Rational Numbers

Concepts Related To Rational Numbers

You have studied fractional numbers in your earlier classes. Some examples of fractional numbers are $\frac{1}{2}, \frac{-4}{7}, \frac{22}{27}$.

These numbers are also known as **rational numbers.**

What comes first to your mind when you hear the word **rational?**

Yes, you are right. It is something related to the ratios.

The ratio 4:5 can be written as 4/5**,** which is a rational number. In ratios, the numerator and denominator both are positive numbers while in rational numbers, they can be negative also.

Thus, rational numbers can be defined as follows.

"**Any number which can be expressed in the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$*,* is called a rational number.**"

For example, 15/19 is a rational number in which the numerator is 15 and the denominator is 19.

Now, is −34 a rational number?

Yes, it is a rational number. −34 can be written as -34/1. It is in the form of p/q and $q \neq 0$.

Thus, we can say that **every integer is a rational number.**

Now, consider the following decimal numbers.

1.6, 3.49, and 2.5

These decimal numbers are also rational numbers as these can be written as $\frac{16}{10}, \frac{349}{100}$, and $\frac{25}{10}$

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If in a rational number, either the numerator or the denominator is a negative integer, then the rational number is negative.

For example, $\frac{-5}{12}$ and $\frac{6}{-7}$ are **negative rational numbers.**

If the numerator and the denominator both are either positive integers or negative integers, then the rational number is positive.

For example, $\frac{-9}{-4}$ and $\frac{56}{5}$ **are positive rational numbers.**

Conventions used for writing a rational number:

We know that in a rational number, the numerator and denominator both can be positive or negative.

Conventionally, rational numbers are written with positive denominators.

For example, –9 can be represented in the form of a rational number as $\frac{-9}{1}$ or $-\frac{9}{1}$ or $\frac{9}{-1}$, but generally we do not write the denominator negative and thus, $\frac{9}{-1}$ is eliminated. So, according to the convention, –9 can be represented in the form of a rational number as $\frac{-9}{1}$ or $-\frac{9}{1}$.

Equality relation for rational numbers:

For any four non-zero integers *p*, *q*, *r* and *s*, we have

$\frac{p}{q} = \frac{r}{s}$ if $ps = qr$

Order relation for rational numbers:

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers such that $q > 0$ and $s > 0$ then it can be said that $\frac{p}{q} > \frac{r}{s}$ if $ps > qr$.

Absolute Value of a Rational Number:

The absolute value of a rational number is its numerical value regardless of its sign. The absolute value of a rational number pqpq is denoted as |p/q|

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Therefore, $\left|-\frac{3}{2}\right| = \frac{3}{2}$, $\left|\frac{12}{-7}\right| = \frac{12}{7}$ etc.

Note: The absolute value of any rational number is always non-negative.

Now, let us go through the given example.

Example:

Write each of the following rational numbers according to the convention.

i) $\frac{8}{-15}$

ii) $\frac{-1131}{-729}$

Solution:
According to the convention used in rational numbers, the denominator must be a positive number.

Let us now write the given numbers according to the convention.

i)
In the number $\frac{8}{-15}$, denominator is negative.

We have,

$$\frac{8}{-15} = \frac{-8}{15} = -\frac{8}{15}$$

According to convention, the given number should be written as $\frac{-8}{15}$ or $-\frac{8}{15}$.

ii)
In the number $\frac{-1131}{-729}$, denominator is negative.

We have,

$$\frac{-1131}{-729} = \frac{1131}{729}$$

According to convention, the given number should be written as $\frac{1131}{729}$.

Example:

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Find the absolute value of the following:

(i) $\left|-\frac{121}{71}\right|$

(ii) $\left|\frac{12}{19}\right|$

Solution:

(i) Absolute value = $\left|-\frac{121}{71}\right| = \frac{121}{71}$

(ii) Absolute value = $\left|\frac{12}{19}\right| = \frac{12}{19}$

Addition And Subtraction Of Rational Numbers

Many times instead of adding whole numbers, we have to add rational numbers in which we have to follow a specific method. This method is explained in the following example.

Vikram went to a market and bought $\frac{3}{4}$ kg apples and $\frac{2}{4}$ kg mangoes.

How much fruits did he buy?

Hence, we can make the following conclusions.

1) Addition of rational numbers having the same denominator

When two rational numbers having the same denominator are added, only their numerators are added keeping the denominator same as that of the given numbers and the resultant number is reduced to the simplest form, if possible.

2) Addition of rational numbers with different denominators

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To add the rational numbers having different denominators, first the rational numbers are converted to their equivalent rational numbers having common denominator. Then, the numerators are added keeping the common denominator same and the resultant number is reduced to the simplest form, if possible.

Now, let us add $\frac{-3}{5}$ and $\frac{3}{5}$.

Now, let us learn about the **subtraction** of rational numbers.

Rahul gave 2/3 of a chocolate to Sonu and Sonu gave 1/4 of it to his younger brother.

Hence, we can conclude that the subtraction of a rational number from another rational number is same as the addition of the additive inverse of the rational number that is being subtracted to the other rational number.

Let us find the value of $\left(23\frac{1}{3}-5\frac{3}{4}-6\frac{1}{6}\right)$.

$$23\frac{1}{3}-5\frac{3}{4}-6\frac{1}{6}=\frac{70}{3}-\frac{23}{4}-\frac{37}{6}$$
$$=\frac{70}{3}+\frac{(-23)}{4}+\frac{(-37)}{6}$$

Now, we have to find the L.C.M. of 3, 4, and 6.

2	3 4 6
2	3 2 3
3	3 1 3
	1 1 1

The L.C.M. of 3, 4, and 6 is 2 × 2 × 3 = 12

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Now, $\frac{70}{3}+\frac{(-23)}{4}+\frac{(-37)}{6}$

$$=\frac{70\times4}{3\times4}+\frac{(-23)\times3}{4\times3}-\frac{(-37)\times2}{6\times2}$$
$$=\frac{280}{12}+\frac{(-69)}{12}+\frac{(-74)}{12}$$
$$=\frac{280-69-74}{12}$$
$$=\frac{137}{12}$$
$$=\frac{(12\times11)+5}{12}$$
$$=11\frac{5}{12}$$

Opposite Rational Numbers: If the sum of two rational numbers is 0 then the numbers are called opposite numbers. Also, each number is said to be the additive inverse of the other.

For example, –11 + 11 = 0. Thus, –11 and 11 are opposite numbers as well as these are additive inverse of each other.

Similarly, $-\frac{2}{3}+\frac{2}{3}=0$. Thus, $-\frac{2}{3}\text{ and }\frac{2}{3}$ are opposite numbers. Also, these are additive inverse of each other.

Let us solve a few more examples to understand the concept better.

Example 1:

Find the value of $\left(\frac{13}{4}+\frac{29}{3}+\frac{11}{2}\right)$**.**

Solution:

Since the denominators of rational numbers are different, we have to take the L.C.M. of the denominators.

The L.C.M. of 4, 3, and 2 is 12.

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$$=\frac{13}{4}+\frac{29}{3}+\frac{11}{2}$$
$$=\frac{13\times 3}{4\times 3}+\frac{29\times 4}{3\times 4}+\frac{11\times 6}{2\times 6}$$
$$=\frac{39}{12}+\frac{116}{12}+\frac{66}{12}$$
$$=\frac{39+116+66}{12}=\frac{221}{12}$$
$$=\frac{(12\times 18)+5}{12}=18+\frac{5}{12}$$
$$=18\frac{5}{12}$$

Thus, the value of the given expression $\left(\frac{13}{4}+\frac{29}{3}+\frac{11}{2}\right)$ is $18\frac{5}{12}$.

Example 2:

Write the additive inverse of each of the following numbers.

(a) 625 **(b)** $-\frac{5}{12}$ **(c)** $\frac{11}{25}$

(d) –15 **(e) 21** **(f)** $\frac{46}{45}$

Solution:

(a)

We have,

625 + (–625) = 625 – 625 = 0

Thus, –625 is the additive inverse of 625.

(b)

We have,

$$-\frac{5}{12}+\frac{5}{12}=0$$

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Thus, $\frac{5}{12}$ is the additive inverse of $-\frac{5}{12}$.

(c)

We have,

$\frac{11}{25}+\left(-\frac{11}{25}\right)=\frac{11}{25}-\frac{11}{25}=0$

Thus, $-\frac{11}{25}$ is the additive inverse of $\frac{11}{25}$.

(d)

We have,

–15 + 15 = 0

Thus, 15 is the additive inverse of –15.

(e)

We have,

21 + (–21) = 21 – 21 = 0

Thus, 21 is the additive inverse of –21.

(f)

We have,

$\frac{46}{45}+\left(-\frac{46}{45}\right)=\frac{46}{45}-\frac{46}{45}=0$

Thus, $-\frac{46}{45}$ is the additive inverse of $\frac{46}{45}$.

Example 3:

The distance between two stations A and B is $3\frac{5}{6}$ km, that of B and C is $6\frac{3}{5}$ km, and that of C and D is $5\frac{1}{2}$ km. The stations are in a straight line as shown in the given figure. Find the distance between A and D. If Ritu went from A to D and

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returned on the same path through a distance of $2\frac{1}{3}$ km, then how much distance is she away from A?

Solution:

Distance between A and D = Distance between A and B + Distance between B and C +

Distance between C and D

$$=\left(3\frac{5}{6}+6\frac{3}{5}+5\frac{1}{2}\right)\text{ km}$$

$$=\left(\frac{23}{6}+\frac{33}{5}+\frac{11}{2}\right)\text{ km}$$

Now, in order to add the above rational numbers, we have to find the L.C.M. of 6, 5, and 2 to make their denominators same.

The L.C.M. of 6, 5, and 2 is 30.

Now,

$$\begin{aligned}\frac{23}{6}+\frac{33}{5}+\frac{11}{2} &= \frac{23\times5}{6\times5}+\frac{33\times6}{5\times6}+\frac{11\times15}{2\times15}\\ &= \frac{115}{30}+\frac{198}{30}+\frac{165}{30}\\ &= \frac{115+198+165}{30}\\ &= \frac{478}{30}\end{aligned}$$

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$$= \frac{(15 \times 30) + 28}{30}$$

$$= 15 + \frac{28}{30}$$

$$= 15 + \frac{14}{15}$$

$$= 15\frac{14}{15}$$

Thus, the distance between A and D is $15\frac{14}{15}$ km.

After reaching D, Ritu returned towards A (opposite direction). Therefore, the distance between A and her current position will be obtained by subtracting $2\frac{1}{3}$ km from the distance AD.

Let this distance be *x* km.

Therefore, $x = 15\frac{14}{15}\text{ km} + \left(-2\frac{1}{3}\right)\text{ km} = \frac{239}{15} + \frac{(-7)}{3}\text{ km}$

The L.C.M. of 15 and 3 is 15.

$$\frac{239}{15} + \frac{(-7)}{3} = \frac{239 \times 1}{15 \times 1} + \frac{(-7) \times 5}{3 \times 5} = \frac{239}{15} + \frac{(-35)}{15} = \frac{239 - 35}{15} = \frac{204}{15}$$

$$= \frac{(13 \times 15) + 9}{15}$$

$$= 13 + \frac{9}{15}$$

$$= 13 + \frac{3}{5}$$

$$= 13\frac{3}{5}$$

Thus, the distance *x* is $13\frac{3}{5}$ km.

Example 4:

Two months ago, weight of Raj was $65\frac{1}{2}$ kg. He reduced $3\frac{3}{4}$ kg weight in two months. How much does he weigh now?

Solution:

Weight of Raj two months ago = $65\frac{1}{2}$ kg

Weight reduced = $3\frac{3}{4}$ kg

∴ Present weight = Weight two months ago − Weight reduced

$$=\left(65\frac{1}{2}-3\frac{3}{4}\right)\text{ kg}=\left(\frac{131}{2}-\frac{15}{4}\right)\text{ kg}=\left[\frac{131}{2}+\frac{(-15)}{4}\right]\text{ kg}$$

The L.C.M. of 2 and 4 is 4.

∴ Present weight $=\frac{131}{2}+\frac{(-15)}{4}$

$$=\frac{131\times2}{2\times2}+\frac{(-15)\times1}{4\times1}$$
$$=\frac{262}{4}+\frac{(-15)}{4}$$
$$=\frac{262-15}{4}$$
$$=\frac{247}{4}$$
$$=\frac{(61\times4)+3}{4}$$
$$=61+\frac{3}{4}$$
$$=61\frac{3}{4}$$

Thus, his present weight is $61\frac{3}{4}$ kg.

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Multiplication And Division Of Rational Numbers

Multiplication and division of rational numbers are required in many real life situations to make the calculations easier. Following examples explain the method used to multiply and divide rational numbers.

Hence, we can conclude the following.

To multiply a rational number by another rational number, the denominator of one rational number is multiplied with the denominator of the other and the numerator of one rational number is multiplied with the numerator of the other. Then, the resultant is simplified, if possible.

The division of a rational number by another rational number is same as the multiplication of the dividend by the reciprocal of the divisor.

Now, what will be $\left(-3\frac{1}{2}\right)\div\frac{3}{7}$?

$$\left(-3\frac{1}{2}\right)\div\frac{3}{7}=\left(-\frac{7}{2}\right)\div\frac{3}{7}=\left(-\frac{7}{2}\right)\times\frac{7}{3}=-\frac{49}{6}$$

Note: Division of a non-zero rational number by itself gives 1.

For example,

$2\div 2=\frac{2}{2}=1,$

$(-10)\div(-10)=\frac{-10}{-10}=1,$

$\frac{4}{5}\div\frac{4}{5}=\frac{4}{5}\times\frac{5}{4}=1$ and

$\left(-\frac{7}{11}\right)\div\left(-\frac{7}{11}\right)=\left(-\frac{7}{11}\right)\times\left(-\frac{11}{7}\right)=1$

Let us solve some examples to understand the concept better.

Example 1:

There are six baskets full of fruits each weighing $1\frac{4}{5}$ kg. What is the total weight of the six baskets?

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Solution:

Number of baskets = 6

Weight of each basket = $1\frac{4}{5}$ kg

Therefore,

Total weight of the six baskets $= 6 \times 1\frac{4}{5}\text{kg}$

$$= \left(6 \times \frac{9}{5}\right)\text{kg}$$
$$= \left(\frac{6 \times 9}{5}\right)\text{kg}$$
$$= \frac{54}{5}\text{kg}$$

Now, converting into standard form, we obtain

$$= \left[\frac{(10 \times 5) + 4}{5}\right]\text{kg}$$
$$= \left(10 + \frac{4}{5}\right)\text{kg}$$
$$= 10\frac{4}{5}\text{kg}$$

Thus, the total weight of the six baskets is $10\frac{4}{5}\text{kg}$.

Example 2:

Amit bought a pipe of length $4\frac{1}{5}$ m. He cut the pipe into smaller pieces each of length $\frac{3}{5}$ m. How many pieces was the pipe cut into?

Solution:

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It is given that length of each piece = $\frac{3}{5}$ m

Total length of pipe = $4\frac{1}{5}$ m = $\frac{21}{5}$ m

∴ Number of pieces $= \frac{21}{5} \div \frac{3}{5} = \frac{21}{5} \times \frac{5}{3} = \frac{21}{3} = 7$

Thus, the pipe was cut into 7 pieces.

Closure Properties Of Rational Numbers

Consider the two rational numbers as 5/6 and 1/4.

What would we get if we add these two rational numbers, i.e. what is the value of $\frac{5}{6}+\frac{1}{4}$?

$$\frac{5}{6}+\frac{1}{4}=\frac{10+3}{12}$$
$$=\frac{13}{12}, \text{ which is again a rational number.}$$

This means that the sum of two rational numbers 5/6 and 1/4 is a rational number. In other words, we can say that rational numbers are closed under addition.

Is this true for all rational numbers?

Yes. We can try for different rational numbers and see that this property is true for all rational numbers. Thus, we can say that the sum of two rational numbers is again a rational number. In other words, we can say that **rational numbers are closed under addition.** This property of rational numbers is known as the closure property for rational numbers and it can be stated as follows.

"If *a* and *b* are any two rational numbers and *a* + *b* = *c*, then *c* will always be a rational number".

Are rational numbers closed under subtraction also?

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Let us find out.

Consider two rational numbers $\frac{-11}{12}$ and $\frac{7}{8}$.

$$\text{Now, } \left(\frac{-11}{12}\right) - \frac{7}{8} = \frac{-22-21}{24}$$
$$= \frac{-43}{24}, \text{ which is a rational number.}$$

Thus, **rational numbers are closed under subtraction also.**

Closure property of rational numbers under subtraction can be stated as follows.

> **"If a and b are any two rational numbers and a − b = c, then c will always be a rational number".**

Now, let us check whether rational numbers are closed under multiplication also. For this, consider two rational numbers $\frac{3}{7}$ and $\frac{-4}{11}$.

Now, $\frac{3}{7} \times \frac{-4}{11} = \frac{-12}{77}$, which is a rational number.

Thus, **rational numbers are closed under multiplication also.**

Closure property of rational numbers under multiplication can be defined as follows.

> **"If *a* and *b* are any two rational numbers, then *a* × *b* = *c*, then *c* will always be a rational number".**

But rational numbers are not closed under division. If we consider the division of 2/5 by 0, then we will not obtain a rational number.

2/5 ÷ 0 is not a rational number because division of a rational number by zero is not defined.

Thus, we can say that ***rational numbers are not closed under division.***

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We can summarize the above discussed facts as follows.

Rational numbers are closed under addition, subtraction and multiplication.

Rational numbers are not closed under division.

Commutative And Associative Properties Of Rational Numbers

Consider the expression $\left(\frac{5}{18}+\frac{77}{25}\right)+\frac{-5}{18}$.

What will be the value of this expression?

If we try to find the value of this expression by usually adding the terms in the bracket first and then by adding the result so obtained to the third term, then it will take a long time. Therefore here, we can make use of commutative and associative properties of addition of rational numbers to make our calculation simpler.

Let us first study these properties for rational numbers and then we will find the value of the above expression.

The commutative property of rational numbers over addition can be stated as follows.

"If *a* and *b* are any two rational numbers, then *a* + *b* = *b* + *a*".

For example, consider the rational numbers $\frac{5}{14}$ and $\frac{-7}{12}$.

$$\frac{5}{14}+\left(\frac{-7}{12}\right)=\frac{30-49}{84}=\frac{-19}{84}$$

$$\left(\frac{-7}{12}\right)+\frac{5}{14}=\frac{-49+30}{84}=\frac{-19}{84}$$

$$\therefore \frac{5}{14}+\left(\frac{-7}{12}\right)=\left(\frac{-7}{12}\right)+\frac{5}{14}$$

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Here, the numbers $\frac{5}{14}$ and $\frac{-7}{12}$ are arbitrary, therefore we can say that **rational numbers are commutative under addition**.

The associative property of rational numbers over addition can be stated as follows.

"**If *a*, *b,* and *c* are any three rational numbers, then *a* + (*b* + *c*) = (*a* + *b*) + *c*".**

For example: consider the rational numbers $\frac{1}{2}, \frac{-2}{3}$, and $\frac{1}{3}$.

Let us first find the value of the expressions $\frac{1}{2}+\left[\left(\frac{-2}{3}\right)+\frac{1}{3}\right]$ and $\left[\frac{1}{2}+\left(\frac{-2}{3}\right)\right]+\frac{1}{3}$.

$$\frac{1}{2}+\left[\left(\frac{-2}{3}\right)+\frac{1}{3}\right]=\frac{1}{2}+\left[\frac{(-2)+1}{3}\right]$$
$$=\frac{1}{2}-\frac{1}{3}=\frac{3-2}{6}=\frac{1}{6}$$

$$\left[\frac{1}{2}+\left(\frac{-2}{3}\right)\right]+\frac{1}{3}=\left[\frac{3-4}{6}\right]+\frac{1}{3}$$
$$=\frac{-1}{6}+\frac{1}{3}=\frac{-1+2}{6}=\frac{1}{6}$$

∴ We have $\frac{1}{2}+\left[\left(\frac{-2}{3}\right)+\frac{1}{3}\right]=\left[\frac{1}{2}+\left(\frac{-2}{3}\right)\right]+\frac{1}{3}$

Here, the numbers $\frac{1}{2}, \frac{-2}{3}$, and $\frac{1}{3}$ are arbitrary, therefore we can say that **rational numbers are associative under addition.**

Now, let us go back to our previous problem with which we started our discussion.

We can write $\left(\frac{5}{18}+\frac{77}{25}\right)+\frac{-5}{18}$ = $\left(\frac{77}{25}+\frac{5}{18}\right)+\frac{-5}{18}$ (using commutative property)

$$= \frac{77}{25} + \left[\frac{5}{18} + \left(\frac{-5}{18}\right)\right] \qquad \text{(using associative property)}$$

$$= \frac{77}{25} + \left(\frac{5}{18} - \frac{5}{18}\right) = \frac{77}{25} + 0 = \frac{77}{25}$$

Thus, in this way, we can make use of commutative and associative properties of rational numbers to make our calculations easier and simpler.

Let us know about commutative and associative properties of rational numbers for other operations such as multiplication, division etc.

Rational numbers are commutative and associative under multiplication also.

We can write these properties for multiplication as follows.

If *a*, *b* and *c* are any three rational numbers, then

(i) $a \times b = b \times a$ (commutative property)

(ii) $a \times (b \times c) = (a \times b) \times c$ (associative property)

Rational numbers are not commutative and associative under subtraction.

For this, let us see the following example.

Consider the rational numbers $\frac{4}{7}$ and $\frac{5}{7}$.

$$\text{Now, } \frac{4}{7} - \frac{5}{7} = \frac{4-5}{7} = \frac{-1}{7}$$

$$\frac{5}{7} - \frac{4}{7} = \frac{5-4}{7} = \frac{1}{7}$$

Therefore, we see that $\frac{4}{7} - \frac{5}{7} \neq \frac{5}{7} - \frac{4}{7}$

Thus, **rational numbers are not commutative under subtraction.**

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In the same way, we can check that rational numbers are not associative under subtraction.

Now, let us check whether rational numbers are commutative and associative under division.

Let us find the value of $\frac{2}{5} \div \frac{1}{6}$ and $\frac{1}{6} \div \frac{2}{5}$.

$$\frac{2}{5} \div \frac{1}{6} = \frac{2}{5} \times 6 = \frac{12}{5}$$

$$\frac{1}{6} \div \frac{2}{5} = \frac{1}{6} \times \frac{5}{2} = \frac{5}{12}$$

$$\therefore \frac{2}{5} \div \frac{1}{6} \neq \frac{1}{6} \div \frac{2}{5}$$

Thus, we find that **rational numbers are not commutative under division.**

To check associative property, let us consider three rational numbers $\frac{1}{2}, \frac{1}{4}$, and $\frac{-1}{5}$.

$$\text{Now, } \frac{1}{2} \div \left[\frac{1}{4} \div \left(\frac{-1}{5}\right)\right] = \frac{1}{2} \div \left[\frac{1}{4} \times \left(-\frac{5}{1}\right)\right]$$

$$= \frac{1}{2} \div \left[-\frac{5}{4}\right]$$

$$= \frac{1}{2} \times \left(-\frac{4}{5}\right) = -\frac{2}{5}$$

$$\left[\frac{1}{2} \div \frac{1}{4}\right] \div \left(\frac{-1}{5}\right) = \left[\frac{1}{2} \times \frac{4}{1}\right] \div \left(-\frac{1}{5}\right)$$

$$= 2 \times \left(-\frac{5}{1}\right) = -10$$

$$\therefore \frac{1}{2} \div \left[\frac{1}{4} \div \left(\frac{-1}{5}\right)\right] \neq \left[\frac{1}{2} \div \frac{1}{4}\right] \div \left(\frac{-1}{5}\right)$$

Thus, we can say that **rational numbers are not associative under division.**

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We can summarize the above facts as follows.

Rational numbers are commutative under addition and multiplication.

Rational numbers are not commutative under subtraction and division.

Rational numbers are associative under addition and multiplication.

Rational numbers are not associative under subtraction and division.

Let us now look at some more examples.

Example 1:

Fill in the blanks using commutative and associative properties of rational numbers.

1. $\frac{2}{11} + \_\_\_\_\_ = \left(\frac{-1}{17}\right) + \frac{2}{11}$
2. $\left(\frac{-5}{11}\right) \times \frac{17}{12} = \frac{17}{12} \times \_\_\_\_\_$
3. $\left(\frac{-1}{9}\right) + \left[\frac{4}{7} + \frac{18}{192}\right] = \left[\frac{(-1)}{9} + \_\_\_\_\_\right] + \frac{18}{192}$
4. $\_\_\_\_\_ \times \left[\frac{25}{26} \times \frac{1}{5}\right] = \left[\frac{7}{15} \times \frac{25}{26}\right] \times \frac{1}{5}$

Solution:

1. Rational numbers are commutative under addition.

$$\frac{2}{11} + \underline{\left(\frac{-1}{17}\right)} = \left(\frac{-1}{17}\right) + \frac{2}{11}$$

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2. Rational numbers are commutative under multiplication.

$$\left(\frac{-5}{11}\right)\times\frac{17}{12}=\frac{17}{12}\times\underline{\left(\frac{-5}{11}\right)}$$

3. Rational numbers are associative under addition.

$$\left(\frac{-1}{9}\right)+\left[\frac{4}{7}+\frac{18}{192}\right]=\left[\frac{(-1)}{9}+\underline{\frac{4}{7}}\right]+\frac{18}{192}$$

4. Rational numbers are associative under multiplication.

$$\underline{\frac{7}{15}}\times\left[\frac{25}{26}\times\frac{1}{5}\right]=\left[\frac{7}{15}\times\frac{25}{26}\right]\times\frac{1}{5}$$

Example 2:

Find the value of the following expressions using properties of rational numbers.

1. $\frac{7}{5}+\frac{2}{3}+\left(\frac{-8}{25}\right)+\left(\frac{-11}{6}\right)$

2. $\frac{9}{7}\times\left(\frac{-8}{11}\right)\times\left(\frac{-49}{3}\right)\times\frac{33}{64}$

Solution:

1. $\frac{7}{5}+\frac{2}{3}+\left(\frac{-8}{25}\right)+\left(\frac{-11}{6}\right)$

$$=\left[\frac{7}{5}+\left(\frac{-8}{25}\right)\right]+\left[\frac{2}{3}+\left(\frac{-11}{6}\right)\right] \quad \text{(by using associativity and commutativity)}$$

$$=\left[\frac{35-8}{25}\right]+\left[\frac{4-11}{6}\right]$$

$$=\left[\frac{27}{25}\right]+\left[\frac{-7}{6}\right]=\frac{162-175}{150}=-\frac{13}{150}$$

2. $\frac{9}{7}\times\left(\frac{-8}{11}\right)\times\left(\frac{-49}{3}\right)\times\frac{33}{64}$

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$$= \left[\frac{9}{7} \times \left(\frac{-49}{3}\right)\right] \times \left[\left(\frac{-8}{11}\right) \times \frac{33}{64}\right] \quad \text{(by using associativity and commutativity)}$$

$$= [3 \times (-7)] \times \left[-\frac{3}{8}\right]$$

$$= (-21) \times \left(-\frac{3}{8}\right) = \frac{63}{8}$$

Additive And Multiplicative Identities For Rational Numbers

What happens if we add 0 to a rational number or multiply a rational number with 1? Is there anything special that you can note in these 2 operations? Let us find that out.

If the product of two rational numbers is 0, then either of both is 0.

Mathematically, for any two rational numbers *a* and *b*, we have

If *a* × *b* = 0, then either *a* = 0 or *b* = 0

Let us now look at an example based on this concept.

Example:

Fill in the blanks.

1. $\frac{7}{8} \times \left(\frac{-5}{6}\right) \times \_\_\_\_ = \frac{-35}{48}$

2. $\frac{15}{21} + 0 = \_\_\_\_$

3. $\frac{(-4)}{17} \times \_\_\_\_ = \frac{(-4)}{17}$

Solution:

1. $\frac{7}{8} \times \left(\frac{-5}{6}\right) \times \underline{\quad 1 \quad} = \frac{-35}{48}$ (By property of multiplicative identity)

2. $\frac{15}{21} + 0 = \underline{\quad \frac{15}{21} \quad}$ (By property of additive identity)

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3. $\frac{(-4)}{17} \times \underline{\quad 1 \quad} = \frac{(-4)}{17}$ (By property of multiplicative identity)

Additive Inverse and Multiplicative Inverse for Rational Numbers

As integers have additive and multiplicative inverse, similarly, rational numbers also have additive and multiplicative inverse. Go through the following video to understand this concept.

Thus, we define additive inverse and multiplicative inverse as:

"If the sum of two rational numbers is 0, then the numbers are called opposite numbers. Also, each of both numbers is said to be the additive inverse or negative of the other".

"If the multiplication of two numbers gives the result as 1, then the two numbers are called reciprocal or multiplicative inverse of each other".

Let us look at some more examples now.

Example 1:

Find the multiplicative inverse of the following rational numbers.

(i) $\frac{5}{6}$

(ii) $\frac{-15}{17}$

(iii) $\frac{3}{-8}$

(iv) $2\frac{1}{4}$

(v) 0.5

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Solution:

(i) The multiplicative inverse of $\frac{5}{6}$ is $\frac{6}{5}$.

(ii) The multiplicative inverse of $\frac{-15}{17}$ is −1715-1715.

(iii) The multiplicative inverse of $\frac{3}{-8}$ is $\frac{-8}{3}$.

(iv) $2\frac{1}{4}=\frac{9}{4}$

Thus, the multiplicative inverse of $2\frac{1}{4}$ is $\frac{4}{9}$.

(v) $0.5=\frac{5}{10}=\frac{1}{2}$

Thus, the multiplicative inverse of 0.5 is 2.

Example 2:

Write the additive inverse of the following rational numbers.

(i) $\frac{1}{7}$

(ii) $-\frac{14}{15}$

(iii) $\frac{7}{-11}$

(iv) $\frac{-2}{-5}$

Solution:

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(i) The additive inverse of $\frac{1}{7}$ is $-\frac{1}{7}$.

(ii) The additive inverse of $-\frac{14}{15}$ is $\frac{14}{15}$.

(iii) $\frac{7}{-11} = -\frac{7}{11}$

Thus, the additive inverse of $\frac{7}{-11}$ is $\frac{7}{11}$.

(iv) $\frac{-2}{-5} = \frac{2}{5}$

Thus, the additive inverse of $\frac{-2}{-5}$ is $-\frac{2}{5}$.

Distributive Property of Multiplication for Rational Numbers

Consider the rational numbers $\frac{2}{5}, \frac{-3}{7}, \text{and } \frac{1}{4}$.

What will be the values of the expressions $\frac{2}{5} \times \left\{\left(-\frac{3}{7}\right) + \frac{1}{4}\right\}$ **and** $\left\{\frac{2}{5} \times \left(-\frac{3}{7}\right)\right\} + \left\{\frac{2}{5} \times \left(\frac{1}{4}\right)\right\}$ **?**

Let us see.

$$\begin{aligned}\frac{2}{5} \times \left\{\left(-\frac{3}{7}\right) + \frac{1}{4}\right\} &= \frac{2}{5} \times \left\{\frac{(-12)+7}{28}\right\} \\ &= \frac{2}{5} \times \left(-\frac{5}{28}\right) \\ &= \frac{2 \times (-5)}{5 \times 28} \\ &= -\frac{1}{14}\end{aligned}$$

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$$\left\{\frac{2}{5}\times\left(-\frac{3}{7}\right)\right\}+\left(\frac{2}{5}\times\frac{1}{4}\right)=\frac{2\times(-3)}{5\times7}+\frac{2\times1}{5\times4}$$

$$=\frac{(-6)}{35}+\frac{1}{10}$$

$$=\frac{(-12)+7}{70}$$

$$=-\frac{5}{70}$$

$$=-\frac{1}{14}$$

Thus, $\frac{2}{5}\times\left\{\left(\frac{-3}{7}\right)+\frac{1}{4}\right\}=\left\{\frac{2}{5}\times\left(-\frac{3}{7}\right)\right\}+\left(\frac{2}{5}\times\frac{1}{4}\right)$

Here we see that the values of both the expressions are same. This is the distributive property of rational numbers for multiplication over addition. This property is true for all rational numbers and mathematically it can be written as follows.

If *x, y* and *z* are any three rational numbers, then *x* × (*y* + *z*) = (*x* × *y*) + (*x* × *z*).

Does this distributive property hold for multiplication over subtraction also?

Let us check it.

Consider the rational numbers $\frac{4}{7},\frac{-2}{3},\text{and}\,\frac{1}{2}$.

Let us find the value of the expressions $\frac{4}{7}\times\left\{\left(\frac{-2}{3}\right)-\frac{1}{2}\right\}\text{ and }\left\{\frac{4}{7}\times\left(\frac{-2}{3}\right)\right\}-\left(\frac{4}{7}\times\frac{1}{2}\right)$

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$$\frac{4}{7}\times\left\{\left(\frac{-2}{3}\right)-\frac{1}{2}\right\}=\frac{4}{7}\times\left\{\frac{(-2)}{3}-\frac{1}{2}\right\}$$

$$=\frac{4}{7}\times\left\{\frac{(-4)-3}{6}\right\}$$

$$=\frac{4}{7}\times\frac{(-7)}{6}$$

$$=\frac{4\times(-7)}{7\times 6}$$

$$=\frac{-4}{6}$$

$$=\frac{-2}{3}$$

$$\left\{\frac{4}{7}\times\left(\frac{-2}{3}\right)\right\}-\left\{\frac{4}{7}\times\frac{1}{2}\right\}=\left\{\frac{4}{7}\times\frac{(-2)}{3}\right\}-\left\{\frac{4}{7}\times\frac{1}{2}\right\}$$

$$=\frac{(-8)}{21}-\frac{4}{14}$$

$$=\frac{(-8)}{21}-\frac{2}{7}$$

$$=\frac{(-8)-6}{21}$$

$$=\frac{-14}{21}$$

$$=\frac{-2}{3}$$

Thus, $\frac{4}{7}\times\left\{\left(-\frac{2}{3}\right)-\frac{1}{2}\right\}=\left\{\frac{4}{7}\times\left(-\frac{2}{3}\right)\right\}-\left(\frac{4}{7}\times\frac{1}{2}\right)$

Hence, we notice that the result is same from both the ways. Thus, the distributive property of rational numbers for multiplication over subtraction can be written as follows.

"If *x*, *y,* and *z* are any three rational numbers, then *x* × (*y* – *z*) = (*x* × *y*) – (*x* × *z*)".

Let us now look at some more examples.

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Example 1:

Find the value of the following expression using appropriate properties.

$$\left(\frac{1}{4}\times\frac{(-3)}{5}-\frac{4}{5}\times\frac{2}{5}-\frac{5}{6}\times\frac{3}{5}\right)$$

Solution:

$$\frac{1}{4}\times\frac{(-3)}{5}-\frac{4}{5}\times\frac{2}{5}-\frac{5}{6}\times\frac{3}{5}$$

$$\frac{1}{4}\times\frac{(-3)}{5}-\left(\frac{4}{5}\times\frac{2}{5}+\frac{5}{6}\times\frac{3}{5}\right)$$

$$=\frac{1}{4}\times\frac{(-3)}{5}-\left(\frac{5}{6}\times\frac{3}{5}+\frac{4}{5}\times\frac{2}{5}\right)$$ [By commutative property for addition]

$$=\frac{(-3)}{5}\times\frac{1}{4}-\left(\frac{3}{5}\times\frac{5}{6}+\frac{4}{5}\times\frac{2}{5}\right)$$ [By commutative property for multiplication]

$$=\frac{(-3)}{5}\times\frac{1}{4}+\frac{(-3)}{5}\times\frac{5}{6}-\frac{4}{5}\times\frac{2}{5}$$

$$=\frac{(-3)}{5}\times\left(\frac{1}{4}+\frac{5}{6}\right)-\frac{4}{5}\times\frac{2}{5}$$ [By distributive property of multiplication over addition]

$$=\frac{(-3)}{5}\times\left(\frac{3+10}{12}\right)-\frac{4}{5}\times\frac{2}{5}$$

$$=\frac{(-3)}{5}\times\frac{13}{12}-\frac{4}{5}\times\frac{2}{5}$$

$$=\frac{(-39)}{60}-\frac{8}{25}$$

$$=\frac{(-39)\times 5-8\times 12}{300}$$

$$=\frac{(-195)-96}{300}$$

$$=\frac{-291}{300}$$

$$=-\frac{97}{100}$$

Example 2:

Solve the following expression.

$$\left\{\left(\frac{-4}{5}\right)\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{3}+\frac{4}{5}\times\frac{3}{7}\right\}$$

Solution:

$$\left(\frac{-4}{5}\right)\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{3}+\frac{4}{5}\times\frac{3}{7}$$

$=\left(\frac{-4}{5}\right)\times\frac{2}{3}+\frac{4}{5}\times\frac{3}{7}+\frac{1}{2}\times\frac{1}{3}$ [By commutative property for addition]

$$=\frac{(-4)}{5}\times\frac{2}{3}-\frac{(-4)}{5}\times\frac{3}{7}+\frac{1}{2}\times\frac{1}{3}$$

$=\frac{(-4)}{5}\times\left(\frac{2}{3}-\frac{3}{7}\right)+\frac{1}{2}\times\frac{1}{3}$ [By distributive property of multiplication over subtraction]

$$=\frac{(-4)}{5}\left(\frac{14-9}{21}\right)+\frac{1}{2}\times\frac{1}{3}$$

$$=\frac{(-4)}{5}\times\frac{5}{21}+\frac{1}{2}\times\frac{1}{3}$$

$$=\frac{(-4)}{21}+\frac{1}{6}$$

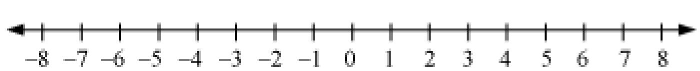
$$=\frac{(-4)\times 2+1\times 7}{42}$$

$$=\frac{(-8)+7}{42}$$

$$=-\frac{1}{42}$$

Rational Numbers on Number Line

A number line has numbers marked at equal distances as shown in the figure.



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Every point on the number line represents a number. We know how to locate integers and positive fractions in which the numerator is less than the denominator on the number line.

Now, how can we represent the rational numbers in which numerator is greater than the denominator?

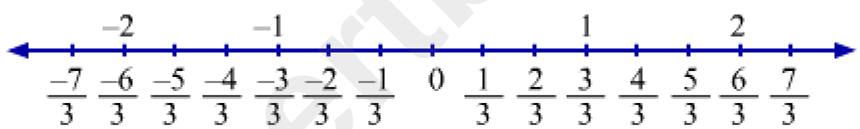
To represent such rational numbers on the number line, we write them as mixed fractions. Let us go through the following video to understand the method of representing $\frac{5}{2}$ and $\frac{-13}{3}$ on the number line.

In this way, we can represent rational numbers on the number line.

The ordinal relationship between rational numbers:

Defining ordinal relationship between two rational numbers is to find out that which number is greater and which one is smaller.

Look at the following number line.



On observing the number line, following points are obtained about the ordinal relationship between rational numbers:

(1) Out of any two numbers on the number line, the number on the left is smaller whereas the number on the right is greater.

(2) On the number line, all negative numbers lie on the left of zero. Thus, all negative numbers are smaller than zero.

(3) All negative numbers along with zero lie on the left of positive numbers. Thus, each positive number is greater than all negative numbers and zero.

(4) Out of the numbers having same denominator, the number having greater numerator is greater.

Let us solve some more examples to understand the concept better.

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Example 1:

Represent the rational numbers $\frac{1}{3}, \frac{-5}{3}$, and 2 on a number line.

Solution:

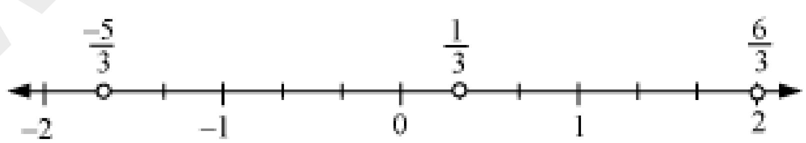
Here, $\frac{1}{3}$ and $\frac{-5}{3}$ have a common denominator 3.

And we can convert 2 into a rational number with denominator 3 by multiplying the numerator and denominator by 3. Therefore, 2 can be written as $\frac{6}{3}$.

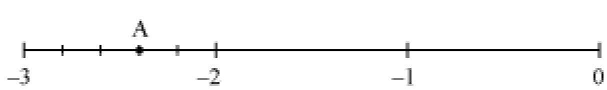
By converting all of them into rational numbers having a common denominator, it will become easier to represent them on the number line.

First, each part of the number line between two integers is divided into three equal parts as shown below.

Then, $\frac{1}{3}$ can be marked between 0 and 1. To mark $\frac{-5}{3}$, we move 5 units to the left of 0 and to mark $2\left(=\frac{6}{3}\right)$, we move 6 units to the right of 0.



Example 2:
A is a point on the following number line.



What is the rational number represented by the point A?

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Solution:

In the given number line, we may note that the number line between −3 and −2 is divided into five equal parts. The point A is 2 units left to −2. Therefore, the rational number represented by the point A is $-2-2\times\frac{1}{5}=\frac{-12}{5}=-2\frac{2}{5}$.

Finding Rational Numbers between Given Rational Numbers

Let's summarize.

We know that each point on the number line represents a number. Thus, between any two rational numbers, there are infinitely many numbers on the number line.

Let us try to find some rational numbers between $\frac{1}{6}$ and $\frac{7}{8}$.

To find the rational numbers between $\frac{1}{6}$ and $\frac{7}{8}$, firstly we have to make their denominators same.

2	6,	8
2	3,	4
2	3,	2
3	3,	1
	1,	1

The L.C.M. of 6 and 8 is 2 × 2 × 2 × 3 = 24

Now, we can write

$$\frac{1}{6}=\frac{1\times4}{6\times4}=\frac{4}{24}$$
$$\frac{7}{8}=\frac{7\times3}{8\times3}=\frac{21}{24}$$

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Therefore, between $\frac{4}{24}\left(\frac{1}{6}\right)$ and $\frac{21}{24}\left(\frac{7}{8}\right)$, we can find many rational numbers.

Some of them are

$$\frac{5}{24},\frac{6}{24},\frac{7}{24},\frac{8}{24},\frac{9}{24},\frac{10}{24},\frac{11}{24},\frac{12}{24},\frac{13}{24},\frac{14}{24},\frac{15}{24},\frac{16}{24},\frac{17}{24},\frac{18}{24},\frac{19}{24},\frac{20}{24}$$

Let us solve some more examples to understand the concept better.

Example 1:

Find three rational numbers between $\frac{-1}{15}$ and $\frac{1}{9}$.

Solution:

The first step is to find the L.C.M. of 15 and 9.

3	15,	9
3	5,	3
5	5,	1
	1,	1

The L.C.M. of 15 and 9 is 3 × 3 × 5 = 45

Now, we can write

$$\frac{-1}{15}=\frac{(-1)\times 3}{15\times 3}=\frac{-3}{45}$$
$$\frac{1}{9}=\frac{1\times 5}{9\times 5}=\frac{5}{45}$$

Therefore, three rational numbers between $\frac{-1}{15}$ and $\frac{1}{9}$ are $\frac{-2}{45},\frac{0}{45}(=0)$, and $\frac{1}{45}$.

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Example 2:

Find 10 rational numbers between $\frac{2}{5}$ and $\frac{5}{7}$.

Solution:

The first step is to find the L.C.M. of 5 and 7.

5	5,	7
7	1,	7
	1,	1

The L.C.M. of 5 and 7 is 5 × 7 = 35

Now, we can write

$$\frac{2}{5} = \frac{2\times7}{5\times7} = \frac{14}{35}$$
$$\frac{5}{7} = \frac{5\times5}{7\times5} = \frac{25}{35}$$

Therefore, 10 rational numbers between

$\frac{2}{5}$ and $\frac{5}{7}$ are $\frac{15}{35}\left(\frac{3}{7}\right), \frac{16}{35}, \frac{17}{35}, \frac{18}{35}, \frac{19}{35}, \frac{20}{35}\left(\frac{4}{7}\right), \frac{21}{35}\left(\frac{3}{5}\right), \frac{22}{35}, \frac{23}{35} \text{ and } \frac{24}{35}$.

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