Understanding shapes

Classification of Curves as Open and Closed

Look at the following figures.

(i) (ii) (iii)

Each of these shapes is an example of a **curve.** In fact, any shape that we draw is a curve. We can define a curve as follows.

Any figure drawn on a paper is known as a curve. A curve may or may not be straight.

Note: In real life, we do not consider straight lines as curves. However, in mathematics, straight lines are also considered as curves.

Can we find any difference among the three curves that we discussed in the beginning?

Curves (i) and (iii) do not intersect themselves, while curve (ii) does. Also, curves (i) and (iii) are not closed figures, while curve (ii) is a closed figure. On the basis of these observations, we classify curves as follows.

1. Simple curves
2. Closed Curves
3. Open curves

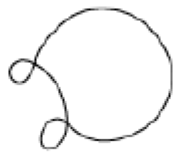
Let us discuss each of these with the help of the following video.

Let us discuss some more examples based on classification of curves.

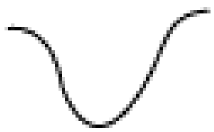
Example 1:

Classify each of the following curves as open or closed.

(a)

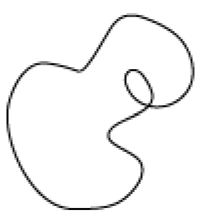


(b)



(c)

(d)



Solution:

(a) Since no end points can be seen in the curve, it is an example of a closed curve.

(b) Since the two end points of the curve can be seen, it is an example of an open curve.

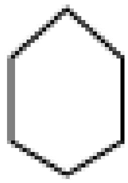
(c) Since the two end points of the curve can be seen, it is an example of an open curve.

(d) Since no end points can be seen in the curve, it is an example of a closed curve.

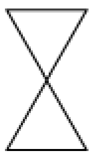
Example 2:

State whether each of the following curves is simple or not.

(a)



(b)



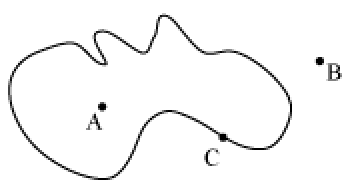
Solution:

(a) Since the curve does not cross itself, it is a simple curve.

(b) Since the curve crosses itself at one point, it is not a simple curve.

Identification Of Regions Of A Curve

Let us consider the following closed curve.



Now, how can we classify the parts of the curve according to the position of points A, B, and C?

From this figure, we can see that point A lies inside the boundary of the curve. In another way, we can say that point A lies in the **interior** of the curve. Point B lies outside the boundary of the curve. We can also say that point B lies in the **exterior** of the curve. Point C lies **on the curve**, that is, on the boundary of the curve.

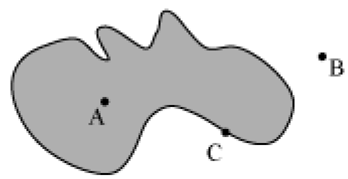
From this observation, we can say that a closed curve has three parts. They are

(i) Interior of the curve

(ii) Exterior of the curve

(iii) Boundary of the curve

If we shade the interior of this closed curve along with its boundary, then we have the following closed curve.



This shaded portion of the curve is known as **region.** Therefore, the region of a closed curve can be defined as:

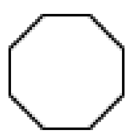
The interior of a closed curve along with its boundary is called its region.

Let us discuss some more examples to understand this concept better.

Example 1:

Shade the interior of the following curves.

(a)

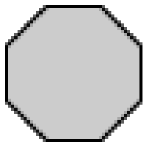


(b)

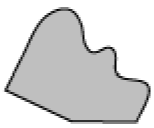
Solution:

The interior region is the region inside the curve. Thus, the interior of the given curves can be shaded as:

(a)

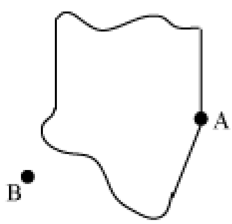


(b)



Example 2:

In which parts of the curve do the points A and B lie?



Solution:

Point A lies on the boundary of the curve, i.e., on the curve. Point B lies outside the curve, i.e., in the exterior of the curve.

Polygons and Their Attributes

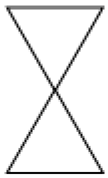
Polygons can be classified on the basis of number of sides. But before we learn about this, we should know the basic properties of polygons such as vertices, adjacent sides, diagonals, etc. This is the foundation of higher polygon concepts and the given video will help us get familiar with these properties.

Let us discuss some examples based on polygon.

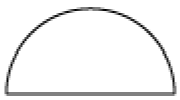
Example 1:

State whether each of the following curves is a polygon or not.

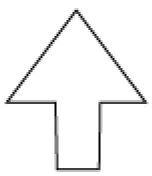
(a)



(b)



(c)



Solution:

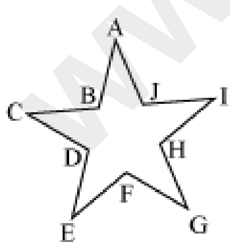
(a) A polygon is always a simple and closed curve, entirely made up of line segments. Since the given curve crosses itself, it is not a simple curve and thus, not a polygon.

(b) The given curve is not entirely made up of line segments. Therefore, it is not a polygon.

(c) The given curve is a simple and closed curve and is entirely made up of line segments. It is, thus, a polygon.

Example 2:

Answer the questions below with respect to the given figure.



(a) Name the vertices of the polygon.

(b) Name the adjacent sides of AJ, GF, and BC.

(c) Name the adjacent vertices of A, G, and E.

Solution:

(a) The point where two sides of a polygon meet is called its vertex. The vertices of the polygon are A, B, C, D, E, F, G, H, I, and J.

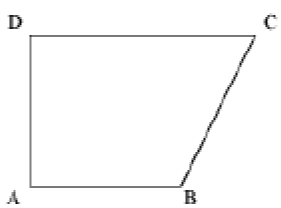
(b) Any two sides of a polygon with a common vertex are called adjacent sides. Thus, the adjacent sides of AJ are AB and IJ; that of GF are GH and EF; and that of BC are AB and CD.

(c) The vertices of a polygon that lie on the same side are called adjacent vertices. Thus, the adjacent vertices of A are B and J; that of G are H and F; and that of E are D and F.

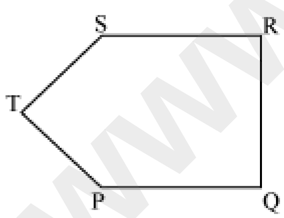
Example 3:

Name all the sides of the following polygons. Also, draw and count all the possible number of diagonals.

(a)

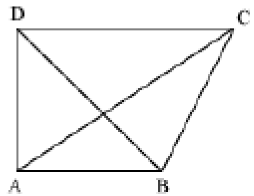


(b)

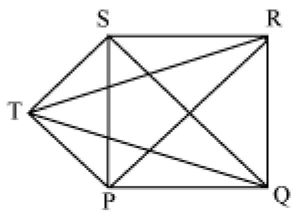


Solution:

(a) The sides of the given polygon are AB, BC, CD, and DA. It has two diagonals, AC and BD.

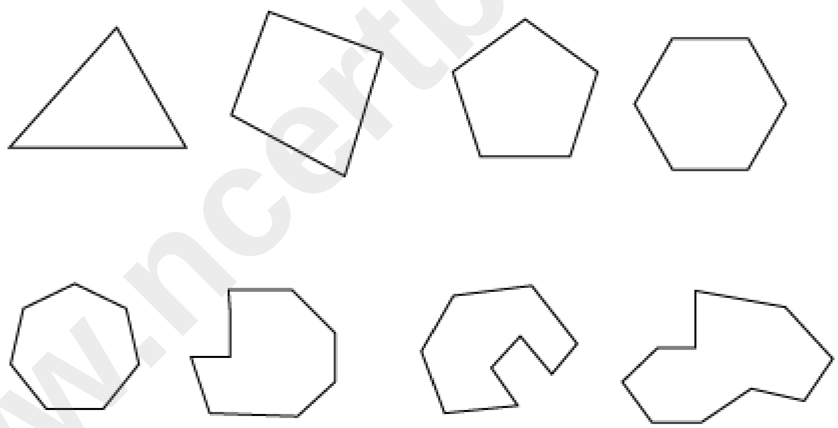


(b) The sides of the given polygon are PQ, QR, RS, ST, and TP. It has five diagonals, PR, PS, QS, QT, and RT.



Classification of Polygons on the Basis of Their Sides

Look at the following figures.



What do we observe in these figures?

We observe that each figure is made up of line segments only and has different number of sides. All these figures are known as **polygons.** We know that polygons with three sides are known as ***triangles*** and polygons with 4 sides are known as ***quadrilaterals.***

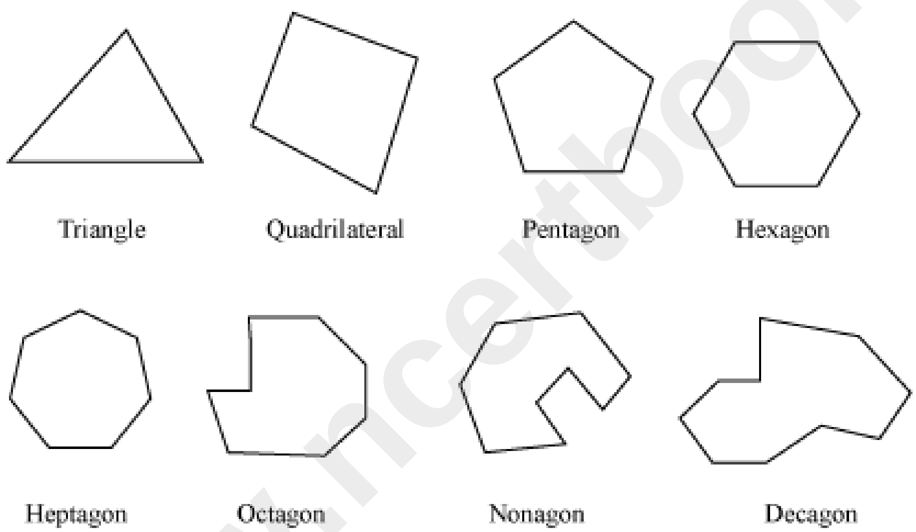
But how do we classify the polygons having more than four sides?

Let us see.

We can classify polygons on the basis of the number of sides as follows:

- **Pentagon:** Polygon having five sides
- **Hexagon:** Polygon having six sides
- **Heptagon:** Polygon having seven sides
- **Octagon:** Polygon having eight sides
- **Nonagon:** Polygon having nine sides
- **Decagon**: Polygon having ten sides

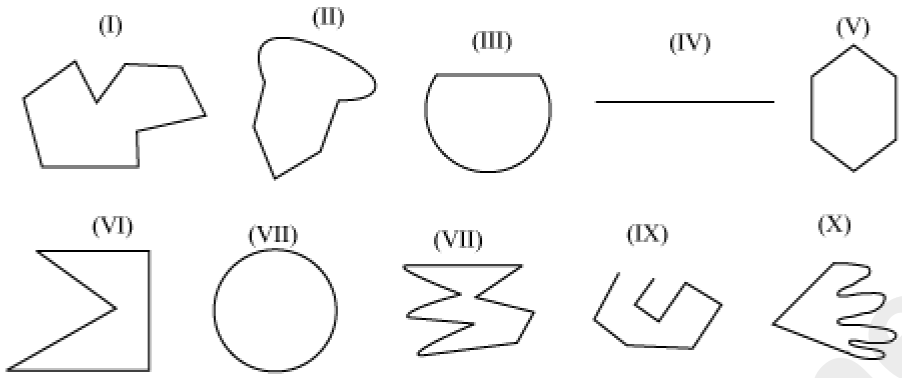
Therefore, now we can classify the polygons in the above given figures.



Let us now look at some more examples to understand this concept better.

Example 1:

Identify and name the polygons out of the following figures.

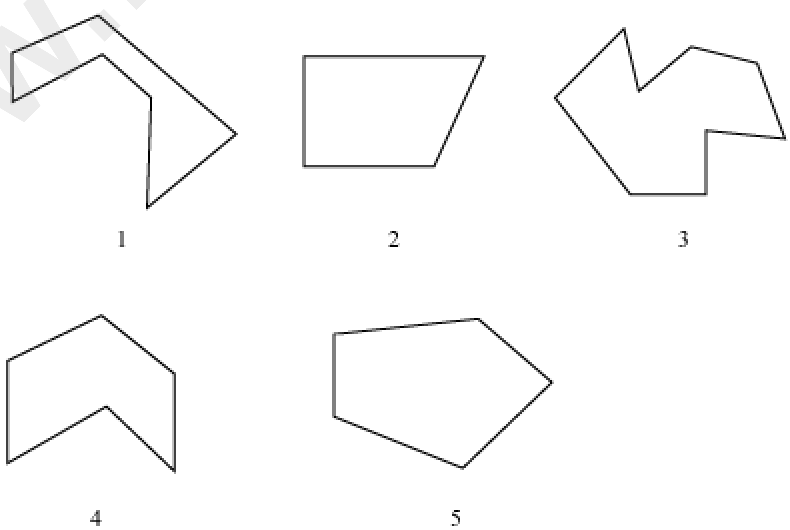


Solution:

1. The closed figure is made of nine line segments. Therefore, it is a nonagon.
2. The closed figure has a curve. Therefore, it is not a polygon.
3. The closed figure has a curve. Therefore, it is not a polygon.
4. The figure has only one line segment. A polygon should have at least three line segments. Therefore, it is not a polygon.
5. The closed figure is made of six line segments. Therefore, it is a hexagon.
6. The closed figure is made of five line segments. Therefore, it is a pentagon.
7. The closed figure is a curve. Therefore, it is not a polygon.
8. The closed figure is made of nine line segments. Therefore, it is a nonagon.
9. The figure is not closed. Therefore, it cannot be a polygon.
10. The closed figure has curves. Therefore, it is not a polygon.

Example 2:

Name the following polygons.



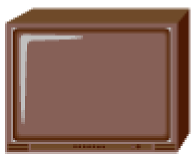
Solution:

1. The given figure has seven sides. Therefore, it is a heptagon.
2. The given figure has four sides. Therefore, it is a quadrilateral.
3. The given figure has nine sides. Therefore, it is a nonagon.
4. The given figure has six sides. Therefore, it is a hexagon.
5. The given figure has five sides. Therefore, it is a pentagon.

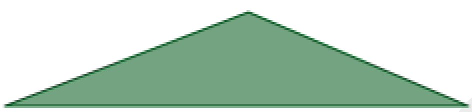
Example 3:

Write the number of sides and the types of polygons represented by the following figures.

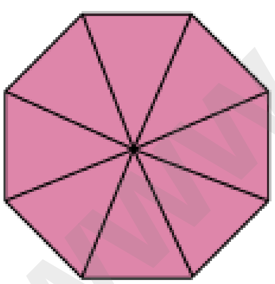
(i)



(ii)



(iii)

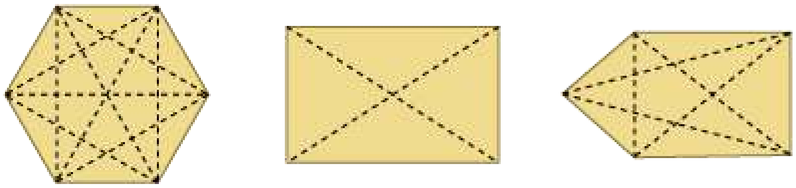


Solution:

1. This figure has four sides. Therefore, it is a quadrilateral.
2. This figure has three sides. Therefore, it is a triangle.
3. This figure has eight sides. Therefore, it is an octagon.

Classification of Polygons as Convex and Concave

Let us consider some polygons such as a hexagon, a quadrilateral, and a pentagon as shown in the following figure.



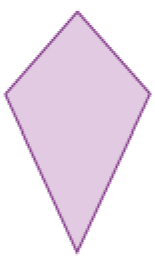
These polygons are known as convex polygons. How can we say these are convex polygons?

Let us now look at some examples.

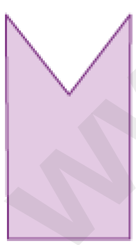
Example:

Classify the following figures as concave or convex polygons.

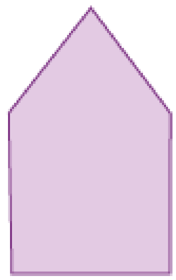
1.



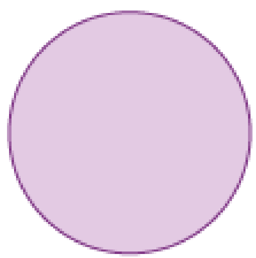
2.



3.



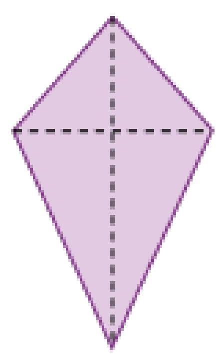
4.



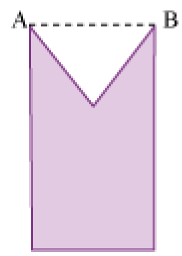
5.

Solution:

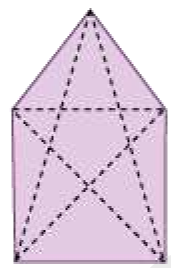
1. In this figure, we can clearly see that all the diagonals of the polygon lie inside the polygon. Therefore, it is a convex polygon.



2. In this polygon, diagonal AB lies in the exterior of the polygon. Therefore, it is a concave polygon.

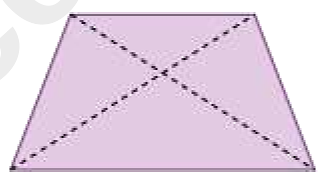


3. In this polygon, all diagonals lie in the interior of the polygon. Therefore, it is a convex polygon.



4. The given figure is a curve. It is not made up of line segments. Therefore, it is not a polygon.

5. In this polygon, all diagonals lie in the interior of the polygon. Therefore, it is a convex polygon.



Angle Sum Property Of Polygons

Let us suppose that we have a quadrilateral and we want to find the sum of all the interior angles made by its sides.

One simple way to find the sum of the angles is to find the measure of the angles and then add them. But how will we find its angles?

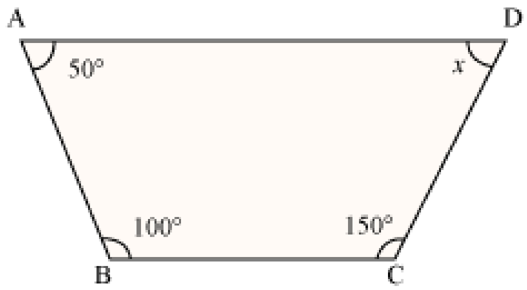
Is it possible to find the sum of all the angles of a quadrilateral without finding the measure of each angle? Is the sum of the interior angles of every quadrilateral same?

Let us find out the answers to these questions.

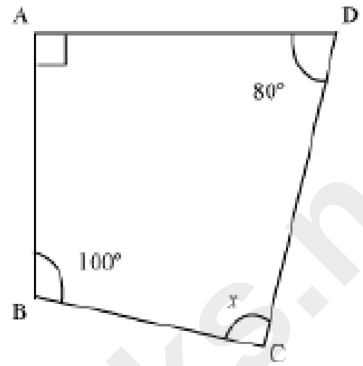
Let us solve some examples now.

Example:

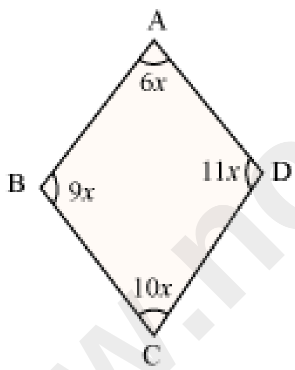
Find the value of *x* in the following figures.



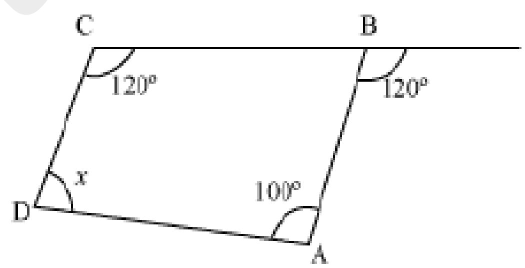
(a)



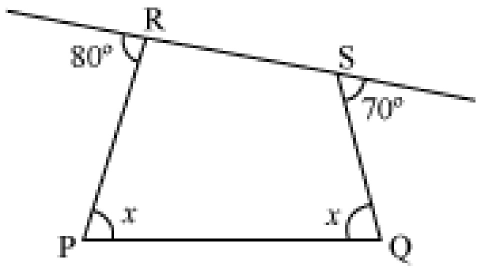
(b)



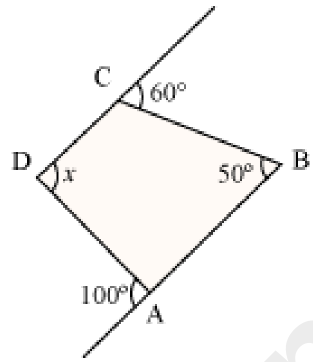
(c)



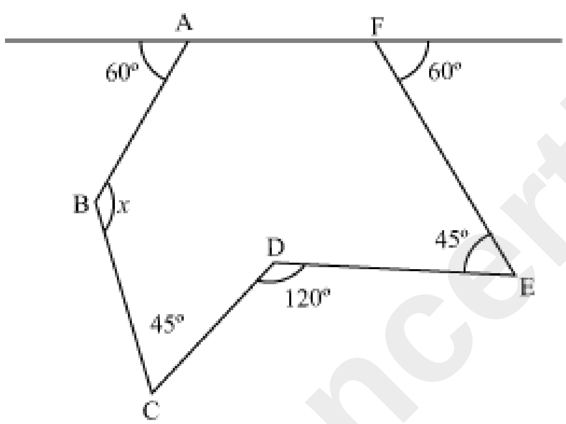
(d)



(e)



(f)



(g)

Solution:

(a) The sum of all the interior angles of a quadrilateral is 360°.

Therefore, from the figure,

$100° + 150° + x + 50° = 360°$

$\Rightarrow 300° + x = 360°$

$\Rightarrow x = 360° - 300°$

$\Rightarrow x = 60°$

(b) The sum of all the interior angles of a quadrilateral is 360°.

Therefore,

$90° + 80° + x + 100° = 360°$

$\Rightarrow 270° + x = 360°$

$\Rightarrow x = 360° - 270°$

$\Rightarrow x = 90°$

(c) The sum of all the interior angles of a quadrilateral is 360°. Therefore, from the figure,

$9x + 6x + 11x + 10x = 360°$

$\Rightarrow 36x = 360°$

On dividing both sides by 36, we obtain $x = 10°$

Thus, the angles of the quadrilateral are 90°, 60°, 110°, and 100°.

(d) The sum of the angles which forms a linear pair is 180°.

$\therefore \angle ABC + 120° = 180°$

$\Rightarrow \angle ABC = 180° - 120° = 60°$

Also, the sum of all the interior angles of a quadrilateral is 360°. Therefore,

$x + 100° + 60° + 120° = 360°$

$\Rightarrow x + 280° = 360°$

$\Rightarrow x = 360° - 280°$

$\Rightarrow x = 80°$

(e) The sum of the adjacent angles on a straight line is 180°.

∴ ∠PSR + 80° = 180°

⇒ ∠PSR = 180° − 80°

⇒ ∠PSR = 100°

Also, ∠SRQ + 70° = 180°

⇒ ∠SRQ = 180° − 70°

⇒ ∠SRQ = 110°

The sum of all the interior angles of a quadrilateral is 360°. Therefore,

$100° + 110° + x + x = 360°$

$\Rightarrow 210° + 2x = 360°$

$\Rightarrow 2x = 360° - 210°$

$\Rightarrow 2x = 150°$

$\Rightarrow x = 75°$

(f) The sum of the angles which forms a linear pair is 180°.

∴ 100° + ∠DAB = 180°

⇒ ∠DAB = 180° − 100°

⇒ ∠DAB = 80°

Similarly, ∠DCB + 60° = 180°

⇒ ∠DCB = 180° − 60°

⇒ ∠DCB = 120°

Now, the sum of all the interior angles of a quadrilateral is 360°. Therefore,

$x + 80° + 50° + 120° = 360°$

$\Rightarrow x + 250° = 360°$

⇒ $x = 360° − 250°$

⇒ $x = 110°$

(g) ∠BAF = 180° − 60° = 120°

Similarly, ∠AFE = 120°

∠CDE = 360° − 120° = 240°

The polygon ABCDEF is a hexagon.

∴ Sum of all the interior angles of a hexagon = 180° × (6 − 2)

= 180° × 4

= 720°

∴ $x = 720° − (120° + 120° + 45° + 240° + 45°)$

⇒ $x = 720° − 570°$

⇒ $x = 150°$

Exterior Angle Sum Property of Polygons

Let us consider a quadrilateral. We know that the sum of all the interior angles of a quadrilateral is 360°.

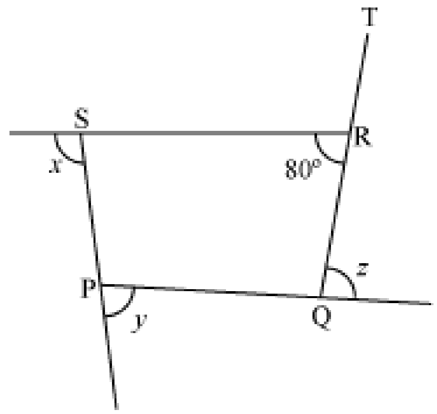
But if we want to find the sum of all the exterior angles of a quadrilateral, then how will we proceed?

In order to understand the answer to this question, let's look at the following video.

Let us look at some examples now.

Example 1:

Find the value of ($x + y + z$) from the following figure.



Solution:

In the given figure, we can see that ∠QRS and ∠SRT form linear pair of angles.

Therefore, their sum should be 180°.

Thus, we obtain

80° + ∠SRT = 180°

⇒ ∠SRT = 180° – 80°

= 100°

The sum of the exterior angles of a quadrilateral is 360°.

∴ $x + y + z$ + 100° = 360°

⇒ $x + y + z$ = 360° – 100°

⇒ $x + y + z$ = 260°

Example 2:

Find the number of sides of a regular polygon in which each exterior angle has a measure of 40°.

Solution:

The measure of all the exterior angles of a polygon is 360°. It is given that the measure of each exterior angle is 40° and the given polygon is a regular polygon, therefore all the exterior angles are same.

Therefore, number of exterior angles = $\frac{360°}{40°}$ = 9

Thus, the polygon has 9 sides.

Example 3:

Find the measure of each exterior angle of a regular polygon having 12 sides.

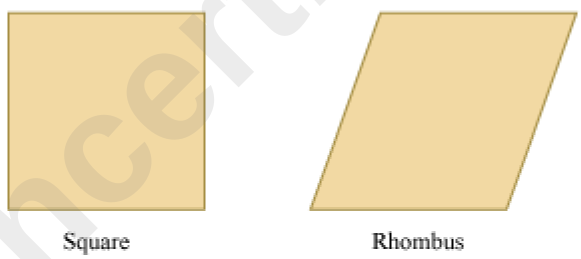
Solution:

The measure of all the exterior angles of a polygon is 360°. It is given that the polygon has 12 sides. Since it is a regular polygon, all its exterior angles are equal.

∴ Measure of each exterior angle = $\frac{360°}{12}$ = 30°

Classification of Polygons as Regular and Irregular

Let us consider a square and a rhombus.



What is the difference between the two figures?

We can see that in a square, all the sides are equal and all the angles are also of equal measure. On the other hand, in a rhombus, all sides are equal; however, the measures of all angles are not equal.

We thus say that a **square is a regular polygon** and a **rhombus is an irregular polygon**.

The **regular** and **irregular polygons** can be defined as follows.

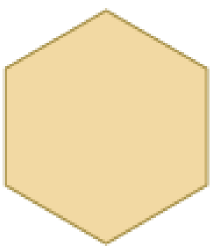
"Polygons in which all sides are of equal length and all interior angles of equal measure are known as regular polygons".

"Polygons in which all sides are not of equal length and all angles are not of equal measure are known as irregular polygons".

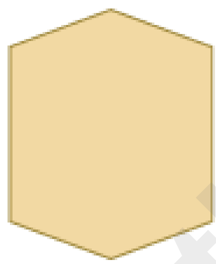
Let us see another example.

A **regular hexagon** has all sides of equal length. Moreover, all the angles are of equal measure 120°.

However, in case of an **irregular hexagon**, all the sides are not of equal length. Also, all the angles are not equal. A regular and an irregular hexagon are shown in the following figure.



Regular hexagon



Irregular hexagon

Formulas Related to Regular Polygons:

(i) The sum of the interior angles of an *n* sided polygon = $2n-4\times 90^\circ$

where each interior angle = $2n - 4 \times 90^\circ n$

(ii) A regular polygon has all its exterior angles equal.

The sum of its exterior angles = 360º
So, the sum of each exterior angle = 360°n

(iii) Number of sides of a regular polygon, n=360°exterior angle

Note: For a polygon, regardless of the fact whether it is regular or non-regular, at each vertex the sum of exterior and interior angle = 180°

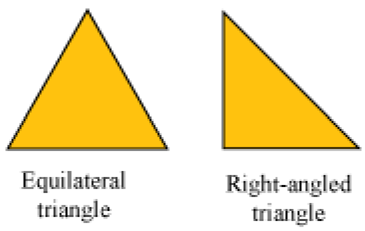
i.e Exterior angle + Interior angle = 180°

Let us now look at some more examples to understand this concept better.

Example 1:

Show that an equilateral triangle is a regular polygon and a right-angled triangle is an irregular polygon.

Solution:



An equilateral triangle is a regular polygon as all the sides of equilateral triangle are of equal length and all angles are of equal measure 60°.

In case of a right-angled triangle, neither all the sides are of equal length nor the measure of all angles are equal. Therefore, right-angled triangle is an example of irregular polygon.

Example 2:

Write the name of a regular polygon having

(i) 3 sides

(ii) 4 sides

Solution:

A regular polygon is a polygon in which all the sides are of equal length and all interior angles are of equal measure.

Therefore, a regular polygon having 3 sides is an equilateral triangle. A regular polygon having 4 sides is a square.

Quadrilaterals and Their Attributes

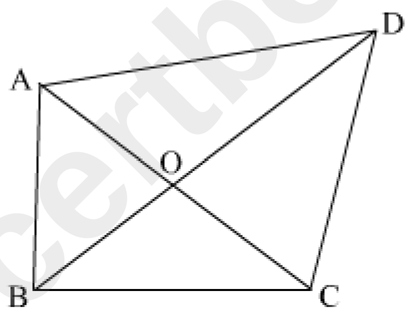
Let us look at the following figures.

These are the figures of a kite, window, door, and a painting. If we notice these figures carefully, then we will observe that each of these figures is a polygon of four sides. Such polygons are known as **quadrilaterals**. We can define it as follows.

A polygon formed by four line segments is called a quadrilateral i.e., a quadrilateral is a four-sided polygon.

Note: Adjacent sides and adjacent angles are also known as consecutive sides and consecutive angles respectively.

Now, look at the following quadrilateral.



Let us name this quadrilateral as ABCD. Symbolically, it can be represented as □ ABCD.

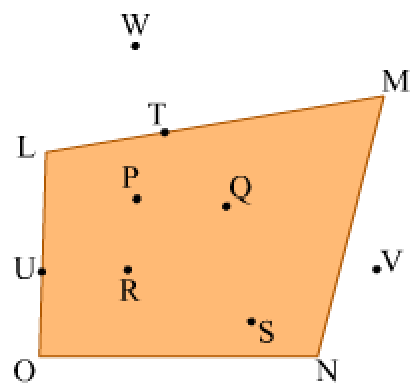
Now, it can be seen that vertices A and C are opposite to each other, so these make a pair of **opposite vertices**. Similarly, B and D also make a pair of opposite vertices.

If we join opposite vertices A and C as well as B and D, we get two line segments such as AC and BD. These are the **diagonals** of □ ABCD. Also, O is the point of intersection of the diagonals.

We can define the diagonals of a quadrilateral as follows:

Line segments obtained after joining the opposite vertices of a quadrilateral are its diagonals.

Now, observe □ LMNO.



The whole coloured part is the **interior** of □ LMNO which means that points P, Q, R and S lie in the interior of the quadrilateral.

Points, T and U lie on the **boundary** of □ LMNO.

The interior and the boundary together form the **region** of the quadrilateral.

The part outside the region of the quadrilateral is **exterior** of the quadrilateral.

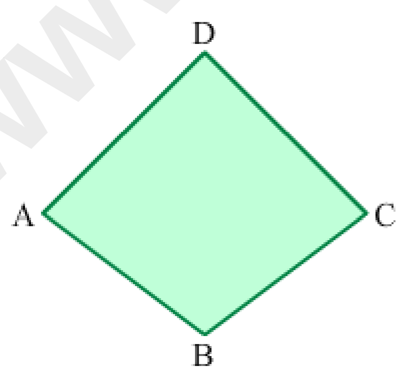
From the figure, it can be seen that the points V and W lie in the exterior of □ LMNO.

On the basis of the internal angles of a quadrilateral, it can be classified into two types: convex and concave quadrilaterals.

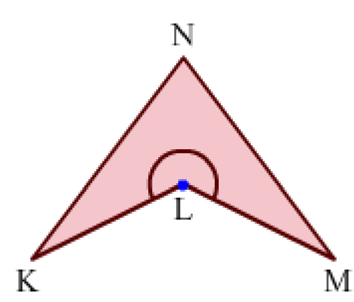
A quadrilateral is said to be **convex**, if each of its internal angles are of the measure less than 180º.

Otherwise, it is a **concave** quadrilateral.

Examples of convex and concave quadrilaterals are shown below:



Convex Quadrilateral



Concave Quadrilateral

It can be seen that each internal angle of quad. ABCD is less than 180º, whereas the marked angle of quad. KLMN is more than 180º.

Thus, quad. ABCD is a convex quadrilateral and quad. KLMN is a concave quadrilateral.

Let us discuss some examples based on these concepts of a quadrilateral.

Example 1:

Draw a rough sketch of a quadrilateral and name it. Also, in the quadrilateral, identify the following.

(i) Sides

(ii) Angles

(iii) Opposite sides

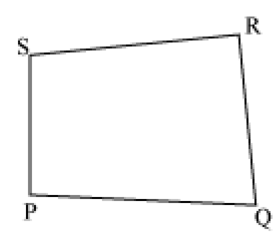
(iv) Adjacent sides

(v) Opposite angles

(vi) Adjacent angles

Solution:

Let us draw a quadrilateral PQRS as



(i) The four sides are $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$, and $\overline{SP}$.

(ii) The four angles are ∠SPQ, ∠PQR, ∠QRS, and ∠RSP. We can also simply say that the four angles are ∠P, ∠Q, ∠R, and ∠S.

(iii) The opposite sides are $\overline{PQ}$ and $\overline{RS}$, $\overline{QR}$ and $\overline{SP}$.

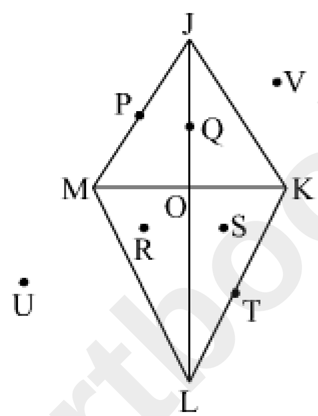
(iv) The adjacent sides are $\overline{PQ}$ and $\overline{QR}$, $\overline{QR}$ and $\overline{RS}$, $\overline{RS}$ and $\overline{SP}$, $\overline{SP}$ and $\overline{PQ}$.

(v) The opposite angles are ∠PQR and ∠RSP, ∠SPQ and ∠QRS.

(vi) The adjacent angles are ∠PQR and ∠QRS, ∠QRS and ∠RSP, ∠RSP and ∠SPQ, ∠SPQ and ∠PQR.

Example 2:

Observe the following figure.



Identify the following from the figure.

(a) Pair of opposite vertices

(b) Diagonals of the quadrilateral

(c) Points lying in the interior of the quadrilateral

(d) Points lying in the exterior of the quadrilateral

(e) Points lying on the boundary of the quadrilateral

Solution:

(a) Pair of opposite vertices are J, L and K, M.

(b) Diagonals of the quadrilateral are JL and KM

(s) Points lying in the interior of the quadrilateral are Q, R, S and O.

(d) Points lying in the exterior of the quadrilateral are V and U.

(e) Points lying on the boundary of the quadrilateral are P and T.

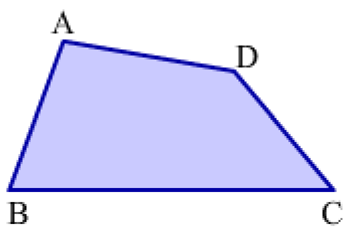
Angle Sum Property of Quadrilaterals

We have learnt about the angle sum property of triangles, which states that the sum of the measures of all three angles of a triangle is180°.Similarly, there is a property about the sum of all angles of a quadrilateral, which states that **the sum of the measures of the four angles of a quadrilateral is 360°.**

This property can be verified by two ways. They are as follows:

1.By measuring the angles of a quadrilateral

Let us consider a quadrilateral ABCD.



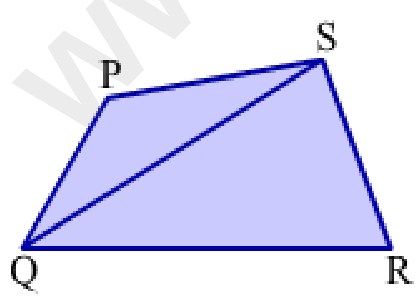
On measuring the angles of this quadrilateral, we found $m\angle A= 100°$, $m\angle B= 70°$, $m\angle C= 50°$and $m\angle D= 140°$.

∴Sum of four angles of quadrilateral ABCD = 100°+ 70°+ 50°+ 140°= 360°

Similarly,we can verify that the sum of the four angles of any quadrilateral is360°.

2.By dividing a quadrilateral into two triangles

Let us consider a quadrilateral PQRS with diagonal QS.



From the figure, it can be seen that the diagonal QS divides the quadrilateral PQRS into two triangles. We know that the sum of all three angles of a triangle is 180°.

So,in ΔPQS, $m\angle P + m\angle PQS + m\angle PSQ = 180°$…(1)

Similarly, inΔQRS,$m\angle SQR + m\angle R + m\angle QSR = 180°$…(2)

On adding (1) and (2):

$m\angle P + m\angle PQS + m\angle PSQ + m\angle SQR + m\angle R + m\angle QSR = 180° + 180°$

$\Rightarrow m\angle P + m\angle PQS + m\angle SQR + m\angle R + m\angle PSQ + m\angle QSR = 360°$

$\Rightarrow m\angle P + m\angle Q + m\angle R + m\angle S = 360°$

[$m\angle PQS + m\angle SQR = m\angle Q$ and $m\angle PSQ + m\angle QSR = m\angle S$]

This verifies that the sum of the four angles of a quadrilateral is 360°.

Now let us have a look at some examples based on this property.

Example1:

Two equal angles of a quadrilateral measure 75°each. What is the sum of the remaining two angles of the quadrilateral?

Solution:

We know that the sum of the measures of the four angles of a quadrilateral is 360°.

∴ Sum of measures of given two angles + Sum of measures of remaining two angles = 360°

⇒75°+ 75°+ Sum of measures of remaining two angles = 360°

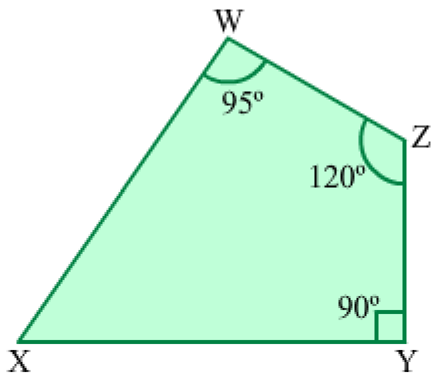
⇒150°+ Sum of measures of remaining two angles = 360°

⇒Sum of measures of remaining two angles = 360°−150°

⇒Sum of measures of remaining two angles = 210°

Example2:

In the given quadrilateral WXYZ, what is the measure of ∠X?



Solution:

In the given quadrilateral, $m\angle W = 95°$, $m\angle Y = 90°$ and $m\angle Z = 120°$.

Since the sum of the measures of the four angles of a quadrilateral is 360°,

$m\angle W + m\angle X + m\angle Y + m\angle Z = 360°$

$\Rightarrow 95° + m\angle X + 90° + 120° = 360°$

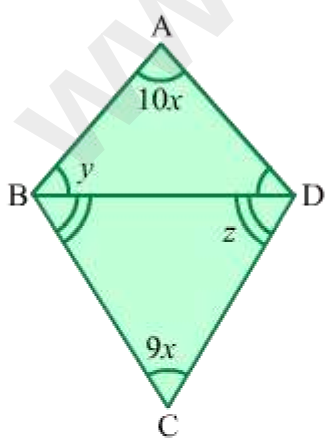
$\Rightarrow m\angle X + 305° = 360°$

$\Rightarrow m\angle X = 360° - 305°$

$\Rightarrow m\angle X = 55°$

Example3:

In the given figure, $\angle ABD = \angle ADB$ and $\angle CBD = \angle CDB$. If $y + z = 132.5°$, then find the measures of all angles of the quadrilateral ABCD.



Solution:

It is given that ∠ABD= ∠ADB= y, ∠CBD= ∠CDB= z and y+ z= 132.5°.

Since the sum of the measures of the four angles of a quadrilateral is360°,

∠A+ ∠B+ ∠C+ ∠D= 360°

⇒∠A + (∠ABD+ ∠DBC)+ ∠C+ (∠CDB+ ∠BDA)= 360°

⇒10x+ (y+ z)+ 9x +(z+ y)= 360°

⇒19x+ 2(y+ z)= = 360°

⇒19x+ 2 ×132.5°= 360°

⇒19x= 360°−265°

⇒19x= 95°

⇒x= 5°

∴∠A= 10 ×5°= 50°and ∠C= 9 ×5°= 45°

Now, using angle sum property in ΔABD:

∠A+ ∠ABD+ ∠ADB= 180°

⇒50°+ y+ y= 180°

⇒2y= 180°−50°

⇒2y= 130°

⇒y= 65°

Also, y+ z= 132.5°

⇒z= 132.5°−65°

⇒z =67.5°

Now, ∠B= ∠ABD+ ∠DBC

$= y + z$

$= 132.5°$

Similarly, $\angle D = 132.5°$

Thus,the measures of four angles of the quadrilateral ABCD are 50°,132.5°,45°and 132.5°.

Various Types of Quadrilaterals and Their Properties

Let us now look at following examples.

Example 1:

State whether the following statements are true or false.

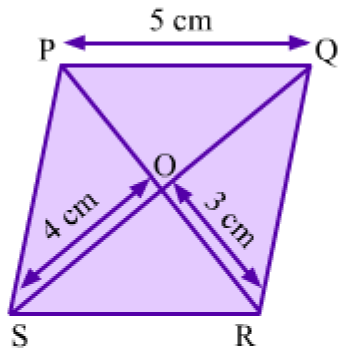
1. **Each angle of a parallelogram is a right angle.**
2. **All the sides of a rectangle are equal in length.**
3. **The opposite sides of a trapezium are parallel.**
4. **The diagonals of a rectangle are perpendicular to one another.**
5. **All the sides of a square are of equal lengths.**
6. **Opposite angles of a parallelogram are equal.**

Solution:

1. False
2. False
3. False
4. False
5. True
6. True

Example 2:

Observe the figure of given rhombus.



Find the following components of this rhombus.

(1) *m*∠SOR

(2) *l*(SQ)

(3) *l*(PO)

(4) *l*(QR), *l*(RS) and *l*(SP)

Solution:

(1) Diagonals of a rhombus intersect each other at the angle of 90°.

∴ *m*∠SOR = 90°

(2) Diagonals of a rhombus bisect each other.

∴ *l*(SQ) = 2 × *l*(SO) = (2 × 4) cm = 8 cm

(3) Diagonals of a rhombus bisect each other.

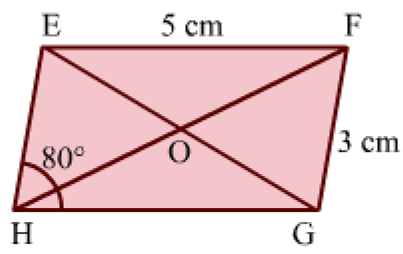
∴ *l*(PO) = *l*(RO) = 3 cm

(4) All sides of a rhombus are of equal length.

l(QR) = *l*(RS) = *l*(SP) = *l*(PQ) = 5 cm

Example 3:

Observe the figure of given parallelogram.



Find the following components of this parallelogram.

(1) *m*∠EFG

(2) *l*(GH)

(3) *l*(HE)

Solution:

(1) Opposite angles of a parallelogram are equal.

∴ *m*∠EFG = *m*∠GHE = 80°

(2) Opposite sides of a parallelogram are equal.

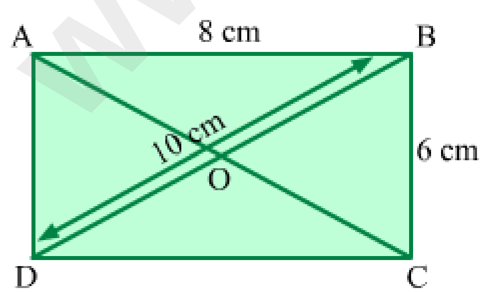
∴ *l*(GH) = *l*(EF) = 5 cm

(3) Opposite sides of a parallelogram are equal.

∴ *l*(HE) = *l*(FG) = 3 cm

Example 4:

Observe the figure of given rectangle.



Find the following components of this rectangle.

(1) *l*(AO)

(2) *m*∠ABC, *m*∠BCD, *m*∠CDA and *m*∠DAB

(3) *l*(CD)

(4) *l*(DA)

Solution:

(1) Diagonals of of rectangle are equal and bisect each other.

∴ *l*(AC) = *l*(BD) = 10 cm

∴ *l*(AO) = $\frac{1}{2}l(\text{AC})$ = 5 cm

(2) All angles of a rectangle are of 90°.

∴ *m*∠ABC = *m*∠BCD = *m*∠CDA = *m*∠DAB = 90°

(3) Opposite sides of a rectangle are equal.

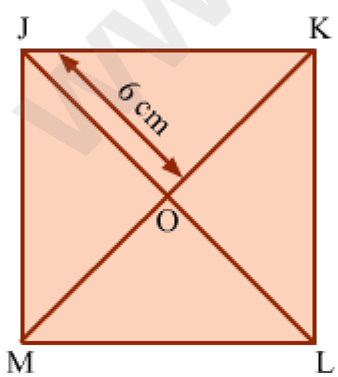
∴ *l*(CD) = *l*(AB) = 8 cm

(4) Opposite sides of a rectangle are equal.

∴ *l*(DA) = *l*(BC) = 6 cm

Example 5:

Observe the figure of given square.



Find the following components of this square.

(1) *m*∠KOL

(2) *l*(LO)

(3) *l*(MK)

Solution:

(1) Diagonals of a square intersect each other at the angle of 90°.

∴ *m*∠KOL = 90°

(2) Diagonals of a square bisect each other.

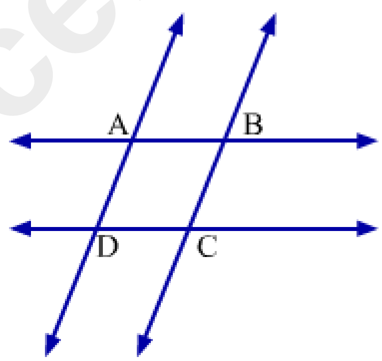
∴ *l*(LO) = *l*(JO) = 6 cm

(3) Diagonals of a square are equal.

∴ *l*(MK) = *l*(JL) = *l*(LO) + *l*(JO) = 6 cm + 6 cm = 12 cm

Property of the Sides of a Parallelogram

Consider the given pairs of parallel lines.



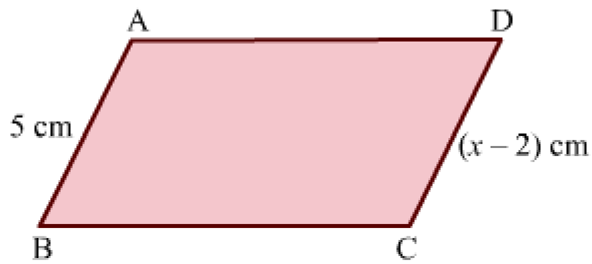
A closed figure ABCD is formed by the intersection of the two pairs of parallel lines. This figure is a parallelogram. A property of parallelograms defines the relation between the sides of a parallelogram as follows:

Opposite sides of a parallelogram are equal.

In this lesson, we will study the above-stated property and its converse. We will also solve some examples related to the same.

Opposite Sides of a Parallelogram Are Equal

Consider the given parallelogram ABCD.



We can find the value of *x* by using the property of parallelograms which states that:

Opposite sides of a parallelogram are equal.

Thus, in the given figure, we have AB = DC and AD = BC.

Since AB = DC, we have:

$x - 2 = 5$

$\Rightarrow x = 7$

Proof of the Property

Concept Builder

- A quadrilateral is a polygon having four sides.
- The sum of the interior angles of a quadrilateral is 360°.

Converse of the Property

Know More

- A pentadecagon is a fifteen-sided polygon. The sum of its interior angles is 2340°.
- An icosagon is a twenty-sided polygon. The sum of its interior angles is 3240°.

Did You Know?

The headquarters of the US Department of Defense is called 'the Pentagon'. It is one of the world's largest office buildings. It is virtually a city in itself.

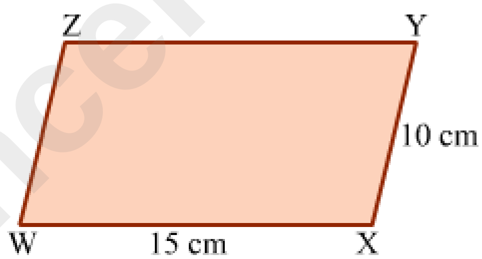
Solved Examples

Easy

Example 1:

What is the perimeter of the given parallelogram WXYZ if WX = 15 cm and XY = 10 cm?

Solution:



We know that the opposite sides of a parallelogram are equal.

∴ WX = ZY = 15 cm and XY = WZ = 10 cm

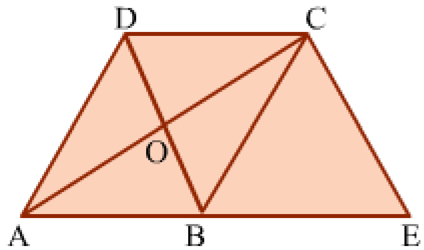
Perimeter of parallelogram WXYZ = WX + XY + YZ + ZW
= (15 + 10 + 15 + 10) cm

= 50 cm

Medium

Example 1:

In the given figure, ABCD is a parallelogram and B is the midpoint of AE. If DB = CE, then prove that BECD is also a parallelogram.



Solution:

We know that the opposite sides of a parallelogram are equal.

∴ AB = DC … (1)

It is given that B is the midpoint of AE.

∴ AB = BE … (2)

From equations 1 and 2, we get:

DC = BE

Also, it is given that DB = CE.

Now, in quadrilateral BECD, the opposite sides are equal (i.e., DC = BE and DB = CE). Therefore, it is a parallelogram.

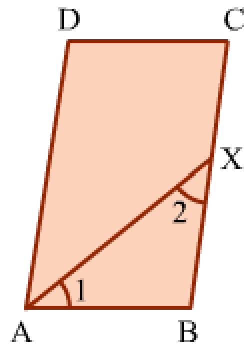
Hard

Example 1:

In a parallelogram ABCD, the bisector of ∠BAD also bisects side BC. Prove that the length of side AD is twice the length of side AB.

Solution:

The parallelogram ABCD according to the given specifications is shown below.



Here, AX is the bisector of ∠BAD.

$$\therefore \angle 1 = \frac{1}{2}\angle BAD \quad \ldots(1)$$

Since ABCD is a parallelogram, AD||BC and AB is the transversal between these lines.

∴ ∠BAD + ∠CBA = 180° … (2)

In ΔABX, by the angle sum property of triangles, we have:

∠1 + ∠2 + ∠ABX = 180°

$$\Rightarrow \frac{1}{2}\angle BAD + \angle 2 + 180^\circ - \angle BAD = 180^\circ \quad (\text{Using equations 1 and 2})$$
$$\Rightarrow \angle 2 + \frac{1}{2}\angle BAD = 0$$
$$\Rightarrow \angle 2 = \frac{1}{2}\angle BAD$$
$$\Rightarrow \angle 2 = \angle 1$$

We know that the sides opposite equal angles are also equal.

∴ AB = BX … (3)

Since ABCD is a parallelogram, AD = BC.

Now, BC = BX + XC

⇒ AD = BX + XC

⇒ AD = 2BX(∵ AX bisects BC)

⇒ ∴ AD = 2AB (Using equation 3)

Thus, in parallelogram ABCD, the length of side AD is twice the length of side AB.

Properties of The Angles of a Parallelogram

Opposite Angles of a Parallelogram

Look at the postage stamp shown below.

Observe how the stamp is shaped like a parallelogram. What can you say about its opposite angles? Is there any relation between them? Are they equal?

A property of parallelograms relates the opposite angles of a parallelogram as follows:

Opposite angles of a parallelogram are equal.

In this lesson, we will study the above-stated property and its converse. We will also solve some examples based on the same.

Proof of the Property

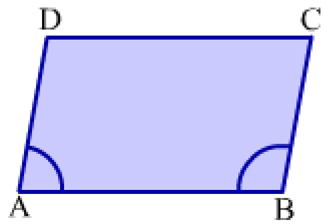
Whiz Kid

The sum of the measures of all the exterior angles of a quadrilateral (i.e., one at each vertex) is equal to the sum of the measures of all the interior angles of the quadrilateral, i.e., 360°.

Concept Builder

Adjacent angles in a parallelogram are supplementary.

In parallelogram ABCD, AD||BC and AB is the transversal intersecting these lines.



Therefore, ∠A and ∠B are **interior angles** on the same side of the transversal and, hence, **supplementary**.

Similarly, we can say that ∠B and ∠C, ∠C and ∠D, and ∠D and ∠A are supplementary angles.

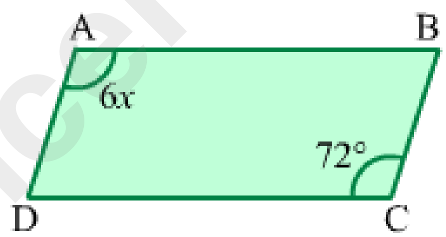
Converse of the Property

Solved Examples

Easy

Example 1:

Find the value of *x* if ABCD is a parallelogram.



Solution:

We know that the opposite angles of a parallelogram are equal.

∴ ∠A = ∠C

⇒ $6x = 72°$

∴ $x = 12°$

Example 2:
Find the measure of all the angles of a parallelogram whose adjacent angles are in the ratio 1:2.

Solution:

In a parallelogram ABCD, let $\angle A = x^\circ$ and $\angle B = 2x^\circ$.
In a parallelogram, the adjacent angles are supplementary.

$$\angle A + \angle B = 180^\circ$$
$$\Rightarrow x^\circ + 2x^\circ = 180^\circ$$
$$\Rightarrow 3x^\circ = 180$$
$$\Rightarrow x^\circ = \frac{180^\circ}{3}$$
$$\Rightarrow x^\circ = 60^\circ$$

Thus, we get

$\angle A = x^\circ$ = 60°
$\angle B = 2x^\circ$ = 2 × 60° = 120°

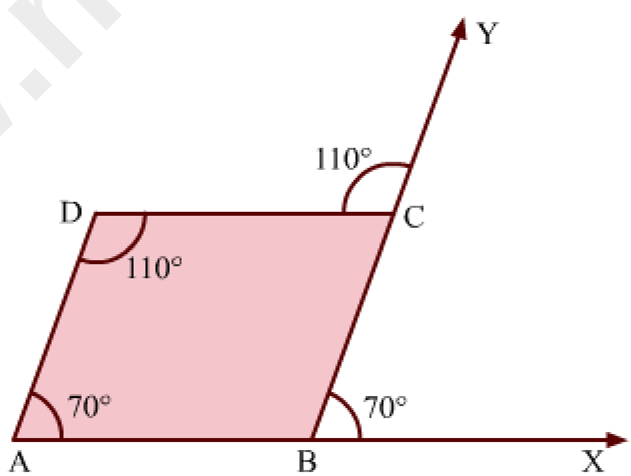
In a parallelogram, the opposite angles are equal.
Thus, we get

∠A = ∠C = 60° and ∠B = ∠D = 120°

Medium

Example 1:

Is the shown quadrilateral ABCD a parallelogram?



Solution:

In the given figure, ∠CBX and ∠CBA form a linear pair.

∴ ∠CBX + ∠CBA = 180°

⇒ ∠CBA = 180° − ∠CBX

⇒ ∠CBA = 180° − 70°

⇒ ∠CBA = 110°

⇒ ∠CBA = ∠CDA

Similarly, ∠DCY and ∠BCD form a linear pair.

∴ ∠DCY + ∠BCD = 180°

⇒ ∠BCD = 180° − ∠DCY

⇒ ∠BCD = 180° − 110°

⇒ ∠BCD = 70°

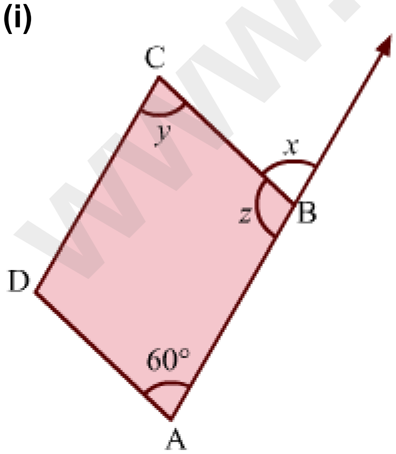
⇒ ∠BCD = ∠BAD

Thus, quadrilateral ABCD has two pairs of equal opposite angles. Hence, it is a parallelogram.

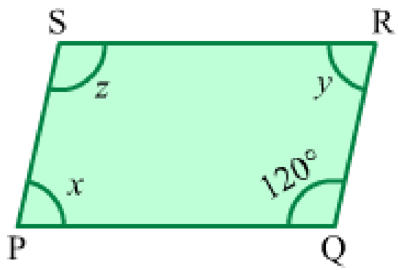
Example 2:

Find the values of *x*, *y* and *z* in the following parallelograms.

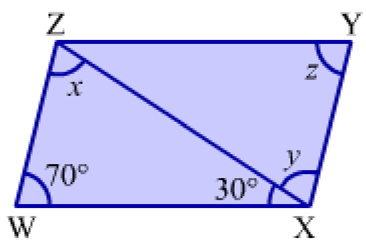
(i)



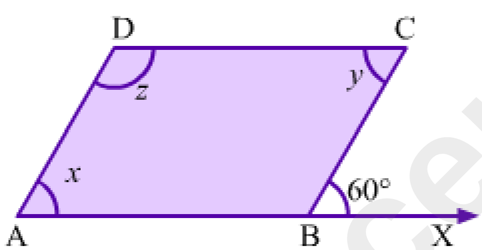
(ii)



(iii)



(iv)



Solution:

1. We know that the opposite angles of a parallelogram are equal.

   So, ∠BCD = ∠DAB

   ∴ $y = 60°$

   We also know that the adjacent angles of a parallelogram are supplementary.

   So, ∠CBA + ∠DAB = 180°

   ⇒ $z + 60° = 180°$

   ⇒ $z = 180° - 60°$

   ⇒ ∴ $z = 120°$

Now, x and z form a linear pair of angles; so, their sum is 180°.

So, $x + z = 180°$

$\Rightarrow x + 120° = 180°$

$\Rightarrow \therefore x = 180° - 120° = 60°$

2. We know that the opposite angles of a parallelogram are equal.

So, ∠PSR = ∠PQR

$\therefore z = 120°$

∠QPS and ∠PQR are adjacent angles.

So, ∠QPS + ∠PQR = 180°

$\Rightarrow x + 120° = 180°$

$\Rightarrow x = 180° - 120°$

$\Rightarrow \therefore x = 60°$

∠QRS and ∠QPS are opposite angles.

So, ∠QRS = ∠QPS

$\Rightarrow y = x$

$\Rightarrow \therefore y = 60°$

3. We know that the opposite angles of a parallelogram are equal.

So, ∠XYZ = ∠XWZ

$\therefore z = 70°$

∠XYZ and ∠WXY are adjacent angles.

$\therefore$ ∠XYZ + ∠WXY = 180°

$\Rightarrow z + 30° + y = 180°$

⇒ 70° + 30° + y = 180°

⇒ 100° + y = 180°

⇒ y = 180° − 100°

⇒ ∴ y = 80°

Now, XY||WZ; so, ∠WZX and ∠YXZ are alternate interior angles.

So, ∠WZX = ∠YXZ

⇒ $x = y$

⇒∴ x = 80°

4. It is given that ∠CBX = 60°.

∠CBA and ∠CBX form a linear pair.

So, ∠CBA + ∠CBX = 180°

⇒ ∠CBA + 60° = 180°

⇒ ∠CBA = 180° − 60°

⇒ ∴ ∠CBA = 120°

∠CDA and ∠CBA are opposite angles.

So, ∠CDA = ∠CBA

⇒ z = ∠CBA

⇒ ∴ z = 120°

∠BCD and ∠CBA are adjacent angles.

So, ∠BCD + ∠CBA = 180°

⇒ y + 120° = 180°

⇒ y = 180° − 120°

$\Rightarrow \therefore y = 60°$

∠BAD and ∠BCD are opposite angles.

So, ∠BAD = ∠BCD

$\Rightarrow x = y$

$\Rightarrow \therefore x = 60°$

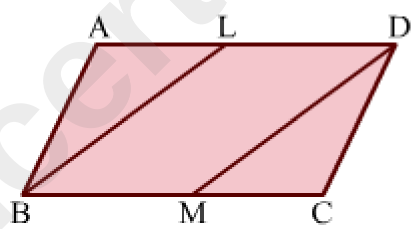
Hard

Example 1:

Show that the bisectors of opposite angles of a parallelogram are parallel to each other.

Solution:

Let ABCD be a parallelogram. Let BL and DM be the bisectors of ∠ABC and ∠ADC respectively.



Since BL and DM are the bisectors of ∠ABC and ∠ADC respectively, we have:

$$\angle LBM = \frac{\angle ABC}{2} \qquad ...(1)$$

$$\angle LDM = \frac{\angle ADC}{2} \qquad ...(2)$$

We know that the opposite angles of a parallelogram are equal.

∴ ∠ABC = ∠ADC

On dividing both sides of the above equation by 2, we obtain:

$$\frac{\angle ABC}{2} = \frac{\angle ADC}{2}$$

Using equations 1 and 2, we obtain:

∠LBM = ∠LDM

Now, LD and BM are parallel.

So, ∠DLB + ∠LBM = 180° (Interior angles on the same side of a transversal)

⇒ ∠DLB = 180° − ∠LBM

Similarly, ∠DMB = 180° − ∠LDM

∴ ∠DLB = ∠DMB (∵ ∠LBM = ∠LDM)

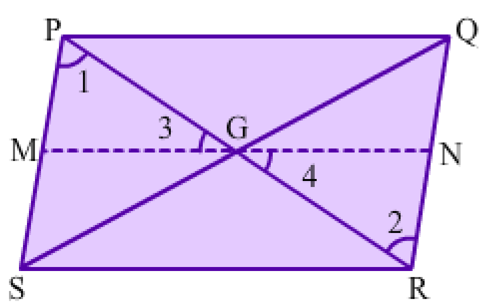
In quadrilateral LDMB, the opposite angles ∠DLB and ∠DMB are equal. Hence, it is a parallelogram.

⇒ BL||DM

We know that BL and DM are the bisectors of opposite angles of parallelogram ABCD. Thus, the bisectors of opposite angles of a parallelogram are parallel.

Relation between the Diagonals of a Parallelogram

Consider the following parallelogram PQRS.



In the figure, GM = GN, but can we prove this?

In order to prove GM = GN, we need to show that ΔGMP is congruent to ΔGNR.

In ΔGMP and ΔGNR, we have two sets of equal angles as follows:

∠3 = ∠4(Vertically opposite angles)

∠1 = ∠2(Alternate interior angles; since PS||QR and PR is the transversal)

Now, to apply the ASA congruence rule, we need to show that GP and GR are equal.

A property of parallelograms helps us establish this equality and it can be stated as follows:

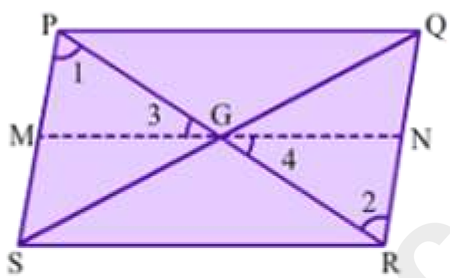
The diagonals of a parallelogram bisect each other.

In this lesson, we will study the above-stated property and solve some problems based on it.

Property of the Diagonals of a Parallelogram

Using the Property

Let us use the property of the diagonals of a parallelogram to solve the problem discussed at the beginning.



Let us once again consider parallelogram PQRS.

We have to prove that GM = GN.

Since diagonals PR and QS bisect each other, we obtain:

GP = GR and GS = GQ… (1)

In ΔGMP and ΔGNR, we have:

∠3 = ∠4(Vertically opposite angles)

∠1 = ∠2(Alternate interior angles; since PS||QR and PR is the transversal)

GP = GR(Using 1)

Thus, by the ASA congruence rule, we obtain:

ΔGMP ≅ ΔGNR

⇒ GM = GN(By CPCT)

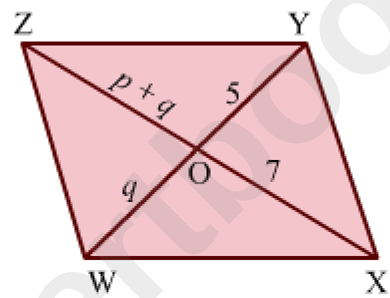
Similarly, we can use the property of the diagonals of a parallelogram to solve other problems.

Solved Examples

Easy

Example 1:

If the shown quadrilateral WXYZ is a parallelogram, then find the values of *p* and *q*.



Solution:

We know that the diagonals of a parallelogram bisect each other.

∴ WO = OY

⇒ $q = 5$

Similarly, XO = OZ

∴ $p + q = 7$

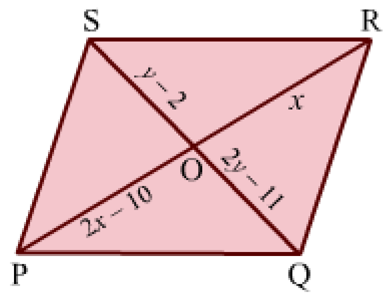
⇒ $p = 7 - q$

⇒ $p = 7 - 5$

⇒ ∴ $p = 2$

Example 2:

In the given parallelogram PQRS, find the lengths of the diagonals PR and QS.



Solution:

We know that the diagonals of a parallelogram bisect each other.

∴ PO = OR

⇒ $2x - 10 = x$

⇒ $2x - x = 10$

⇒ ∴ $x = 10$

Similarly, QO = OS

⇒ $y - 2 = 2y - 11$

⇒ $2y - y = -2 + 11$

⇒ ∴ $y = 9$

Now, PR = PO + OR

$= 2x - 10 + x$

$= 3x - 10$

$= 3 \times 10 - 10$

$= 30 - 10$

$= 20$

Similarly, QS = QO + OS

$= y - 2 + 2y - 11$

$= 3y - 13$

$= 3 \times 9 - 13$

$= 27 - 13$

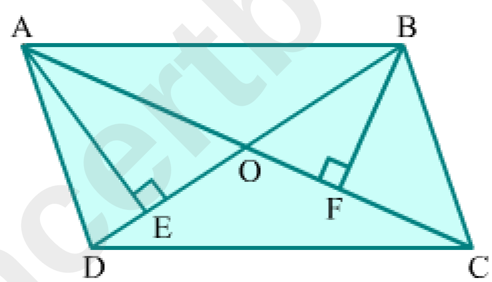
$= 14$

Thus, the lengths of the diagonals PR and QS are 20 units and 14 units respectively.

Medium

Example 1:

ABCD is a parallelogram with diagonals AC and BD of lengths 10 cm and 8 cm respectively. If the perpendiculars on DO and OC are 5 cm each, then find the sum of the areas of ΔAOD and ΔBOC.



Solution:

We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, we have:

AO = OC = $\frac{AC}{2}$ and BO = OD = $\frac{BD}{2}$

⇒ AO = OC = 10/2cm = 5 cm and BO = OD = 8/2cm = 4 cm

Now, area of ΔAOD= 1/2 × Base × Height

= 1/2 × OD × AE

= 1/2 × 4 × 5 cm^2

$= 10\ cm^2$

Similarly, area of ΔBOC= 1/2 × OC × BF

$= 1/2 \times 5 \times 5\ cm^2$

$= 12.5\ cm^2$

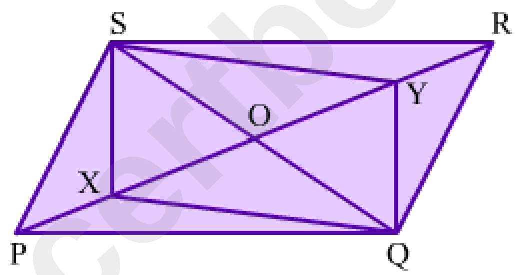
Therefore, sum of the areas of ΔAOD and ΔBOC = $(10 + 12.5)\ cm^2 = 22.5\ cm^2$

Hard

Example 1:

In parallelogram PQRS, X and Y are points on PR such that PX = YR. Prove that:

1. **XQYS is a parallelogram**
2. **ΔSXP ≅ΔQYR**



Solution:

1. We know that the diagonals of a parallelogram bisect each other.

∴ OS = OQ … (1)

And OP = OR… (2)

Also, PX = YR… (3) [Given]

On subtracting equation 3 from equation 2, we obtain:

OP − PX = OR − YR

⇒ OX = OY … (4)

In quadrilateral XQYS, XY and QS are the diagonals.

We know from equations 1 and 4 that the diagonals bisect each other.

Thus, XQYS is a parallelogram.

2. In ΔSXP and ΔQYR, we have:

   PS = QR (Opposite sides of parallelogram PQRS)

   SX = QY (Opposite sides of parallelogram XQYS)

   PX = YR (Given)

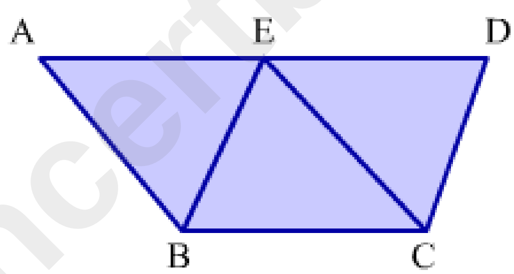
   ∴ ΔSXP ≅ ΔQYR (By the SSS congruence rule)

Simulation

A Quadrilateral is a Parallelogram if a Pair of Opposite Sides is Equal and Parallel

Necessary Condition for a Quadrilateral to Be a Parallelogram

Consider the given figure.



Here, ABCE is a parallelogram and AE is extended to D such that AE = ED.

Using an important property of parallelograms, we can prove that BCDE is also a parallelogram.

The property used for proving the above can be stated as follows:

A quadrilateral is a parallelogram if it has one pair of parallel and equal (or congruent) opposite sides.

Since ABCE is a parallelogram, AE||BC and AE = BC.

Also, AE = ED ⇒ ED = BC

And AE||BC ⇒ AD||BC ⇒ ED||BC

Hence, by the above-stated property, BCDE is a parallelogram.

In this lesson, we will understand and prove this property of parallelograms. We will also solve some examples based on the same.

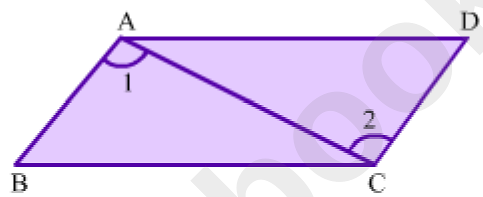
Proof of the Property

Solved Examples

Easy

Example 1:

In the given figure, ∠1 = ∠2 and AB = DC. Is quadrilateral ABCD a parallelogram?



Solution:

In quadrilateral ABCD, ∠1 = ∠2(Given)

These angles are alternate angles made by the transversal AC between lines AB and DC.

We know that if equal alternate angles are made by a transversal between two lines, then the lines intersected by the transversal are parallel.

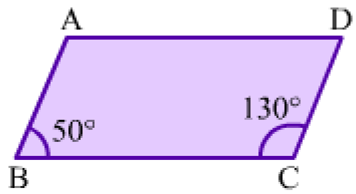
∴ AB||DC

Also, AB = DC (Given)

We know that a quadrilateral is a parallelogram if it has a pair of parallel and equal opposite sides. Thus, ABCD is a parallelogram.

Example 2:

In the given figure, AB = DC. Prove that AD||BC.



Solution:

In quadrilateral ABCD, ∠B + ∠C = 50° + 130° = 180°

We know that if the interior angles on the same side of a transversal are supplementary, then the lines intersected by the transversal are parallel.

⇒ AB||DC

Also, AB = DC (Given)

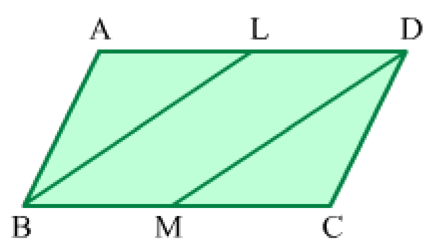
We know that a quadrilateral is a parallelogram if it has a pair of parallel and equal opposite sides. Thus, ABCD is a parallelogram.

⇒ AD||BC (∵ Opposite sides of a parallelogram are parallel)

Medium

Example 1:

ABCD is a parallelogram in which L and M are the midpoints of AD and BC respectively. Prove that BMDL is a parallelogram.



Solution:

It is given that L and M are the midpoints of AD and BC respectively.

$\therefore BM = \frac{1}{2}BC \text{ and } LD = \frac{1}{2}AD$

ABCD is a parallelogram. (Given)

So, BC = AD (∵ Opposite sides of a parallelogram are equal)

$\therefore \frac{1}{2}BC = \frac{1}{2}AD$

⇒ BM = LD … (1)

Also, BC||AD (∵ Opposite sides of a parallelogram are parallel)

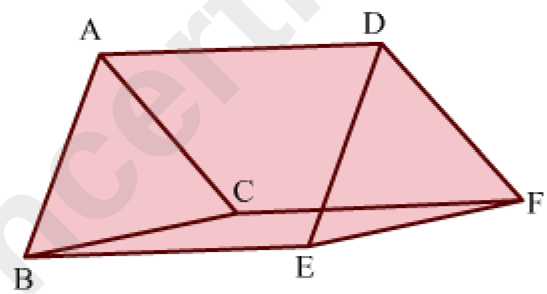
⇒ BM||LD … (2)

We know that a quadrilateral is a parallelogram if it has a pair of parallel and equal opposite sides. From 1 and 2, we conclude that BMDL is a parallelogram.

Hard

Example 1:

Sides AB and BC of ΔABC are parallel and equal to the corresponding sides DE and EF of ΔDEF. Prove that ACFD is a parallelogram.



Solution:

Consider quadrilateral ABED.

We have AB = DE and AB||DE (Given)

Hence, ABED is a parallelogram. (∵ There is one pair of equal and parallel opposite sides)

⇒ AD = BE and AD||BE… (1)

Now, consider quadrilateral BCFE.

We have BC = EF and BC||EF (Given)

Hence, BCFE is a parallelogram. (∵ There is one pair of equal and parallel opposite sides) ⇒ CF = BE and CF||BE… (2)

From 1 and 2, we have:

AD = CF and AD||CF

Hence, ACFD is a parallelogram. (∵ There is one pair of equal and parallel opposite sides)