Algebraic Expressions

Introduction of Variables and Expressions

We know that there are infinitely many numbers. However, there are only 26 alphabets in the English language.

Can we represent numbers with the help of letters?
Yes, we can represent the numbers with the help of alphabets(letters).
Alphabets like *a*, *b*, *c*, *x*, *y*, *z*, *l*, *m*, *n*, etc. are used in Mathematics to denote variables.

Let us take an example.

In a class, the number of boys are 20 more than the number of girls. How many boys are there in the class?

The number of boys in the class varies with the number of girls.
If there are 10 girls in the class, then the number of boys = 10 + 20 = 30

If there are 55 girls in the class, then the number of boys = 55 + 20 = 75
Therefore, we can write a rule to find the number of boys as follows:

Number of boys = Number of girls + 20

Here, number of girls in the class can vary, and can take different values. On the other hand, number of boys in the class varies according to the number of girls in the class. If we replace the number of boys and girls in the class with letters *m* and *n* respectively, then we get following expression:
$m = n + 20$

Here, *n* can take any value such as 0, 1, 2, 3... etc., and the value of *m* will change accordingly. Since the values of *m* and *n* can vary, these are known as variables.

A variable is something that does not have a fixed value. The value of a variable varies.

Also, a symbol with a fixed numeric value is known as a constant.

For example, 2, –4, $\sqrt{8}$, –3.4, $\frac{1}{2}$ etc., are constants as each of them have a fixed numeric value.

A combination of variables, numbers, and operators (+, –, ×, and ÷) is known as an algebraic expression.

For example,

1. $x + 7$
2. $2 – y$
3. $(5 × y) + 9$
4. $11xyz + ab^2 – 2p^4q^3 + \frac{3}{4}$
5. $\frac{a^2b}{cd} + 2p^5 – 1.5z$

Let us try to form few simple mathematical expressions by applying the four operations on numbers.

1. 42 is added to 58 and then the result is divided by 25 = $\frac{58+42}{25}$
2. The product of 24 and 15 is subtracted from the product of 30 and 43 = (30 × 43) – (24 ×15)

In the same way, we can form algebraic expressions by applying the four operations on variables.

1. z divided by 5 and 5 added to the result = $\frac{z}{5}+5$
2. x multiplied by 6 and 4 added to the product = $6x + 4$
3. 39 subtracted from $5m$ = $5m – 39$
4. 5 added to $6m$ and the result is subtracted from $8n$ = $8n – (6m + 5)$
5. 16 subtracted from $7x$ and the result is subtracted from $–2y$ = $–2y – (7x – 16)$

To understand the concept better, look at the following video.

In this way, we can represent a given real-life situation by using variables.
Let us now solve some more examples to understand the concept better.

Example 1:
Write down the following expressions in words.

1. $5x + 19$
2. $6y – 2$
3. $\frac{z}{2}+2y$
4. $(2y + 5) – 8$
5. $6 – (m + 7)$
6. $\frac{2x-5}{5}$

Solution:

(i) x is multiplied with 5 and then 19 is added to the product.
(ii) y is multiplied by 6 and then 2 is subtracted from the product.
(iii) z is divided by 2 and then $2y$ is added to the result.
(**iv)** 5 is added to $2y$ and 8 is subtracted from the result.
Or, 5 is added to the product of 2 and y and then 8 is subtracted from the result.
(v) 7 is added to m and the result is subtracted from 6.
(vi) 5 is subtracted from the product of 2 and x and then the result is divided by 5.

Example 2:
Form six expressions using two numbers 8 and 11 and variable *a.*

Solution:

1. $8a + 11$
2. $11a + 8$
3. $8a - 11$
4. $\frac{a}{8}+11$
5. $\frac{a}{11}+8$
6. $\frac{a}{11}-8$

Example 3:
Write down the expressions for the following situations.

1. **Diganta's age is 2 years more than 4 times Arjun's age.**
2. **What is the length of a rectangular field, if its length is 3 m less than twice its breadth?**
3. **The number of boys in a class is 8 less than 3 times the number of girls. Find the total number of students in the class.**
4. **Sonu is two times taller than Monu. Find the height of Sonu.**
5. **What will be the age of Aman after 14 years from now?**

Solution:

1. Let Arjun's age be z years. Therefore, four times Arjun's age is $4z$.

   ∴ Diganta's age = $4z + 2$

2. Let the breadth of the rectangular field be a m.

∴ Length of the rectangular field = (2*a* – 3) m

3. Let the number of girls be *p.*

   ∴ Number of boys = 3*p* – 8

   Thus, total number of students in the class $= p + 3p - 8 = 4p - 8$

4. Let the height of Monu be *h* cm.

   ∴ The height of Sonu will be 2*h* cm.

4. Let the present age of Aman be *y* years.

   ∴ The age of Aman after 14 years will be *y* + 14 years.

Example 4:
Sonu is twice as old as Monu. Find the rule to find Sonu's age if Monu's age is taken as *x*.

Solution:
It is given that Sonu is twice as old as Monu.

The rule can be written as:
Sonu's age = 2 × Monu's age = 2*x*, where *x* = 1, 2, 3, 4, 5 …

Example 5:
The price of Mohit's book is Rs 3 less than 3 times the price of Rohit's book. Find the rule to find the price of Mohit's book.

Solution:

Here, the price of Mohit's book is given in terms of the price of Rohit's book.
Let the price of Rohit's book be Rs *x*.

∴ Price of Mohit's book = Rs (3 × price of Rohit's book – 3)
= Rs (3*x* – 3)

Example 6:
Sachin has 5 apples more than Suhaan. Find the rule to find the number of apples with Suhaan.

Solution:
Sachin has 5 apples more than Suhaan. This means that Suhaan has 5 apples less than Sachin.

Let the number of apples with Sachin be n.

∴ Number of apples with Suhaan = Number of apples with Sachin – 5
= $n - 5$

Example 7:
The speed of a train is 70 km/h. Find a rule for the total distance covered by the train in *x* hours.

Solution:

Speed of the train = 70 km/h
Total time taken by the train to cover the distance = x h
∴ Total distance covered by the train = speed × total time taken
= 70 × total time taken
= $70 \times x$
= $70x$ km

Degree of Polynomial

More about Polynomials

We know that a polynomial comprises a number of terms, which may have variables or numbers or both. Also, each term can be represented with a variable having some **exponent**. Exponents of the variables in a given polynomial can be the same or different.

Let us consider a polynomial $2x^5 + 4x^2 + 9$.

The terms of this polynomial and their exponents are as follows:

First term = $2x^5$; exponent in the first term = 5

Second term = $4x^2$; exponent in the second term = 2

Third term = 9 = $9x^0$; exponent in the third term = 0

Note that all the exponents in the above polynomial are different. These exponents help us to identify the degrees of polynomials. Polynomials are categorized based on their degrees.

In this lesson, we will learn about the degrees of polynomials and the classification of polynomials based on the same.

The Degree of a Polynomial

Whiz Kid

When a polynomial has an equals sign (=), then it becomes an equation. The maximum number of solutions of an equation is less than or equal to the degree of that equation.

Solved Examples

Easy

Example 1:

Find the degree of each term of the polynomial $3x^6 + 3x^4 - 6x + 3$. Also find the degree of the polynomial.

Solution:

The degree of the term $3x^6$ is 6.

The degree of the term $3x^4$ is 4.

The degree of the term $-6x$ is 1.

The degree of the term 3 is 0.

Here, the highest degree is 6. Hence, the degree of the polynomial is 6.

Medium

Example 1:

Write the degree of each of the following polynomials.

i) $\frac{x^2}{2x} - 9x^7 + \frac{1}{x^{-4}} + 7$

ii) $\frac{x^2}{3} + \frac{4x^{-1}}{x^{-2}} - 5x^2 + \frac{x}{2} - 9$

Solution:

i) $\frac{x^2}{2x} - 9x^7 + \frac{1}{x^{-4}} + 7$

$$= \frac{x^{2-1}}{2} - 9x^7 + x^4 + 7 \qquad \left(\because \frac{a^m}{a^n} = a^{m-n}, \text{where } m > n;\ \frac{1}{a^{-m}} = a^m \right)$$

$$= \frac{x}{2} - 9x^7 + x^4 + 7$$

In the given polynomial, the highest degree is 7. Hence, the degree of the polynomial is 7.

ii) $\frac{x^2}{3} + \frac{4x^{-1}}{x^{-2}} - 5x^2 + \frac{x}{2} - 9$

$$= \frac{x^2}{3} + 4x^{2-1} - 5x^2 + \frac{x}{2} - 9 \qquad \left(\because \frac{a^m}{a^n} = a^{n-m}, \text{where } m < n \right)$$

$$= \frac{x^2}{3} + 4x - 5x^2 + \frac{x}{2} - 9$$

$$= -\frac{14x^2}{3} + \frac{9x}{2} - 9$$

In the given polynomial, the highest degree is 2. Hence, the degree of the polynomial is 2.

The Degree of a Polynomial in more than one Variable

In case of the polynomials in one variable, the degree of a polynomial is the highest exponent of the variable in the polynomial, but what about the degree of the polynomial in more than one variable?

In this case, the sum of the powers of all variables in each term is obtained and the highest sum among all is the degree of the polynomial.

For example, find the degree of the polynomial $2xy + 3y^2z + 4x^2yz^2 - xyz - 2x^3$. Let us find the sum of the powers of all variables in each term of this polynomial.

Sum of the powers of all variables in the term $2xy = 1 + 1 = 2$

Sum of the powers of all variables in the term $3y^2z = 2 + 1 = 3$

Sum of the powers of all variables in the term $4x^2yz^2 = 2 + 1 + 2 = 5$

Sum of the powers of all variables in the term $-xyz = 1 + 1 + 1 = 3$

Sum of the powers of all variables in the term $-2x^3 = 3$

Among all the sums, 5 is the highest and thus, the degree of the polynomial $2xy + 3y^2z + 4x^2yz^2 - xyz - 2x^3$ is 5.

Similarly, we can find the degree of any polynomial in more than one variable.

Solved Examples

Easy

Example 1:

Write the degree of each of the following polynomials.

(i) $24a^2b - abc + 11abc^2 + ab^2c^3 - 7a^2b^2$

(ii) $5p^5 + 10p^2qr - 9p^2qr^2 - p^2r^2 + 2pq^2 - 2p^3q^2r$

Solution:

(i) Let us find the sum of the powers of all variables in each term of $24a^2b - abc + 11abc^2 + ab^2c^3 - 7a^2b^2$.

Sum of the powers of all variables in the term $24a^2b = 2 + 1 = 3$

Sum of the powers of all variables in the term $-abc = 1 + 1 + 1 = 3$

Sum of the powers of all variables in the term $11abc^2 = 1 + 1 + 2 = 4$

Sum of the powers of all variables in the term $ab^2c^3 = 1 + 2 + 3 = 6$

Sum of the powers of all variables in the term $-7a^2b^2 = 2 + 2 = 4$

Among all the sums, 6 is the highest and thus, the degree of the polynomial $24a^2b - abc + 11abc^2 + ab^2c^3 - 7a^2b^2$ is 6.

(ii) Let us find the sum of the powers of all variables in each term of $5p^5 + 10p^2qr - 9p^2qr^2 - p^2r^2 + 2pq^2 - 2p^3q^2r$.

Sum of the powers of all variables in the term $5p^5 = 5$

Sum of the powers of all variables in the term $10p^2qr = 2 + 1 + 1 = 4$

Sum of the powers of all variables in the term $-9p^2qr^2 = 2 + 1 + 2 = 5$

Sum of the powers of all variables in the term $–p^2r^2$ = 2 + 2 = 4

Sum of the powers of all variables in the term $2pq^2$ = 1 + 2 = 3

Sum of the powers of all variables in the term $–2p^3q^2r$ = 3 + 2 + 1 = 6

Among all the sums, 6 is the highest and thus, the degree of the polynomial $5p^5 + 10p^2qr – 9p^2qr^2 – p^2r^2 + 2pq^2 – 2p^3q^2r$ is 6.

Classification of Polynomials According to Their Degrees

Whiz Kid

If all the terms in a polynomial have the same exponent, then the expression is referred to as a **homogenous polynomial**.

Did You Know?

The graphs oflinear polynomials are always straight lines. This is why these polynomials are called 'linear' polynomials.

Solved Examples

Easy

Example 1:

Classify each of the given polynomials according to its degree.

i) $11x^3 + 7x + 3$

ii) $8x^2 + 3x$

iii) $x + 5$

iv) $9t^3$

Solution:

i) $11x^3 + 7x + 3$

The degree of this polynomial is 3. Hence, it is a cubic polynomial.

ii) $8x^2 + 3x$

The degree of this polynomial is 2. Hence, it is a quadratic polynomial.

iii) $\boldsymbol{x + 5}$

The degree of this polynomial is 1. Hence, it is a linear polynomial.

iv) $\boldsymbol{9t^3}$

The degree of this polynomial is 3. Hence, it is a cubic polynomial.

Example 2:

Give an example of each of the following polynomials.

i) A monomial of degree 50

ii) A binomial of degree 17

iii) A trinomial of degree 99

Solution:

i) A monomial of degree 50 means a polynomial having one term and 50 as the highest exponent. An example of such a polynomial is $23y^{50}$.

ii) A binomial of degree 17 means a polynomial having two terms and 17 as the highest exponent. An example of such a polynomial is $41t^{17} + 53t$.

iii) A trinomial of degree 99 means a polynomial having three terms and 99 as the highest exponent. An example of such a polynomial is $p^{99} + 5p - 12$.

Medium

Example 1:

Classify each of the given polynomials according to its degree.

i) $\boldsymbol{\frac{x^2}{3} + 4x^3 - (5x^2 + 4x^3) + \frac{x}{2} - 9}$

ii) $\boldsymbol{x + 3x^2 + (x + 2)(x^2 + 4 - 2x) + 54}$

Solution:

i) $\frac{x^2}{3} + 4x^3 - (5x^2 + 4x^3) + \frac{x}{2} - 9$

$$= \frac{x^2}{3} + 4x^3 - 5x^2 - 4x^3 + \frac{x}{2} - 9$$
$$= \frac{x^2}{3} - 5x^2 + \frac{x}{2} - 9$$
$$= -\frac{14x^2}{3} + \frac{x}{2} - 9$$

The degree of this polynomial is 2. Hence, it is a quadratic polynomial.

ii) $x + 3x^2 + (x + 2)(x^2 + 4 - 2x) + 54$

$= x + 3x^2 + (x^3 + 2^3) + 54$ [$\because a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$]

$= x + 3x^2 + x^3 + 8 + 54$

$= x + 3x^2 + x^3 + 62$

The degree of this polynomial is 3. Hence, it is a cubic polynomial.

Factors, Coefficients and Terms of Algebraic Expressions

Shiva and Somesh are brothers. Shiva's age is 3 years less than Somesh's age.

Now, how can we represent this situation with an algebraic expression?

Let us assume Somesh's age as x years. Therefore, Shiva's age = $(x - 3)$ years

Here, $(x - 3)$ is an algebraic expression that represents Shiva's age.

Here, we can notice one thing. The ages of both Somesh and Shiva can vary, but the difference between the ages, i.e. 3 years, is always constant. In this algebraic expression $(x - 3)$, x can vary but the number 3 does not. Hence, x is known as a **variable (or algebraic number)** and 3 is called a **constant (absolute term**).

Let us consider some algebraic expressions given below.

(i) $3x + 5$

(ii) $4x^2 - 21$

Here, the first expression $(3x + 5)$ is formed by adding $3x$ and 5. In this case, $3x$ and 5 are called **algebraic terms or simply terms** of the expression. The terms are always added to form an algebraic expression.

They are never subtracted to form an algebraic expression. However, an expression may have positive or negative terms. In the expression ($3x - 5$), the terms of the expression are $3x$ and (-5), and not $3x$ and 5. Thus, we added the terms $3x$ and (-5) to get the expression ($3x - 5$).

In expression (ii), $4x^2$ and (-21) are added to form $4x^2 - 21$. Therefore, $4x^2$ and (-21) are terms of the expression $4x^2 - 21$.

Let us again consider the expression $3x + 5$. Here, the term $3x$ is a product of 3 and x. We cannot factorise 3 and x further. Hence, 3 and x are called **factors** of the term $3x$.

The term 5 cannot be expressed as the product of variables and constant. Therefore, 5 is itself a factor of 5.

In expression (ii), the term $4x^2$ can be written as

$4x^2 = 4 \times x^2$

$4x^2 = 4x \times x$

But x^2 and $4x$ cannot be the factors of $4x^2$ as they can be factorised further.

$x^2 = x \times x$ and $4x = 4 \times x$

We can write $4x^2$ as the product of 4, x, and x as shown below.

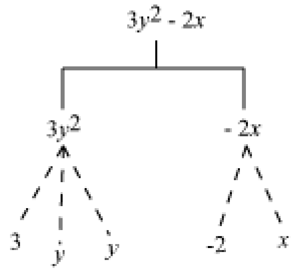
$4x^2 = 4 \times x \times x$

Therefore, 4, x and x are called factors of $4x^2$.

We can also represent the factors and terms of an algebraic expression by a tree diagram.

Let us understand the concept of a tree diagram with the help of an example in the given video.

Similarly, we can represent the tree diagram of an expression ($3y^2 - 2x$) as shown below.



In the expression $3y^2 - 2x$, we can see that the term $3y^2$ is the product of a numerical, i.e. 3, and other variables. This numerical, i.e. 3, is known as the **numerical coefficient** of the term, $3y^2$. Similarly, the coefficient of the term $-2x$ is -2. Generally, we define the numerical coefficient or efficient as

The numerical factor of a term is called the numerical coefficient (or constant coefficient) of the term.

Using this definition, we can say that the numerical coefficient of $-15xy$ is -15, since

$-15xy = -15 \times x \times y$

We can also write $-15xy$ as $-15xy = x \times (-15y) = y \times (-15x)$

Thus, we can say that the coefficient of x is $-15y$ and the coefficient of y is $-15x$.

Can we find the numerical coefficients of x in the expression $(x - 5)$ and that of xy in the expression $(7 - xy)$?

In case of $(x - 5)$, 1 is the numerical coefficient of x. In case of $(7 - xy)$, -1 is the numerical coefficient of xy.

Note: In the expression $-15xy$, xy is said to be the algebraic coefficient.

Thus, we can say that

If the coefficient of a term is 1, then it is not written before the term. If the coefficient of the term is −1, then only the '−' sign is put before the term.

Let us look at the factorization of the terms $5x^3yz^2$ and $-23x^3yz^2$.

$$5x^3yz^2 = 5\underbrace{x \times x \times x}_{3} \times \underbrace{y}_{1} \times \underbrace{z \times z}_{2}$$

$$-23x^3yz^2 = -23 \times \underbrace{x \times x \times x}_{3} \times \underbrace{y}_{1} \times \underbrace{z \times z}_{2}$$

Here, we can see that the two terms have different numerical factors 5 and −23, but same algebraic factors (each of these term contains the same variable, i.e. *x*, *y*, and *z*. Also, powers of these variables of each term are the same, i.e. power of *x*, *y*, and *z* are 3, 1, and 2 respectively). These terms are known as **like terms.** We can define them as

The terms having the same algebraic factors are called like terms. Like terms may have different numerical factors.

Let us consider the terms 6*xy* and 6*x*. Now, 6*xy* = 6 × *x* × *y* and 6*x* = 6 × *x*

Here, we can see that the two terms have the same numerical factor 6. Their algebraic factors *xy* and *x* are different. Such type of terms having different algebraic factors are said to be **unlike terms.** We define them as

The terms having different algebraic factors are called unlike terms.

Let us discuss some examples to understand these concepts better.

Example 1:

Find the terms in the algebraic expression $\left(-\frac{xy}{7}+14xy^2-3\right)$**.**

Solution:

The terms of the expression are $-\frac{xy}{7}, 14xy^2$, and −3.

Example 2:

Find the factors of $\left(-3x^2yz^3\right)$**.**

Solution:

$-3x^2yz^3 = -3 \times x \times x \times y \times z \times z \times z$

Therefore, the factors of $-3x^2yz^3$ are -3, x, x, y, z, z, and z.

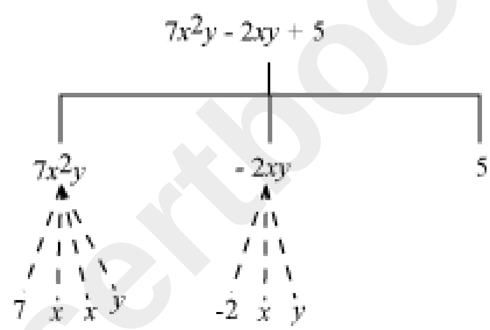
Example 3:

Represent the terms and factors of the algebraic expression $7x^2y - 2xy + 5$ through a

tree diagram.

Solution:

The tree diagram representation of the algebraic expression, $7x^2y - 2xy + 5$ is



Example 4:

Find the like terms in the algebraic expression $51x^2y - 21x^2y^2 + \frac{x^2y}{2} + 31 - 6xy - x^2y$.

Solution:

Here, the like terms are $51x^2y$, $\frac{x^2y}{2}$, and $-x^2y$.

Example 5:

Find the coefficients of *pq* in the following terms.

pq^2, $-3pq$, $15p^2q^2$, $-\frac{31}{5}p^2q$

Solution:

Terms	Coefficients of pq
pq^2	q
−3pq	−3
$15p^2q^2$	15pq
$-\frac{31}{5}p^2q$	$-\frac{31}{5}p$

Addition and Subtraction of Polynomials

Just like natural numbers, we can even perform mathematical operations on algebraic expressions. The process is very much similar to the process required for natural numbers. You can understand the process by going through the following video.

The most important point to remember in this topic is as follows.

We can add and subtract like terms. In case of addition or subtraction of like terms, only their numerical coefficients are added or subtracted. The algebraic part of the terms remains as it is.

In the video given above, we learned adding polynomials by arranging them horizontally. We can add polynomials by arranging them vertically as well.

Adding and subtracting like monomials by arranging them vertically:
In this method, we have to write given like monomials one below other to perform the addition or subtraction. Also, write the coefficients for pure variable terms as 1. To add or subtract like monomials, we just need to add or subtract the coefficients and write the result with the variable.

For example, let us add monomials a^2, $-2a^2$ and $5a^2$. These monomials can be added by arranging vertically as follows:

$$\begin{array}{rr} & 1a^2 \\ + & -2a^2 \\ + & 5a^2 \\ \hline & 4a^2 \\ \hline \end{array}$$

Here, the sum of 1 and –2 is obtained as –1. Further, the sum of –1 and 5 is obtained as 4. So, we wrote 4 with a^2 and obtained the sum of given polynomials as $4a^2$.

Similarly, we can perform subtraction for the given monomials.

$$\begin{array}{rr} & 1a^2 \\ - & -2a^2 \\ - & 5a^2 \\ \hline & -2a^2 \\ \hline \end{array}$$

When we subtracted –2 from 1, we got 3 [1 – (–2) = 3]. Further, on subtracting 5 from 3, we got –2 (3 – 5 = –2). So, we wrote –2 with a^2 and obtained the result as $-2a^2$.

Adding and subtracting polynomials by arranging them vertically:
To add the polynomials, we can arrange them vertically such that each term of lower polynomial is written below its like term in the upper polynomial. Also, write the coefficients for pure variable terms as 1.

Let us add the polynomials $-3x^2 + 4xy - z$ and $2xy + x^2 - 3z$ to learn the concept.
We can observe that $-3x^2$ and x^2 are like terms as they have same variable having same powers. Similarly, $4xy$ and $2xy$, and $-z$ and $-3z$ are other pairs of like terms.

These polynomials can be arranged vertically as follows:

$$\begin{array}{rrrr} & -3x^2 & +4xy & -z \\ + & 1x^2 & +2xy & -3z \\ \hline & & & \\ \hline \end{array}$$

Now, we just need to add the coefficients of like terms and write the variables as they are.

$$\begin{array}{rrrr} & -3x^2 & +4xy & -z \\ + & 1x^2 & +2xy & -3z \\ \hline & -2x^2 & +6xy & -4z \\ \hline \end{array}$$

Thus, the sum of the given polynomials is $-2x^2 + 6xy - 4z$.

Let us now study the concept of subtraction of polynomials by subtracting $2xy + x^2 - 3z$ from $-3x^2 + 4xy - z$.

Let us arrange them vertically first as we have done before.

$$\begin{array}{rrrr} & -3x^2 & +4xy & -z \\ - & 1x^2 & +2xy & -3z \\ \hline & & & \\ \hline \end{array}$$

To subtract a polynomial from other, we add its opposite.
Now, we get

$$\begin{array}{rl} & -3x^2 + 4xy - z \\ + & -1x^2 - 2xy + 3z \\ \hline & \\ \hline \end{array}$$

Now, we perform the addition as we have done before.

$$\begin{array}{rl} & -3x^2 + 4xy - z \\ + & -1x^2 - 2xy + 3z \\ \hline & -4x^2 + 2xy + 2z \\ \hline \end{array}$$

Thus, the required difference is $-4x^2 + 2xy + 2z$.

Let us discuss some more examples to understand the concept better.

Example 1:

Add the following monomials by arranging them horizontally as well as vertically.

(a) $-3p^2$, $6p^2$ **and** $-11p^2$

(b) $8x^2y$, $-10x^2y$ **and** $-2x^2y$

Solution:

Addition by arranging horizontally:

(a) $-3p^2 + 6p^2 + (-11p^2) = (-3 + 6 - 11)p^2 = -8p^2$

(b) $8x^2y + (-10x^2y) + (-2x^2y) = (8 - 10 - 2)x^2y = -4x^2y$

Addition by arranging vertically:

(a)

$$\begin{array}{rr} & -3p^2 \\ + & 6p^2 \\ + & -11p^2 \\ \hline & -8p^2 \\ \hline \end{array}$$

(b)

$$\begin{array}{rr} & 8x^2y \\ + & -10x^2y \\ + & -2x^2y \\ \hline & -4x^2y \\ \hline \end{array}$$

Example 2:

Subtract the following monomials by arranging them horizontally as well as vertically.

(a) 25mn^2 from the sum of 14mn^2 and –mn^2

(b) (–x^2y^2 + 12x^2y^2) from 19x^2y^2

Solution:

Subtraction by arranging horizontally:

(a) $(14mn^2 – mn^2) – 25mn^2 = 13mn^2 – 25mn^2 = –12mn^2$

(b) $19x^2y^2 – (–x^2y^2 + 12x^2y^2) = 19x^2y^2 – 11x^2y^2 = 8x^2y^2$

Subtraction by arranging vertically:

(a)

$$\begin{array}{rr} & 14m^2n^2 \\ + & -1m^2n^2 \\ \hline & 13m^2n^2 \\ \hline \end{array} \qquad \begin{array}{rr} & 13m^2n^2 \\ - & 25m^2n^2 \\ \hline & -12m^2n^2 \\ \hline \end{array}$$

(b)

$$\begin{array}{rr} & -x^2y^2 \\ + & 12x^2y^2 \\ \hline & 11x^2y^2 \\ \hline \end{array} \qquad \begin{array}{rr} & 19x^2y^2 \\ - & 11x^2y^2 \\ \hline & 8x^2y^2 \\ \hline \end{array}$$

Example 3:

Add the expressions 5x^2 + 6xy – 11, 7 x^2y – 3y, and 12x^2y – 3xy + 4.

Solution:

$(5x^2 + 6xy - 11) + (7x^2y - 3y) + (12x^2y - 3xy + 4)$

$= 5x^2 + 6xy - 11 + 7x^2y - 3y + 12x^2y - 3xy + 4$

$= 5x^2 + 6xy - 3xy + 7x^2y + 12x^2y - 3y - 11 + 4$ (Rearranging the terms)

$=5x^2 + (6 - 3)xy + (7 + 12)x^2y - 3y + (-11 + 4)$

$= 5x^2 + 3xy + 19x^2y - 3y - 7$

Example 4:

Which expression when subtracted from the expression $(7x - 3y + 45xy + 7)$ gives

$(2x - 21y - 42xy)$?

Solution:

To get the required expression, we have to subtract $(2x - 21y - 42xy)$ from $(7x - 3y + 45xy + 7)$.

$(7x - 3y + 45xy + 7) - (2x - 21y - 42xy)$

$= 7x - 3y + 45xy + 7 - 2x + 21y + 42xy$

$= 7x - 2x - 3y + 21y + 45xy + 42xy + 7$

$= (7 - 2)x + (-3 + 21)y + (45 + 42)xy + 7$

$= 5x + 18y + 87xy + 7$

Example 5:

Subtract the sum of $(4y^2 - 6y)$ and $(-2y^2 + 3y - 3)$ from the sum of $(5y + 7)$ and

$(3y^2 - 9y + 2)$.

Solution:

$(5y + 7)+ (3y^2 - 9y + 2)$

$= 5y + 7+ 3y^2 - 9y + 2$

$= 3y^2 + 5y – 9y + 7 + 2$ [Rearranging the terms]

$= 3y^2 + (5y – 9y) + (7 + 2)$

$= 3y^2 + (5 – 9)\ y + 9$

$= 3y^2 + (–4)\ y + 9$

$= 3y^2 – 4y + 9$

$(4y^2 – 6y) + (–2y^2 + 3y – 3)$

$= 4y^2 – 6y – 2y^2 + 3y – 3$

$= 4y^2 – 2y^2 – 6y + 3y – 3$ [Rearranging the terms]

$= (4y^2 – 2y^2) + (–6y + 3y) – 3$

$= (4 – 2)\ y^2 + (–6 + 3)\ y – 3$

$= 2y^2 –3y – 3$

Now, subtracting the sum of $(4y^2 – 6y)$ and $(–2y^2 + 3y – 3)$ from the sum of $(5y + 7)$ and $(3y^2 – 9y + 2)$ is the same as subtracting $(2y^2 – 3y – 3)$ from $(3y^2 – 4y + 9)$.

This can be done as

$(3y^2 – 4y + 9) – (2y^2 – 3y – 3)$

$= 3y^2 – 4y + 9 – 2y^2 + 3y + 3$

$= 3y^2 – 2y^2 – 4y + 3y + 9 + 3$ [Rearranging the terms]

$= (3 – 2)\ y^2 + (–4 + 3)\ y + (9 + 3)$

$= y^2 – y + 12$

Multiplication of Monomials with Polynomials

Let us discuss another example based on the above concept.

Suppose we have two monomials $4x$ and $5y$. By multiplying them, we get

$4x \times 5y = (4 \times x) \times (5 \times y)$

$= (4 \times 5) \times (x \times y)$

$= 20xy$

Hence, we can say that $4x \times 5y = 20xy$

Now, $10x \times 5x^2z = (10 \times x) \times (5 \times x^2 \times z)$

This becomes,

$(10 \times 5) \times (x \times x^2 \times z) = 50x^3z$

Now, what if you have three monomials and you want to multiply them? How will you do so?

Suppose you want to multiply $10x$, $2xy$, and $5z$.

$10x \times 2xy \times 5z = (10 \times x) \times (2 \times x \times y) \times (5 \times z)$

First, we multiply the first two monomials.

$= \{(10 \times 2) \times (x \times x \times y)\} \times (5 \times z)$

$= (20 \times x^2 \times y) \times (5 \times z)$ $\quad [\because x \times x = x^2]$

$= (20 \times 5) \times (x^2 \times y \times z)$

$= 100x^2yz$

This method of multiplication of three monomials can be extended to find out the product of any number of monomials.

Let us discuss some more examples based on multiplication of monomials.

Example 1:

Find the product of the following:

(a) $-2x^2y$ and $15xy^2z^3$

(b) $7ap$, $2qa^2x^3$ and $-5rx$

(c) ab, $-2bc$, $-3cd$ and $4ad$

Solution:

(**a**) $-2x^2y$ and $15\ xy^2z^3$

$= (-2x^2y) \times (15\ xy^2z^3)$

$= (-2 \times 15) \times (x^2y \times xy^2z^3)$

$= -30x^3y^3z^3 \qquad \{\text{as } x^2 \times x = x^3 \text{ and } y \times y^2 = y^3\}$

(**b**) $7ap$, $2qa^2x^3$, and $-5rx$

$= (7ap) \times (2qa^2x^3) \times (-5rx)$

$= \{(7 \times 2) \times (ap \times qa^2x^3)\} \times (-5rx)$

$= (14a^3pqx^3) \times (-5rx)$

$= (14 \times -5) \times (a^3pqx^3 \times rx)$

$= -70a^3pqrx^4 \qquad [\text{as } x^3 \times x = x^4]$

(**c**) ab, $-2bc$, $-3cd$, and $4ad$

$= (ab) \times (-2bc) \times (-3cd) \times (4ad)$

$= [(1 \times -2) \times (ab \times bc)] \times [(-3 \times 4) \times (cd \times ad)]$

{Multiplying the 1st to 2nd and 3rd to 4th term}

$= (-2ab^2c) \times (-12cd^2a)$

$= (-2 \times -12) \times (ab^2c \times cd^2a)$

$= 24a^2b^2c^2d^2$

Example 2.

If the side of a square is $4x$ cm, what is its area?

Answer:

Side of square = $4x$ cm

∴ Area of square = side × side = $4x \times 4x = 16x^2$ cm^2

Example 3:

The length, breadth, and height of three cuboids are given below in the table. Find the volume and area of the base of these cuboids.

	Length	Breadth	Height
(i)	$3ab$	$2bx$	$5xy$
(ii)	a^2b	b^2c	c^2a
(iii)	$3x$	$9x^2$	$27x^3$

Solution:

We know that, area of the base = Length × Breadth

Volume of cuboid = Length × Breadth × Height

(i) Area of the base = $3ab \times 2bx = (3 \times 2) \times (ab \times bx) = 6ab^2x$

Volume of the cuboid = $3ab \times 2bx \times 5xy$

$$= \{(3 \times 2) \times (ab \times bx)\} \times (5xy)$$
$$= (6ab^2x) \times (5xy)$$
$$= (6 \times 5) \times (ab^2x \times xy)$$
$$= 30\ ab^2x^2y$$

(ii) Area of the base = $a^2b \times b^2c = a^2b^3c$

Volume of the cuboid = $(a^2b \times b^2c) \times c^2a = a^2b^3c \times c^2a = a^3\,b^3\,c^3$

(iii) Area of the base = $3x \times 9x^2 = (3 \times 9) \times (x \times x^2) = 27x^3$

Volume of the cuboid = $3x \times 9x^2 \times 27x^3$

$= \left[(3 \times 9) \times (x \times x^2)\right] \times 27x^3$
$= 27x^3 \times 27x^3$
$= (27 \times 27)(x^3 \times x^3)$
$= 729\ x^6$

So far, we know how two multiply any number of monomials. But, what if we need to multiply a monomial with a binomial or a trinomial, etc.?

Can we multiply them?

We can multiply them easily.

To understand the method, look at the following video.

The method discussed in the above video shows the **horizontal arrangement** of multiplying monomials with polynomials.

Let us now learn about the **vertical arrangement** for the same by performing the multiplication of ($4x^2$ + $2x$) and $3x$.

This is similar to vertical method of multiplication of whole numbers.

Here, we will first multiply $3x$ with $2x$ and write the product with sign at the bottom. After doing this, we will multiply $3x$ with $4x^2$ and write the product with sign at the bottom. The expression obtained at the bottom will be the required product.

This can be done as follows:

$4x^2 + 2x$
$\times \quad 3x$
$12x^3 + 6x^2$

Similarly, we can multiply a trinomial with monomial as follows:

$$\begin{array}{r} 2y^3 - 5y + 1 \\ \times \quad 2y \\ \hline 4y^4 - 10y^2 + 2y \\ \hline \end{array}$$

Let us discuss some more examples based on the multiplication of a monomial with polynomials.

Example 4:

Multiply the following in horizontal and vertical arrangements:

(a) 3/5 *p* and *p* − 6*q*

(b) (1.5*x* + *y* + 3*z*) and 7*z*

Also find the values of the above expressions if *p* = − 5, *q* = − 1, *x* = 2, *y* = − 3, and *z* = − 1.

Solution:

(a) Horizontal arrangement:

$$\frac{3}{5}p \times (p - 6q) = \frac{3}{5}p \times p - \frac{3}{5}p \times 6q = \frac{3}{5}p^2 - \frac{18}{5}pq$$

Vertical arrangement:

$$\begin{array}{r} p - 6q \\ \times \quad \frac{3}{5}p \\ \hline \frac{3}{5}p^2 - \frac{18}{5}pq \\ \hline \end{array}$$

Substituting the values of *p* and *q*, we get

$$= \frac{3}{5} \times (-5)^2 - \frac{18}{5} \times (-5)(-1)$$

$= \frac{3}{5} \times 25 - 18$
$= 15 - 18$
$= -3$

(b) Horizontal arrangement:

$(1.5x + y + 3z) \times 7z = 1.5x \times 7z + y \times 7z + 3z \times 7z = 10.5xz + 7yz + 21z^2$

Vertical arrangement:

$$\begin{array}{r} 1.5x + y + 3z \\ \times \qquad\qquad 7z \\ \hline 10.5xz + 7yz + 21z^2 \\ \hline \end{array}$$

Substituting the values of *x*, *y*, and *z*, we get

$10.5xz + 7yz + 21z^2$
$= 10.5 \times 2 \times (-1) + 7 \times (-3) \times (-1) + 21 \times (-1)^2$

$= -21 + 21 + 21$

= 21

Example 5:

(a) Add $a(b - c), b(c - a)$, **and** $c(a - b)$

(b) **Subtract** $5x(x - y + z) - 2z(-3x + 4y + 5z)$ **from** $3y(4x + 3y - 2z)$

Solution:

(a) Addition of *a* (*b* − *c*), *b* (*c* − *a*), and *c* (*a* − *b*)

$= a(b-c) + b(c-a) + c(a-b)$
$= ab - ac + bc - ab + ac - bc$
$= 0$

(b) Subtraction of $5x(x-y+z)-2z(-3x+4y+5z)$ from $3y(4x+3y-2z)$

$$= 3y(4x+3y-2z)-\{5x(x-y+z)-2z(-3x+4y+5z)\}$$

$$= (3y)(4x)+(3y)(3y)+(3y)(-2z)-\left\{\begin{array}{l}(5x)(x)+(5x)(-y)+(5x)(z)\\ -2z(-3x)-2z(4y)-2z(5z)\end{array}\right\}$$

$$= 12xy+9y^2-6yz-\{5x^2-5xy+5zx+6zx-8yz-10z^2\}$$

$$= 12xy+9y^2-6yz-\{5x^2-5xy+11zx-8yz-10z^2\}$$

$$= 12xy+9y^2-6yz-5x^2+5xy-11zx+8yz+10z^2$$

$$= -5x^2+9y^2+10z^2+17xy+2yz-11zx$$

Multiplication of Two Polynomials

Suppose you want to buy $(2x + y)$ metres of rope at the rate of Rs $(a - 3b)$ per metre.

Can you calculate the amount of money you require?

The amount you require is $(2x + y) \times (a - 3b)$.

Now, how will you carry out this type of multiplication?

The above expression is the multiplication of a binomial with a binomial. Let us see how we will multiply a binomial with a binomial with the help of the following video.

In the video, we have multiplied binomial with binomial in **horizontal arrangement**. Let us now multiply the binomials $(3x - y)$ and $(x + 3y)$ in **vertical arrangement**.

Here, first multiply $3x - y$ with $3y$ and then multiply $3x - y$ with x. After doing so, add the like terms as shown below:

$$\begin{array}{rrr} & 3x & -y \\ \times & x & +3y \\ \hline & 9xy & -3y^2 \\ +3x^2 & -xy & \\ \hline 3x^2 & +8xy & -3y^2 \\ \hline \end{array}$$

The process of multiplying a binomial with a trinomial is not too different from that of multiplying two binomials. Let us learn more about it.

In the video, we have multiplied binomial with trinomial in **horizontal arrangement**. Let us now multiply the binomial ($x + y$) with trinomial ($2x + 3y + 1$) in **vertical arrangement**.

$$\begin{array}{rr} & 2x + 3y + 1 \\ \times & x + y \\ \hline & 2xy + 3y^2 + y \\ +2x^2 + 3xy + x & \\ \hline 2x^2 + 5xy + 3y^2 + x + y & \\ \hline \end{array}$$

Thus, we can perform multiplication of binomials with binomials and trinomials using any of the horizontal or vertical arrangement method.

Let us now solve examples based on the above concepts.

Example 1:

Multiply the following using horizontal and vertical arrangement:

(a) 2 ($x + y$) and $x - 3y$

(b) ($l + 3m$) and ($l + 6m + 7n$)

Solution:

(a) Horizontal arrangement:

$2(x + y) = 2x + 2y$

Now, we have to multiply ($2x + 2y$) and $x - 3y$.

$(2x + 2y) \times (x - 3y) = 2x \times (x - 3y) + 2y \times (x - 3y)$

(Using distributive property)

$= 2 \times x \times x - 2 \times 3 \times x \times y + 2 \times x \times y - 2 \times 3 \times y \times y$

$= 2x^2 - 6xy + 2xy - 6y^2$

$= 2x^2 - 4xy - 6y^2$ (Combining the like terms)

Vertical arrangement:

$$\begin{array}{r} x + y \\ \times \quad 2 \\ \hline 2x + 2y \\ \hline \end{array} \qquad \begin{array}{r} x - 3y \\ \times \quad 2x + 2y \\ \hline 2xy - 6y^2 \\ +2x^2 - 6xy \quad \\ \hline 2x^2 - 4xy - 6y^2 \\ \hline \end{array}$$

(b) Horizontal arrangement:

$(l + 3m) \times (l + 6m + 7n)$

$= l \times (l + 6m + 7n) + 3m \times (l + 6m + 7n)$

(Using distributive property)

$= l \times l + l \times 6m + l \times 7n + 3m \times l + 3m \times 6m + 3m \times 7n$

$= l^2 + 6lm + 7ln + 3ml + 18m^2 + 21mn$

$= l^2 + 9ml + 7ln + 21mn + 18m^2$

[Combining the like terms $6lm$ and $3ml$]

Vertical arrangement:

$$\begin{array}{r} l + 6m + 7n \\ \times \quad l + 3m \\ \hline 3lm + 18m^2 + 21mn \\ +l^2 + 6lm + 7nl \qquad\qquad \\ \hline l^2 + 9lm + 18m^2 + 21mn + 7nl \\ \hline \end{array}$$

Example 2:

Simplify the following:

(a) $(x^2 + y^2) \times (x^3 + y + z^2) + 2(z^2 + 5z)$

(b) $(l - m)(l + m) + (m - n)(m + n) - (l - n)(n + l)$

(c) $(x - 4)(y - 4) - 16$

(d) $(a + b + c)(a - b + c)$

Solution:

(a) $(x^2 + y^2) \times (x^3 + y + z^2) + 2(z^2 + 5z)$

$= x^2(x^3 + y + z^2) + y^2(x^3 + y + z^2) + 2(z^2 + 5z)$

(Using distributive property)

$= x^2 \times x^3 + x^2 \times y + x^2 \times z^2 + y^2 \times x^3 + y^2 \times y + y^2 \times z^2 + 2 \times z^2 + 2 \times 5z$

$= x^5 + x^2y + x^2z^2 + x^3y^2 + y^3 + y^2z^2 + 2z^2 + 10z$

(b) $(l - m)(l + m) + (m - n)(m + n) - (l - n)(n + l)$

$= l(l + m) - m(l + m) + m(m + n) - n(m + n) - l(n + l) + n(n + l)$

(Using distributive property)

$= l^2 + lm - ml - m^2 + m^2 + mn - nm - n^2 - ln - l^2 + n^2 + nl$

$= (l^2 - l^2) + (lm - lm) + (-m^2 + m^2) + (mn - mn) + (-n^2 + n^2) + (-ln + ln)$

{*lm* and *ml*, *ln* and *nl*, *mn* and *nm* are like terms}

$= 0$

(c) $(x - 4) \times (y - 4) - 16$

$= x \times (y - 4) - 4(y - 4) - 16$ (Using distributive property)

$= xy - 4x - 4y + 16 - 16$

$= xy - 4x - 4y$

(d) $(a + b + c)(a - b + c)$

$= a(a - b + c) + b(a - b + c) + c(a - b + c)$ (Using distributive property)

$= a^2 - ab + ac + ba - b^2 + bc + ca - cb + c^2$ (Combining the like terms)

$= a^2 + c^2 - b^2 + 2ac$

Division of Polynomials by Monomials Using Factorization Method

Division is exactly the opposite of multiplication. For example, if $4 \times 5 = 20$, then it is also correct to say that $20 \div 5 = 4$ and $20 \div 4 = 5$.

We can use the same concept to divide algebraic expressions.

Let us try to divide the expression $3x$ by 3 and x.

We can factorize $3x$ as $3 \times x$.

This means that $3x$ is a product of 3 and x.

$\therefore 3x \div 3 = x$ and $3x \div x = 3$

Each of the expressions i.e., 3, x, and $3x$ is a monomial. Hence, these were examples of division of monomials by monomials.

When we divide a monomial by another monomial, we first need to factorise each monomial. Next, we divide the monomial by cancelling the common factors.

To understand the concept better, look at the following video.

How do we divide a polynomial by a monomial? For this, we have two methods.

To understand both the methods, look at the following video.

Let us discuss some more examples based on the two methods we just discussed.

Example 1:

Divide the following expressions:

(i). $27x^2y^2z \div 27xyz$

(ii). $144pq^2r \div (-48qr)$

Solution:

(i). $27x^2y^2z \div 27xyz$

Dividend $= 27x^2y^2z$

$= 3 \times 3 \times 3 \times x \times x \times y \times y \times z$

Divisor = $27xyz$

$= 3 \times 3 \times 3 \times x \times y \times z$

$$\Rightarrow \frac{27x^2y^2z^2}{27xyz} = \frac{3\times3\times3\times x\times x\times y\times y\times z}{3\times3\times3\times x\times y\times z}$$

$= xy$

$\therefore 27x^2y^2z \div 27xyz = xy$

(ii). $144pq^2r \div (-48qr)$

Dividend = $144\ pq^2r$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times p \times q \times q \times r$

Divisor = $-48\ qr$

$= -2 \times 2 \times 2 \times 2 \times 3 \times q \times r$

$$\Rightarrow \frac{144pq^2r}{-48qr} = \frac{2\times2\times2\times2\times3\times3\times p\times q\times q\times r}{-2\times2\times2\times2\times3\times q\times r}$$

$$= -3\times p\times q$$

$$= -3pq$$

$\therefore 144pq^2r \div (-48qr) = -3pq$

Example 2:

Carry out the following divisions:

(i). $(x^3y^6 - x^6y^3) \div x^3y^3$

(ii). $26xy\ (x + 5) \div 13xy$

(iii). $27\ (-a^2bc + ab^2c - abc^2) \div (-3abc)$

Solution:

(i). $(x^3y^6 - x^6y^3) \div x^3y^3$

Dividend = $x^3y^6 - x^6y^3$

= $x^3y^3(y^3 - x^3)$

Divisor = x^3y^3

$$\Rightarrow \frac{x^3y^6 - x^6y^3}{x^3y^3} = \frac{x^3y^3\left(y^3 - x^3\right)}{x^3y^3}$$

= $y^3 - x^3$

Another method of simplifying this expression is

$$\frac{x^3y^6 - x^6y^3}{x^3y^3} = \frac{x^3y^6}{x^3y^3} - \frac{x^6y^3}{x^3y^3}$$
$$= y^3 - x^3$$

∴ $(x^3y^6 - x^6y^3) \div x^3y^3 = y^3 - x^3$

(ii). Dividend = $26xy(x + 5)$

= $2 \times 13 \times x \times y \times (x + 5)$

Divisor = $13xy$

= $13 \times x \times y$

$$\therefore \frac{26xy(x+5)}{13xy} = \frac{2\times13\times x\times y\times(x+5)}{13\times x\times y}$$

= $2(x + 5)$

= $2x + 10$

∴ $26xy(x + 5) \div 13xy = 2x + 10$

(iii). $27\left(-a^2bc + ab^2c - abc^2\right) \div (-3abc)$

$$= \frac{27\left(-a^2bc + ab^2c - abc^2\right)}{\left(-3abc\right)}$$

$$= \frac{-27a^2bc + 27ab^2c - 27abc^2}{-3abc}$$

$$= \left(\frac{-27a^2bc}{-3abc}\right) + \left(\frac{27ab^2c}{-3abc}\right) + \left(\frac{-27abc^2}{-3abc}\right)$$

$$= 9a - 9b + 9c$$

Division of Polynomials by Polynomials

We know how to divide a given polynomial by a monomial. But how do we go about dividing a polynomial by another polynomial. The given video will help you understand this type of division.

Let us discuss some more examples, in which we will not only divide binomials, but also divide other types of polynomials.

Example 1:

Factorise the following expressions and divide as directed.

(i) $6ab\,(9a^2 - 16b^2) \div 2ab\,(3a + 4b)$

(ii) $(x^2 - 14x - 32) \div (x + 2)$

(iii) $36abc\,(5a - 25)\,(2b - 14) \div 24(a - 5)\,(b - 7)$

Solution:

(i) We can factorise the given expressions as

$6ab\,(9a^2 - 16b^2) = 2 \times 3 \times a \times b\,[(3a)^2 - (4b)^2]$

$= 2 \times 3 \times a \times b\,[(3a + 4b)\,(3a - 4b)]$ $[a^2 - b^2 = (a + b)(a - b)]$

And, $2ab\,(3a + 4b) = 2 \times a \times b \times (3a + 4b)$

$$\Rightarrow \frac{6ab\left(9a^2 - 16b^2\right)}{2ab\left(3a + 4b\right)} = \frac{2 \times 3 \times a \times b \times \left(3a + 4b\right) \times \left(3a - 4b\right)}{2 \times a \times b \times \left(3a + 4b\right)}$$

$= 3 \times (3a - 4b)$

$= 9a - 12b$

$\therefore 6ab\,(9a^2 - 16b^2) \div 2ab\,(3a + 4b) = 9a - 12b$

(ii) We can factorise the given expression as

$x^2 - 14x - 32 = x^2 - (16 - 2)\,x - 32$

$= x^2 - 16x + 2x - 32$

$= x\,(x - 16) + 2\,(x - 16)$

$= (x - 16)\,(x + 2)$

$= (x + 2) = (x + 2)$

$$\Rightarrow \frac{x^2 - 14x - 32}{x + 2} = \frac{(x-16)(x+2)}{(x+2)}$$

$= x - 16$

$\therefore (x^2 - 14x - 32) \div (x + 2) = (x - 16)$

(iii) We can factorise the given expression as

$$\begin{aligned} 36abc(5a-25)\ (2b-14) &= 2\times2\times3\times3\times a\times b\times c(5\times a-5\times5)\ (2\times b-2\times7) \\ &= 2\times2\times3\times3\times5\times a\times b\times c\ (a-5)\times2(b-7) \\ &= 2\times2\times2\times3\times3\times5\times a\times b\times c\ (a-5)\ (b-7) \end{aligned}$$

$$24(a-5)(b-7) = 24\ (a-5)\ (b-7) = 2\times2\times2\times3(a-5)\ (b-7)$$

$$\therefore \frac{36abc(5a-25)\ (2a-14)}{24(a-5)(b-7)}$$

$$= \frac{2\times2\times2\times3\times3\times5\times a\times b\times c(a-5)(b-7)}{2\times2\times2\times3(a-5)(b-7)}$$

$$= 3\times5\times a\times b\times c$$

$$= 15abc$$

Hence, $36abc\,(5a - 25)\,(2b - 14) \div 24(a - 5)\,(b - 7) = 15abc$

Division Of Polynomials by Polynomials (Degree More than 1) Using Long Division Method

All of us have already studied the method of long division of polynomials by other polynomials in earlier classes. But there, our divisor was just of degree one. Now we will revise long division method of polynomials with other polynomials and extend the discussion to do the division when the divisor is of a higher degree.

Let us discuss the method of division by taking an example. Let us divide ($x^3 - 7x + 6$) by ($x^2 + 2x - 3$).

Now let us solve some examples to practice the above method.

Example 1:

Divide the polynomial by monomial and write the quotient and remainder.

(i) $36n^5 - 3n^3 + 12n^2 \div 9n^2$

(ii) $7p^7 + 3p^5 - 42p^3 + 10 \div 21p^3$

Solution:

(i) $36n^5 - 3n^3 + 12n^2$ can be divided by $9n^2$ using long division method as follows:

$$
\begin{array}{r|l}
 & 4n^3 - \frac{1}{3}n + \frac{4}{3} \\
\hline
9n^2 & 36n^5 - 3n^3 + 12n^2 \\
 & \underline{\overset{-}{}36n^5} \\
 & \quad -3n^3 + 12n^2 \\
 & \quad \underline{-3n^3} \quad (+) \\
 & \qquad\quad 12n^2 \\
 & \qquad\quad \underline{12n^2} \quad (-) \\
 & \qquad\qquad 0
\end{array}
$$

Therefore,

Quotient = $4n^3 - \frac{1}{3}n + \frac{4}{3}$

Remainder = 0

(ii) $7p^7 + 3p^5 - 42p^3 + 10$ can be divided by $21p^3$ using long division method as follows:

$$\begin{array}{r|l} & \frac{1}{3}p^4 + \frac{1}{7}p^2 - 2 \\ \hline 21p^3 & 7p^7 + 3p^5 - 42p^3 + 10 \\ & \underline{_{-}7p^7} \\ & \quad 3p^5 - 42p^3 + 10 \\ & \quad \underline{_{-}3p^5} \\ & \qquad -42p^3 + 10 \\ & \qquad \underline{_{+}-42p^3} \\ & \qquad\qquad 10 \end{array}$$

Therefore,

Quotient = $\frac{1}{3}p^4 + \frac{1}{7}p^2 - 2$

Remainder = 10

Example 2:

Check whether ($x^2 - 3x + 2$) is a factor of ($x^3 - 7x^2 + 14x - 8$) or not.

Solution:

($x^2 - 3x + 2$) will be a factor of ($x^3 - 7x^2 + 14x - 8$), if the polynomial ($x^3 - 7x^2 + 14x - 8$) on dividing by ($x^2 - 3x + 2$) gives remainder zero.

The division has been shown below:

$$\begin{array}{r|l} & x - 4 \\ \hline x^2 - 3x + 2 & x^3 - 7x^2 + 14x - 8 \\ & \underline{x^3 - 3x^2 + 2x} \\ & _{-}\quad_{+}\qquad_{-} \\ & \quad -4x^2 + 12x - 8 \\ & \quad \underline{-4x^2 + 12x - 8} \\ & \quad_{+}\qquad_{-}\qquad_{+} \\ & \qquad 0 \end{array}$$

The remainder is zero.

∴ $(x^2 - 3x + 2)$ is a factor of $(x^3 - 7x^2 + 14x - 8)$.

Example 3:

Find the quotient and remainder when $(15x - 4 + x^3 - 6x^2)$ is divided by

$(x^2 - 3x + 2)$.

Solution:

Firstly, we arrange the terms of the dividend and the divisor in the standard form.

The dividend in the standard form will be

$x^3 - 6x^2 + 15x - 4$.

The divisor is already in standard form.

The division has been shown below:

$$
\begin{array}{r|l}
 & x-3 \\
\hline
x^2-3x+2 & x^3-6x^2+15x-4 \\
 & x^3-3x^2+2x \\
 & -\quad+\qquad- \\
\hline
 & -3x^2+13x-4 \\
 & -3x^2+9x-6 \\
 & +\qquad-\qquad+ \\
\hline
 & 4x+2 \\
\hline
\end{array}
$$

Therefore,

Quotient = $x - 3$

Remainder = $4x + 2$

Example 4:

$\frac{\sqrt{5}}{2}$ and $-\frac{\sqrt{5}}{2}$ are the zeroes of the polynomial $16x^4 - 64x^3 + 40x^2 + 80x - 75$. Find the other zeroes of the polynomial.

Solution:

Let $p(x) = 16x^4 - 64x^3 + 40x^2 + 80x - 75$

$\frac{\sqrt{5}}{2}$ and $-\frac{\sqrt{5}}{2}$ are two zeroes of $p(x)$.

$\Rightarrow$ $\left(x - \frac{\sqrt{5}}{2}\right)$ and $\left(x + \frac{\sqrt{5}}{2}\right)$ are the factors of $p(x)$

$\Rightarrow$ $x^2 - \frac{5}{4}$ is a factor of $p(x)$

To find the other factors, we have to divide $p(x)$ by $x^2 - 5/4$ as follows:

$$
\begin{array}{r|l}
 & 16x^2 - 64x + 60 \\
x^2 - \frac{5}{4} & 16x^4 - 64x^3 + 40x^2 + 80x - 75 \\
 & 16x^4 \qquad\quad - 20x^2 \\
 & (-) \qquad\quad\;\; (+) \\
\hline
 & -64x^3 + 60x^2 + 80x - 75 \\
 & -64x^3 \qquad\quad + 80x \\
 & (+) \qquad\quad\;\; (-) \\
\hline
 & 60x^2 - 75 \\
 & 60x^2 - 75 \\
 & (-) \qquad (+) \\
\hline
 & 0
\end{array}
$$

Now, $16x^2 - 64x + 60$ is the factor of $p(x)$.

So, we can write $16x^2 - 64x + 60 = 4(4x^2 - 16x + 15)$

$= 4(4x^2 - 10x - 6x + 15)$ (Splitting the middle term)

$= 4\{2x(2x - 5) - 3(2x - 5)\}$

$= 4(2x - 5)(2x - 3)$

To find the zeroes, put $4(2x - 5)(2x - 3) = 0$

We have $2x-5=0 \text{ or } 2x-3=0$

$$x=\frac{5}{2} \text{ or } x=\frac{3}{2}$$

Thus, the remaining zeroes of the polynomial are x = 5/2 and 3/2.

Simplification of Expressions Involving Brackets

An algebraic expression may contain some brackets, namely line bracket, common bracket, curly bracket, or rectangular brackets, and some mathematical operations.

An expression enclosed within a bracket is considered as a single quantity even though it may consist of many terms.

Therefore, for simplifying an expression, we remove the bracket by the following rules.

(i) If '+' sign occurs before a bracket, then the signs of all the terms inside the bracket do not change.

(ii) If '−' sign occurs before a bracket, then the signs of all the terms inside the bracket change.

Brackets are removed in order of

(a) line brackets (b) common brackets

(c) curly brackets and lastly (d) rectangular brackets

It can be noted that the above rules apply when we insert a bracket.

Let us see how we simplify an algebraic expression by taking an example.

$$x-\left[2y+2\left\{y-\left(z-\overline{x+y}\right)\right\}\right]$$

Change the signs of the terms inside the line bracket as '−' sign occurs before the line bracket.

$$=x-\left[2y+2\left\{y-\left(z-x-y\right)\right\}\right]$$

Similarly, change the signs of terms inside the common bracket as '−' sign occurs before the common bracket.

$= x - \left[2y + 2\left\{y - z + x + y\right\}\right] = x - \left[2y + 2\left\{2y - z + x\right\}\right]$

Signs of terms inside the curly bracket remain unchanged as '+' sign occurs before it.

$= x - \left[2y + 4y - 2z + 2x\right] = x - \left[6y - 2z + 2x\right]$

Change the signs of terms inside rectangular bracket as '−' sign occurs before it.

$= x - 6y + 2z - 2x$
$= -x - 6y + 2z$

Example 1:

Simplify the following:

(a) $2y - \left\{y - \left(x - \overline{y + z}\right)\right\}$

(b) $4\left(2p - q\right) - 3\left(p - \overline{q + 2p}\right)$

(c) $3e^2 - \left[d^2 - 4\left\{f^2 - \left(2e^2 - \overline{f^2 + d^2}\right)\right\}\right]$

Solution:

(a) $2y - \left\{y - \left(x - \overline{y + z}\right)\right\}$

= 2y − {y − (x − y − z)}[Line bracket is removed]

= 2y − {y − x + y + z} [Line bracket is removed]

= 2y − {2y − x + z}

= 2y − 2y + x − z = x − z

(b) $4\left(2p - q\right) - 3\left(p - \overline{q + 2p}\right)$

= 8p − 4q − 3 (p − q − 2p)[One common bracket is removed and line bracket is removed in the other common bracket]

$= 8p - 4q - 3(-q - p)$

$= 8p - 4q + 3q + 3p$[Common bracket is removed]

$= 11p - q$

(c) $3e^2 - \left[d^2 - 4\left\{f^2 - \left(2e^2 - \overline{f^2 + d^2}\right)\right\}\right]$

$= 3e^2 - [d^2 - 4\{f^2 - (2e^2 - f^2 - d^2)\}]$[Line bracket is removed]

$= 3e^2 - [d^2 - 4\{f^2 - 2e^2 + f^2 + d^2\}]$[Common bracket is removed]

$= 3e^2 - [d^2 - 4\{2f^2 - 2e^2 + d^2\}]$

$= 3e^2 - [d^2 - 8f^2 + 8e^2 - 4d^2]$[Curly bracket is removed]

$= 3e^2 - [-3d^2 - 8f^2 + 8e^2]$

$= 3e^2 + 3d^2 + 8f^2 - 8e^2$[Rectangular bracket is removed]

$= 3d^2 - 5e^2 + 8f^2$