = -14$ or $x = 12$

Since x is a positive even integer, $x = 12$

The other positive even integer will be $x + 2 = 12 + 2 = 14$

Hence, 12 and 14 are the required integers.

(c) Let B's income be Rs 100.
Then, A's income = Rs 160
If A's income is Rs 160, then B's income is Rs 100.

If A's income is Rs 100, then B's income $= \left(\frac{100 \times 100}{160}\right)$ = Rs 62.50

Therefore, B's income is less than A's income by (100 – 62.50)% = 37.5%

Question 3

(a) Area of parallelogram ABCD = 20 cm^2

Since parallelogram ABCD and ΔABC are on the same base AB and between the same parallels AB parallel to DC, we have

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Area of parallelogram } ABCD = \frac{1}{2} \times 20 = 10 \text{ cm}^2$$

$$\text{Also, area}(\Delta ABC) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\therefore 10 = \frac{1}{2} \times AB \times BC$$

$$\therefore 10 = \frac{1}{2} \times 5 \times BC$$

$$\therefore BC = \frac{10 \times 2}{5} = 4 \text{ cm}$$

Thus, the distance between the sides AB and CD is 4 cm.

(b) Given: A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}

$\therefore A \times B = \{1, 2, 3\} \times \{3, 4\} = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$

And, $B \times C = \{3, 4\} \times \{4, 5, 6\} = \{(3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$

$\therefore (A \times B) \cap (B \times C) = \{3, 4\}$

(c) $\dfrac{\left(2x^2y^3\right)^5 \times \left(2x^2y^2\right)^3}{\left(5x^4y\right)^6}$

$$= \frac{\left[(2)^5.(x^2)^5.(y^3)^5\right] \times \left[(2)^3.(x^2)^3.(y^2)^3\right]}{(5)^6.(x^4)^6.(y)^6}$$

$$= \frac{2^5 \times x^{10} \times y^{15} \times 2^3 \times x^6 \times y^6}{5^6 \times x^{24} \times y^6}$$

$$= \frac{(2^5 \times 2^3) \times (x^{10} \times x^6) \times (y^{15} \times y^6)}{5^6 \times x^{24} \times y^6}$$

$$= \frac{2^8 \times x^{16} \times y^{21}}{5^6 \times x^{24} \times y^6}$$

$$= \frac{2^8}{5^6} \times \frac{1}{x^{24-16}} \times y^{21-6}$$

$$= \frac{2^8 y^{15}}{5^6 x^8}$$

Question 4

(a) Square root of $5\frac{19}{25}$: [3]

$$5\frac{19}{25} = \frac{25 \times 5 + 19}{25} = \frac{144}{25}$$

$$\frac{144}{25} = \frac{2 \times 2 \times 2 \times 2 \times 3 \times 3}{5 \times 5}$$

$$\therefore \sqrt{\frac{144}{25}} = \sqrt{\frac{2^2 \times 2^2 \times 3^2}{5^2}}$$

$$\therefore \sqrt{\frac{144}{25}} = \frac{2 \times 2 \times 3}{5} = \frac{12}{5}$$

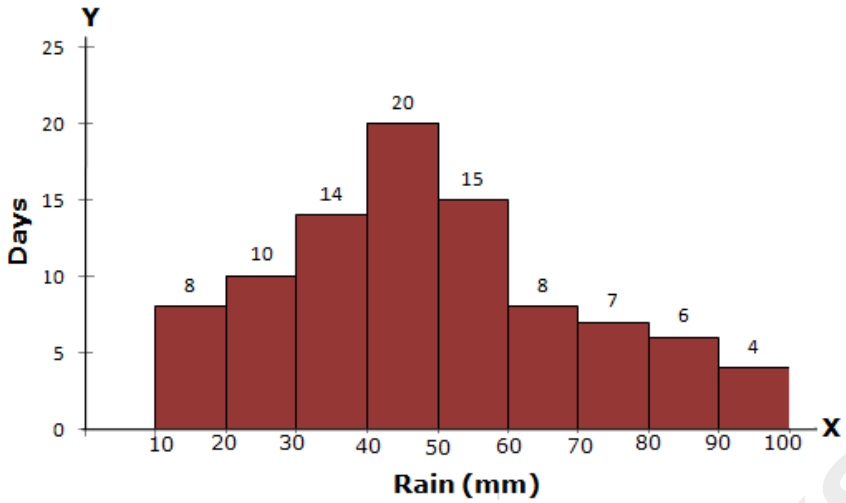
$$\therefore \sqrt{5\frac{19}{25}} = \frac{12}{5} = 2\frac{2}{5}$$

2	144
2	72
2	36
2	18
3	9
3	3
	1

5	25
5	5
	1

(b) Taking suitable scales, mark the class-intervals (Rain in mm) on the X-axis and frequency (No. of days) on Y-axis.

Construct rectangles with class-intervals as bases and the corresponding frequencies as heights.



(c) Let the numerator of a fraction be x and denominator be y.

Then, the fraction is $\frac{x}{y}$.

According to given information,

$\frac{x+6}{y} = \frac{1}{2}$

$\therefore 2(x+6) = y$

$\therefore 2x + 12 = y$

$\therefore 2x - y = -12$ ....(1)

And, $\frac{x}{y+7} = \frac{1}{3}$

$\therefore 3x = y + 7$

$\therefore 3x - y = 7$ ....(2)

Subtracting equation (2) from (1), we get

$-x = -19$

$\therefore x = 19$

Substituting the value of x in equation (1), we get

$2(19) - y = -12$

$\therefore 38 - y = -12$

$\therefore -y = -12 - 38$

$\therefore -y = -50$

$\therefore y = 50$

Hence, required fraction is $\frac{19}{50}$.

Section B (40 Marks)

Question 5

(a) $5+6-3\times(2+70)-\frac{50}{2}+(3+7\times2-9)$

= 5 + 6 – 3 × 72 – 25 + (3 + 14 – 9)

= 5 + 6 – 3 × 72 – 25 + 8

= 5 + 6 – 216 – 25 + 8

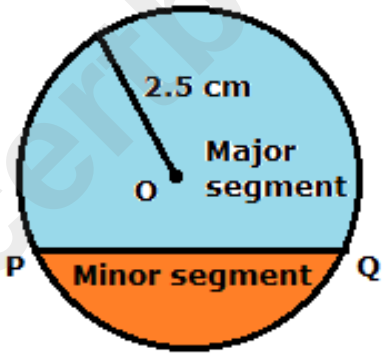
= –222

(b) Let PQ be a chord of the circle with centre O and radius 2.5 cm.

Then PQ divides the circle into two parts. Each of these parts is called a segment of the circle.

The segment containing the major arc, and also the centre of the circle, is called the Major segment.

The segment containing the minor arc is called the Minor segment.



(c) In right-angled triangles BEC and CFB,

∠BEC = ∠CFB ….(90^o each)

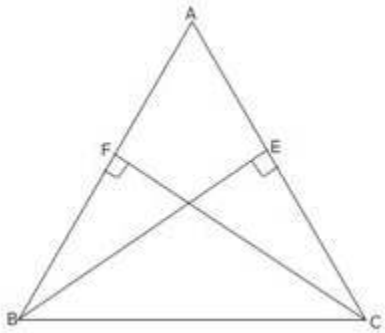
BE = CF ….(given)

BC = BC ….(Common)

∴ ΔBEC ≅ ΔCFB ….(RHS congruence)

∴∠BCE = ∠CBF ….(Corresponding angles of congruent triangles)

∴∠BCA = ∠CBA ….(same angle)



In ΔABC, we have

∠BCA = ∠CBA (Proved above)

∴ AB = AC ….(sides opposite to equal angles are equal)

Question 6

(a) Speed of journey by bus = 30 km/hr

Speed of journey by train = 50 km/hr

$$\text{Average speed} = \frac{50+30}{2} = \frac{80}{2} = 40 \text{ km/hr}$$

Distance covered for entire journey = Time x Average speed

Distance = 40 x 6 = 240 km

Thus, the distance covered by Raj for the entire journey is 240 km.

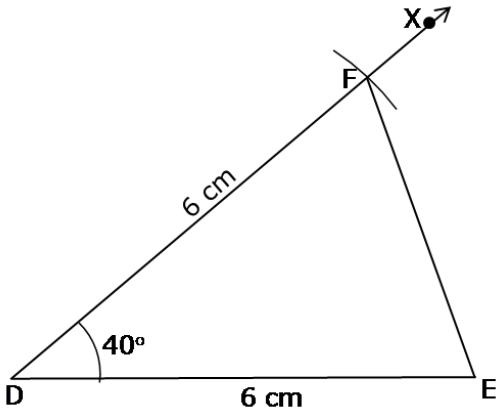
(b) $\frac{x^2-3x-10}{x^2-x-20} \times \frac{x^2-2x+4}{x^3+8}$

$$= \frac{x^2-5x+2x-10}{x^2-5x+4x-20} \times \frac{x^2-2x+4}{(x+2)(x^2-2x+4)}$$
$$= \frac{x(x-5)+2(x-5)}{x(x-5)+4(x-5)} \times \frac{1}{(x+2)}$$
$$= \frac{(x-5)(x+2)}{(x-5)(x+4)} \times \frac{1}{(x+2)}$$
$$= \frac{1}{x+4}$$

(c) Steps of construction:

i. Draw segment DE of length 6 cm.
ii. Placing a protractor at vertex D, draw ∠XDE of measure 40°.
iii. Taking D as centre and radius 6 cm, draw an arc to cut ray XD at F.
iv. Join the points F and E.

Hence, ΔDEF is required triangle.



Question 7

(a) For the first year, the simple interest and the compound interest both are equal.

Hence, to find compound interest on Rs.18,000 at 7% for 1 year, find simple interest on the same amount for 1 year.

Here, Principal (P) = Rs. 18,000, Rate (R) = 7% and Time(T) = 1 year

Now, $I = \frac{P \times R \times T}{100}$

$\therefore I = \frac{18000 \times 7 \times 1}{100} = 1260$

Thus, Rs. 1260 compound interest is earned.

(b) $S = \frac{n}{2}\{2a + (n-1)d\}$

$\therefore 2S = 2an + n(n-1)d$

$\therefore 2S - 2an = n(n-1)d$

$\therefore n(n-1)d = 2(S - an)$

$\therefore d = \frac{2(S - an)}{n(n-1)}$

(c) Here, l = 15 cm and b = 8 cm

Let the height of box be h cm.

Now, Surface area of box = 2(lb + bh + hl)

But surface area is given 470 cm^2.

$\therefore 2(15 \times 8 + 8h + 15h) = 470$

$\therefore$ 2(120 + 23h) = 470

$\therefore$ 120 + 23h = 235

$\therefore$ 23h = 115

$\therefore$ h = 5

Thus, the height of a cuboidal wooden box is 5 cm.

Question 8

(a) $20x - [15x^3 + 5x^2 - \{8x^2 - (4 - 2x - x^3) - 5x^3\} - 2x]$

$= 20x - [15x^3 + 5x^2 - \{8x^2 - 4 + 2x + x^3 - 5x^3\} - 2x]$

$= 20x - [15x^3 + 5x^2 - \{8x^2 - 4 + 2x - 4x^3\} - 2x]$

$= 20x - [15x^3 + 5x^2 - 8x^2 + 4 - 2x + 4x^3 - 2x]$

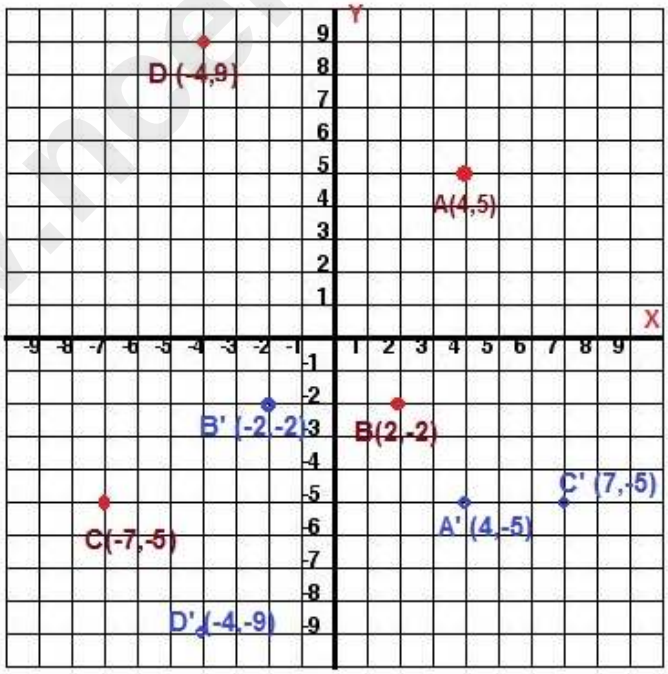
$= 20x - [19x^3 - 3x^2 + 4 - 4x]$

$= 20x - 19x^3 + 3x^2 - 4 + 4x$

$= 24x - 19x^3 + 3x^2 - 4$

$= -19x^3 + 3x^2 + 24x - 4$

(b) The image points are A', B', C' and D'.



(c)

C.I.	Tally Marks	Frequency
10-15	||||	4
15-20	𝍸		6
20-25	𝍸 ||	7
25-30	𝍸𝍸		11
30-35	𝍸 |||	8
35-40	||||	4
40-45	|||	3
45-50	𝍸 ||	7

Question 9

(a) Sum of interior angles of n-sided polygon = $(2n - 4) \times 90°$

i. Sum of interior angles of 6-sided polygon $= (2 \times 6 - 4) \times 90°$
$= 8 \times 90°$
$= 720°$

ii. Sum of interior angles of 8-sided polygon $= (2 \times 8 - 4) \times 90°$
$= 12 \times 90°$
$= 1080°$

iii. Sum of interior angles of 13-sided polygon $= (2 \times 13 - 4) \times 90°$
$= 22 \times 90°$
$= 1980°$

(b) Given: Area of trapezium = 105 cm^2 and h = 7 cm

Let the length of shorter side be x cm.

Then the length of longer side is (x + 6) cm.

Now, sum of parallel sides = x + x + 6 = (2x + 6) cm

$$\text{Area of trapezium} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$$

$$\therefore 105 = \frac{1}{2} \times (2x + 6) \times 7$$

$$\therefore \frac{105 \times 2}{7} = 2x + 6$$

$$\therefore 30 = 2x + 6$$

$$\therefore 2x = 24$$

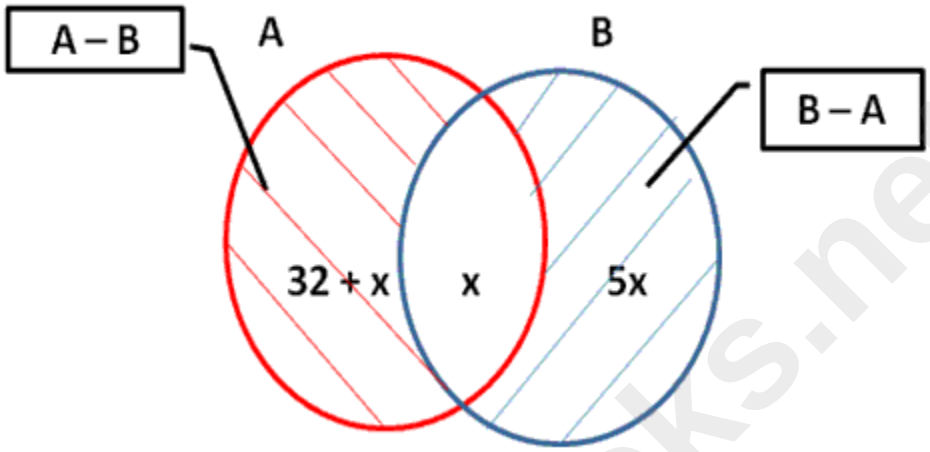
$$\therefore x = 12 \text{ cm}$$

Hence, the length of shorter side = x = 12 cm

And the length of longer side = x + 6 = 12 + 6 = 18 cm.

(c) Given, $n(A - B) = 32 + x$, $n(B - A) = 5x$ and $n(A \cap B) = x$

The Venn-diagram representation is as below:



i. $n(A) = (32 + x) + x = 32 + 2x$

$n(B) = x + 5x = 6x$

Now, $n(A) = n(B)$ ….(given)

$\therefore 32 + 2x = 6x$

$\therefore 4x = 32$

$\therefore x = 8$

ii. We know, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Here, $n(A) = 32 + 2x = 32 + 2(8) = 32 + 16 = 48$

$n(B) = 6x = 6(8) = 48$

$n(A \cap B) = x = 8$

$\therefore n(A \cup B) = 48 + 48 - 8 = 48 + 40 = 88$