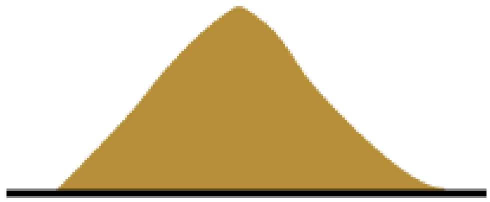
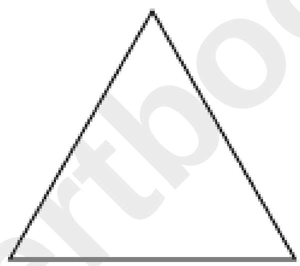
Triangles

Triangles and Their Attributes

Let us look at the figure of a heap of sand given below.



If we draw the above figure in a simple way, then it will look similar to the following figure.



We must have seen these types of shapes before. This is a three-sided polygon. It is called a **triangle.** We define a triangle as follows.

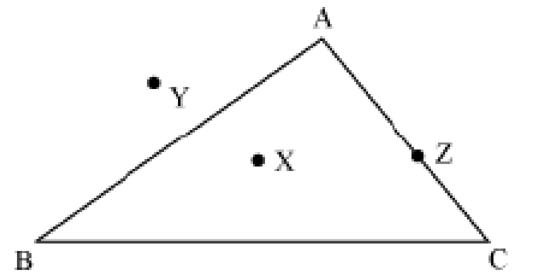
A polygon formed by three line segments is called a triangle i.e., a triangle is a polygon having three sides.

Let us discuss some characteristics of a triangle through various observations.

We know that a polygon has three regions.

1. Interior
2. Exterior
3. Boundary

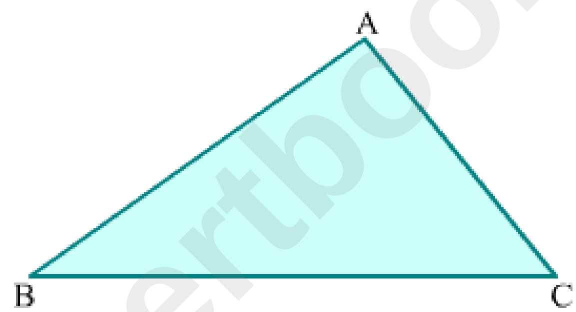
A triangle is a polygon, therefore, it also has the above three regions. It can be clearly understood with the help of the following figure.



In the above figure, point X lies in the interior of ΔABC; point Y lies in the exterior of ΔABC, while points A, B, C, and Z lie on the boundary of ΔABC.

The triangular area:

Look at the following triangle.

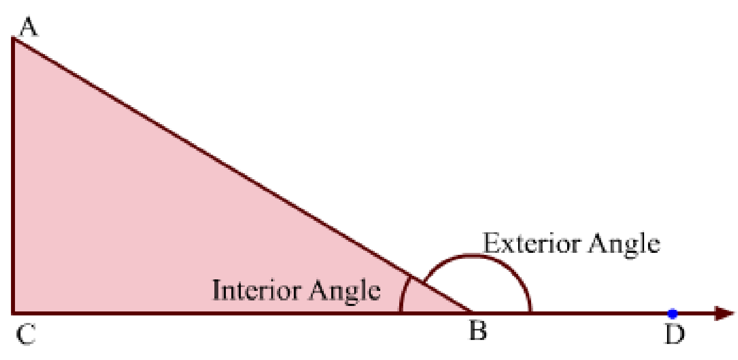


The interior of ΔABC is shaded. The whole shaded part of ΔABC along with its boundary is its area.

Thus, **interior and boundary of a triangle together form the area of triangle or triangular area.**

Exterior angle of a triangle:

Look at the triangle shown below.



It can be seen that in **Δ**ABC, side CB is extended up to point D. This extended side forms an angle with side AB, i.e., ∠ABD. This angle lies exterior to the triangle. Hence, ∠**ABD is an exterior angle of ΔABC.**

An exterior angle of a triangle can be defined as follows:

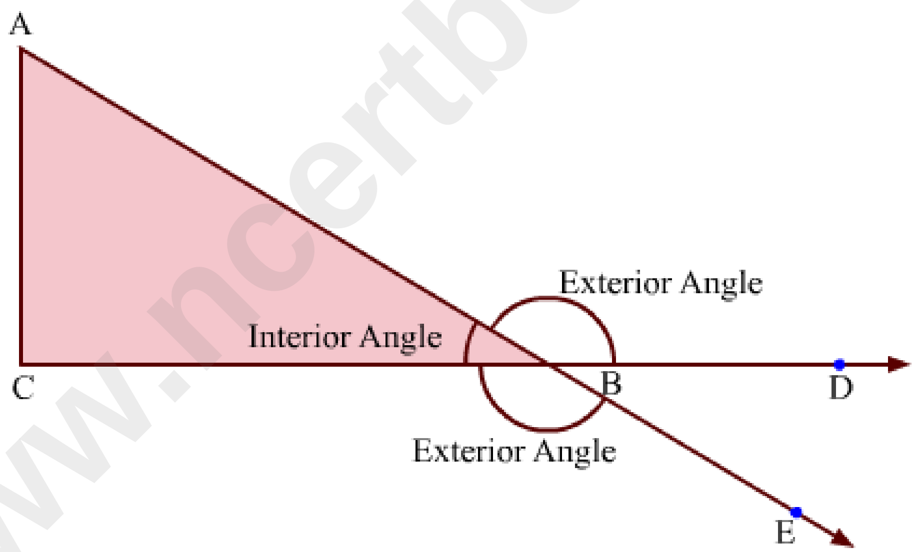
The angle formed by a side of a triangle with an extended adjacent side is called an exterior angle of the triangle.

Also, it can be seen that CBD is a line and hence, ∠ABC and ∠ABD form a linear pair at vertex B.

So, an exterior angle of a triangle can be defined in another way as follows:

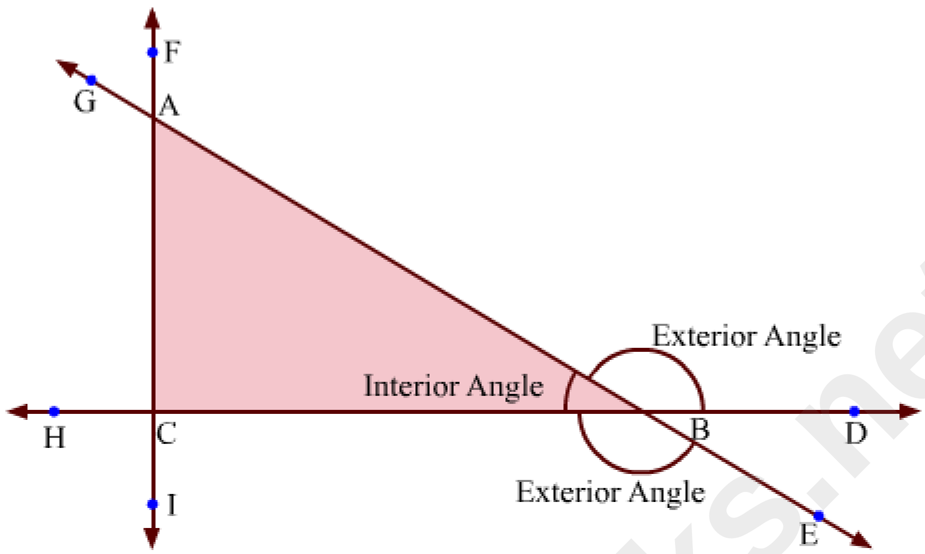
An angle forming linear pair with an interior angle of a triangle is known as exterior angle of that triangle.

Now, look at the following figure.



Here, two exterior angles such as ∠ABD and ∠CBE are formed at the vertex B.

Similarly, two exterior angles can be formed at each vertex of a triangle.



It can be seen that ∠BAF and ∠CAG are exterior angles formed at vertex A whereas, ∠ACH and ∠BCI are exterior angles formed at vertex C.

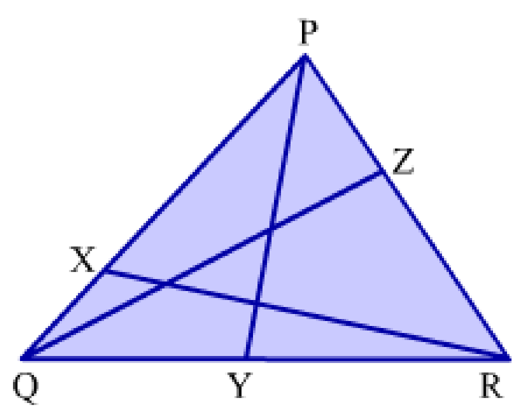
Thus, a triangle has six exterior angles.

In the above figure, it can be seen that there are three more angles such as ∠FAG, ∠DBE and ∠ICH which are vertically opposite angles of interior angles ∠BAC, ∠ABC and ∠ACB respectively. These angles are neither interior nor exterior angles of **Δ**ABC.

Cevians of a triangle:

A line segment joining a vertex of the triangle to any point on the opposite side (or its extension) is known as a cevian of that triangle.

Observe the give figure.



Here, line segments PY, QZ and RX all are cevians.

Note: Infinitely many cevians can be drawn from each vertex of a triangle. In other words, a triangle can have infinitely many cevians.

In a triangle there are few cevians such as **medians, altitudes** and **angle bisectors** are very special as these exhibit interesting properties.

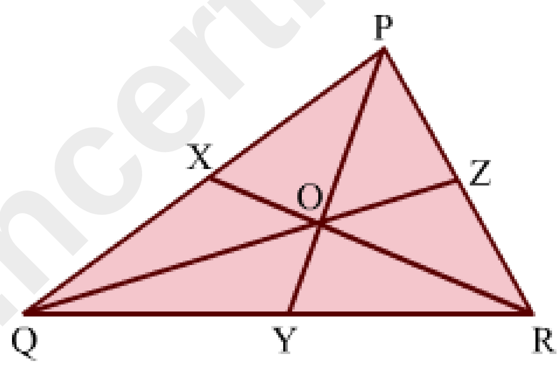
We know that all three medians are concurrent and the point of concurrence is known as centroid which divides each median in the ratio 2 : 1. Thus, centroid trisect each median. Also, centroid is the centre of gravity of any triangular lamina (cut out).

Also, all three altitudes are also concurrent and point of concurrence is known as ortho centre.

Similarly, all three angle bisectors are concurrent and the point of concurrence is known as incentre.

Ceva's Theorem:

A great Italian mathematician **Giovanni Ceva (Dec. 7, 1647 - June 15, 1734)** derived a very interesting property of cevians which is known as Ceva's theorem.



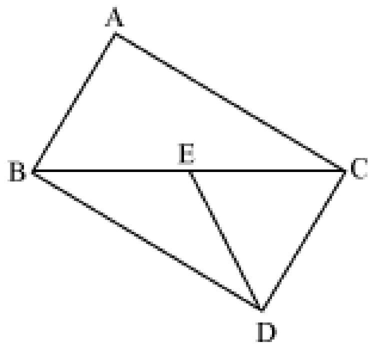
According to the theorem,
In ΔPQR, any three points X, Y and Z on the sides PQ, QR and RP respectively, the line segments PY, QZ and RX will be concurrent, if and only if

$$\frac{PX}{XQ} \cdot \frac{QY}{YR} \cdot \frac{RZ}{ZP} = 1$$

Let us discuss the examples based on these concepts of a triangle.

Example 1:

With respect to the given figure, answer the following questions.



(a) How many triangles are there in the figure?

(b) Write the names of any nine angles.

(c) Write the names of all the line segments.

(d) Which two triangles have ∠DBE common?

(e) Which two triangles have line segment ED as the common side?

(f) Which two triangles have line segment BC as the common side?

Solution:

(a) There are four triangles, namely, ΔABC, ΔBCD, ΔBED, and ΔCED in the given figure.

(b) ∠BAC, ∠ABC, ∠ACB, ∠DBE, ∠BED, ∠BDE, ∠CED, ∠EDC, and ∠DCE are 9 angles of the given figure.

(c) The line segments are $\overline{AB}$, $\overline{BC}$, $\overline{CA}$, $\overline{BD}$, $\overline{DC}$, $\overline{BE}$, $\overline{ED}$, and $\overline{EC}$.

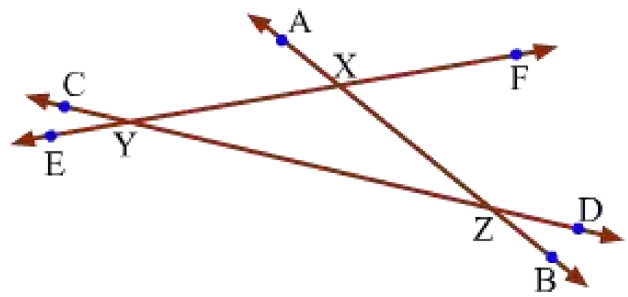
(d) ΔBCD and ΔBED have ∠DBE as common angle.

(e) ΔBED and ΔCED have line segment ED as the common side.

(f) ΔABC and ΔBCD have line segment BC as the common side.

Example 2:

With respect to the given figure, answer the following questions.



(a) Name all the exterior angles of ΔXYZ along with their respective vertices.

(b) Name all the angles which are neither exterior nor interior angles of ΔXYZ.

(c) Name the angles forming linear pair with ∠YXZ.

(d) Name the angle of ΔXYZ forming linear pair with ∠YZB.

Solution:

(a) The exterior angles of ΔXYZ along with their respective vertices are as follows:

∠YXA and ∠ZXF are formed at vertex X.

∠XYC and ∠ZYE are formed at vertex Y.

∠YZB and ∠XZD are formed at vertex Z.

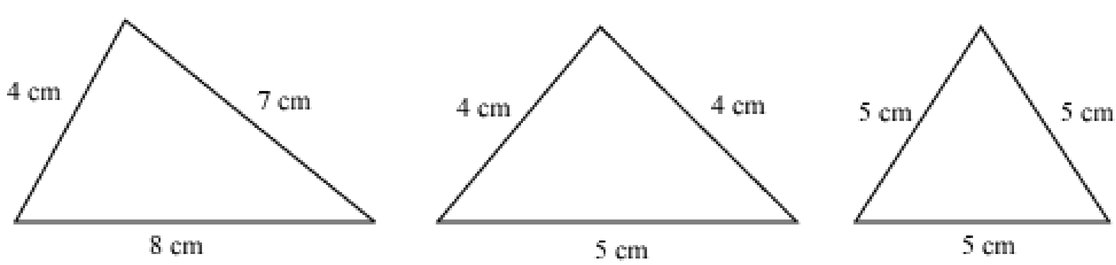
(b) ∠AXF, ∠DZB and ∠EYC are neither exterior nor interior angles of ΔXYZ.

(c) The angles forming linear pair with ∠YXZ are ∠YXA and ∠ZXF.

(d) ∠YZX forms linear pair with ∠YZB.

Classification of Triangles

Consider the following triangles.



Do you observe any difference among the given triangles?

In the first triangle, all sides are of different lengths.

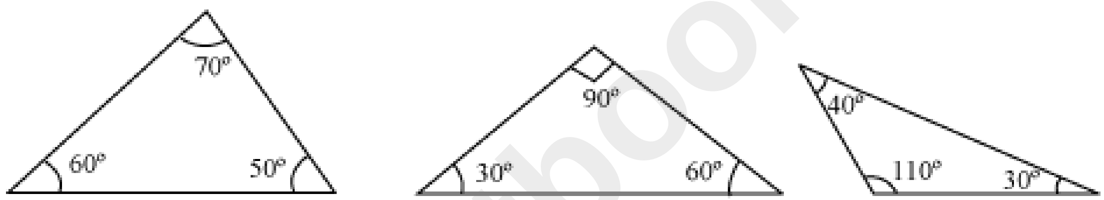
In the second triangle, two sides are equal and the third one is of a different length.

In the third triangle, all the sides are of equal lengths.

These are actually different types of triangles. One of the ways of classifying triangles is on the basis of the lengths of their sides.

Is the length of the sides the only criterion for the classification of triangles?

No. Let us first consider the following triangles.



What do you observe in these triangles?

Observe the following points.

In the first triangle, all angles are less than 90°.

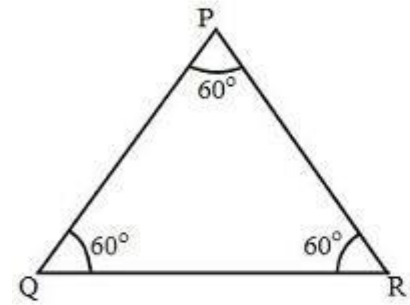
In the second triangle, only one angle is a right angle and the other two angles are less than 90°.

In the third triangle, one angle is more than the right angle and the other two angles are less than 90°.

These are all different types of triangles, which can be classified according to the measures of their angles.

Equiangular triangle:

An equiangular triangle is the triangle whose each angle is equal to each other, or we can say that whose each angle is equal to 60°.

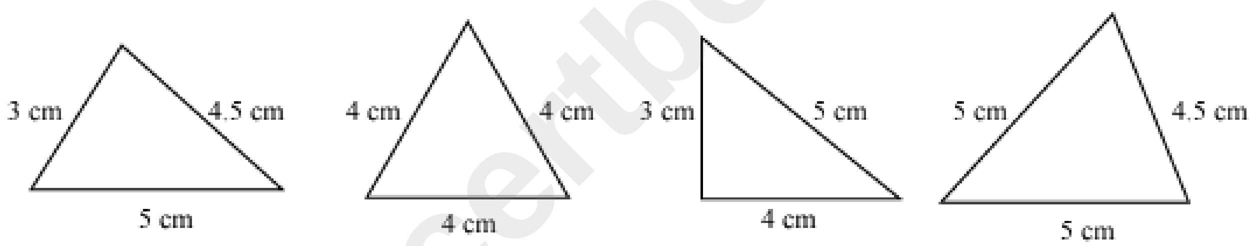


ΔPQR is an equiangular triangle as its each angle measures 60°.
i.e., ∠P = ∠Q = ∠R = 60°

Let us now look at some more examples to understand the concept better.

Example 1:

Classify the following triangles based on the nature of their sides.



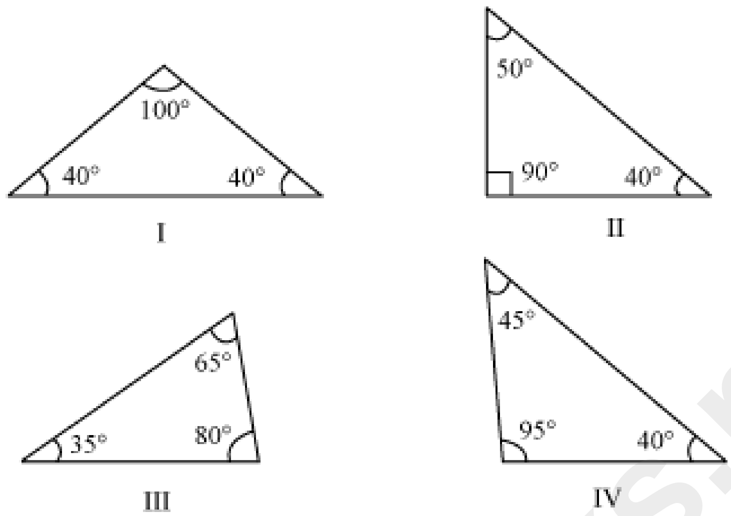
(i) **(ii)** **(iii)** **(iv)**

Solution:

1. Since all the sides are of different lengths, it is a scalene triangle.
2. Since all the sides are of same lengths, it is an equilateral triangle.
3. Since all the sides are of different lengths, it is a scalene triangle.
4. Since two sides of the given triangle are equal, it is an isosceles triangle.

Example 2:

Classify the following triangles based on the nature of their angles.

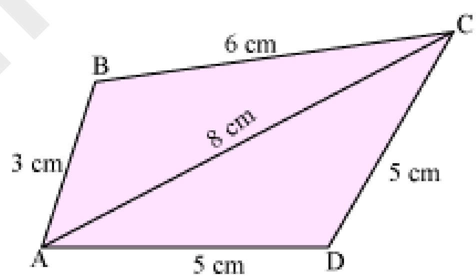


Solution:

1. Since one of the three angles is greater than a right angle, it is an obtuse-angled triangle.
2. Since one angle is a right angle, it is a right-angled triangle.
3. Since all the angles are less than 90°, it is an acute-angled triangle.
4. Since one of the angles is greater than a right angle, it is an obtuse-angled triangle.

Example 3:

Classify the triangles ABC and ADC based on the nature of their sides.



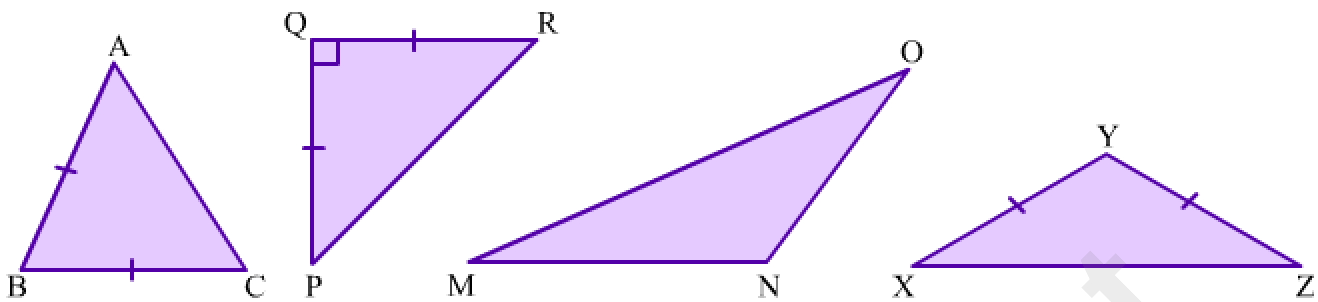
Solution:

ΔABC is a scalene triangle because all the three sides are of different lengths.

ΔACD is an isosceles triangle because two of its sides, AD and CD, are of equal lengths.

Example 4:

Classify the following triangles based on the nature of their angles and sides both.



Solution:

ΔABC has two equal sides and all of its angles are acute angles, so it is an acute angled isosceles triangle.

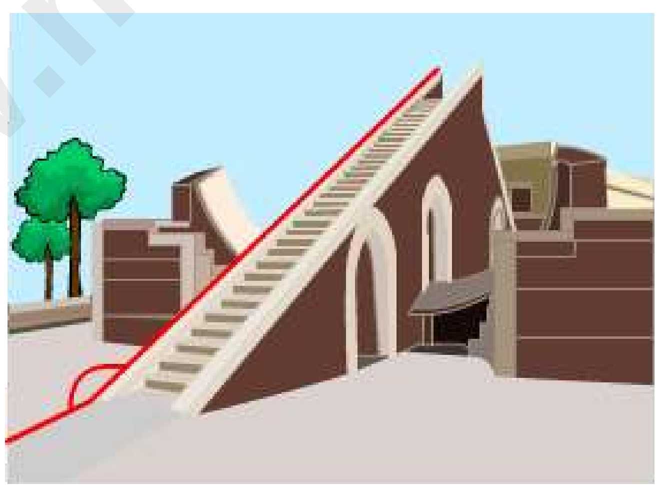
ΔPQR is a right angled triangle having two equal sides, so it is a right angled isosceles triangle.

ΔMNO is an obtuse angled triangle having three unequal sides, so it is an obtuse angled scalene triangle.

ΔXYZ is an obtuse angled triangle having two equal sides, so it is an obtuse angled isosceles triangle.

Exterior Angles in Real Life

Look at the triangular structure in the figure.

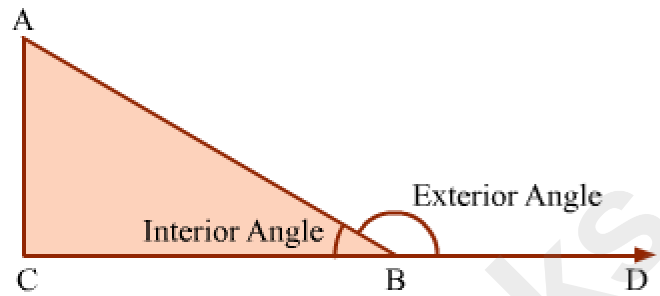


In the given figure, an open angle formed by the edge of the triangular structure with the horizontal plane is marked. This angle lies outside the triangle. Such angles are known as **exterior angles**.

In this lesson, we will study about exterior angles of triangles and the theorem based on them.

Exterior Angles of Triangles

Look at the triangle shown.



It can be seen that in **Δ**ABC, side CB is extended up to point D. This extended side forms an angle with side AB, i.e., ∠ABD. This angle lies exterior to the triangle. Hence, ∠**ABD is an exterior angle of ΔABC.**

An exterior angle of a triangle can be defined as follows:

The angle formed by a side of a triangle with an extended adjacent side is called an exterior angle of the triangle.

It can be seen that exterior ∠ABD forms linear pair with interior ∠ABC of **Δ**ABC. The other two interior angles of the triangle such as ∠ACB and ∠CAB do not form linear pair with ∠ABD.

Such angles are known as the **remote interior angles** of an exterior angle.

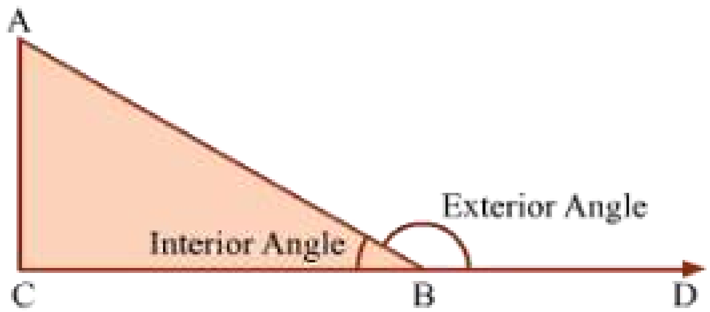
So, ∠ACB and ∠CAB are remote interior angles of exterior ∠ABD.

Corollary Related to Exterior Angle Theorem

There is a corollary related to exterior angle theorem which states that:

An exterior angle is always greater than each of its remote interior angles.

Let us prove this corollary with the help of **Δ**ABC shown in the figure.



Here, ∠ABD is an exterior angle of the triangle and its interior opposite or remote interior angles are ∠ACB and ∠CAB.

In a triangle, no interior angle can be zero angle or straight angle.

Thus, 0° <∠ABC < 180°, 0° < ∠ACB < 180° and 0° < ∠CAB < 180°

Now, ∠ABC < 180°

∴ 180° – ∠ABC > 0°

⇒ ∠ABD > 0°

In triangle **Δ**ABC, we have

∠ABD = ∠ACB +∠CAB (By exterior angle theorem)

And, ∠ABD > 0, ∠ACB > 0° and ∠CAB > 0° (Property of triangle)

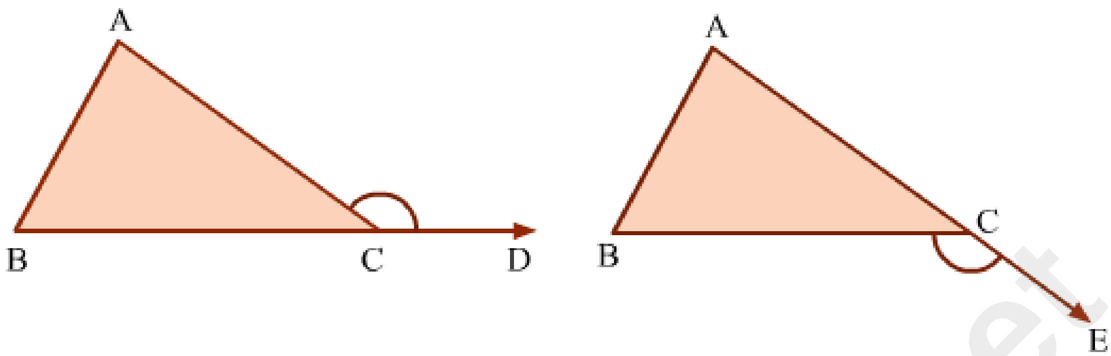
Therefore, ∠ABD > ∠ACB and ∠ABD >∠CAB

Thus, an exterior angle is always greater than each of its remote interior angles.

Two Exterior Angles at the Same Vertex are Equal

At any vertex, two exterior angles can be drawn by extending each of the two sides forming that vertex. These exterior angles are always of equal measure.

Let us prove this using the ΔABC shown in the figure.



The figure clearly shows that two exterior angles can be drawn at vertex C—one by producing BC up to point D and the other by producing AC up to point E. The exterior angles thus obtained are ∠ACD and ∠BCE.

According to the exterior angle theorem, the measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles of the triangle.

∴ ∠ACD = ∠ABC + ∠BAC … (1)

And, ∠BCE = ∠ABC + ∠BAC … (2)

Using equations 1 and 2, we get:

∠ACD = ∠BCE

So, we can conclude that two exterior angles can be drawn at any vertex. The two angles thus drawn have an equal measure and are equal to the sum of the two opposite interior angles.

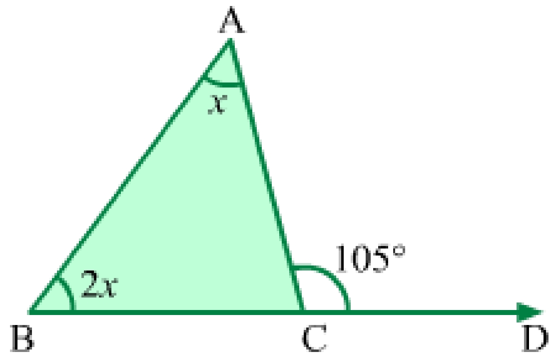
Practical Verification of Exterior Angle Property

Solved Examples

Easy

Example 1:

Find the value of *x* in the given figure.



Solution:

According to the exterior angle property of triangles, the measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles of the triangle.

So, we have:

$x + 2x = 105°$

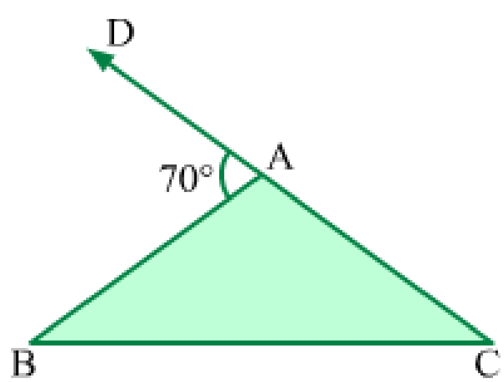
$\Rightarrow 3x = 105°$

On dividing both sides of the equation by 3, we obtain:

$$\frac{3x}{3} = \frac{105°}{3}$$
$$\Rightarrow x = 35°$$

Example 2:

If ∠ABC = ∠ACB in ΔABC, then find the measure of ∠ABC.



Solution:

∠ABC and ∠ACB are interior angles opposite to the exterior angle at vertex A, i.e., ∠BAD.

Therefore, by the exterior angle property of triangles, we obtain:

∠ABC + ∠ACB = ∠BAD

⇒ ∠ABC + ∠ACB = 70°

It is given that ∠ABC = ∠ACB

So, we obtain:

∠ABC + ∠ABC = 70°

⇒ 2∠ABC = 70°

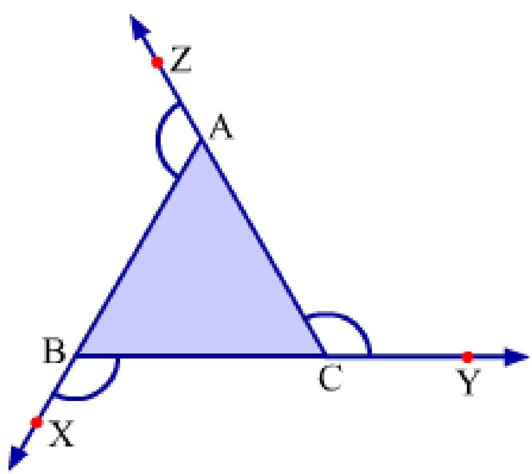
On dividing both sides of the equation by 2, we obtain:

$$\frac{2\angle ABC}{2} = \frac{70^\circ}{2}$$
$$\Rightarrow \angle ABC = 35^\circ$$

Medium

Example 1:

The sides AB, BC and CA of ΔABC are produced up to points X, Y and Z respectively. Find the sum of the three exterior angles so formed.



Solution:

Using the exterior angle property, we obtain:

∠BAZ = ∠ABC + ∠ACB … (1)

∠CBX = ∠BAC + ∠ACB … (2)

∠ACY = ∠BAC + ∠ABC … (3)

On adding equations 1, 2 and 3, we obtain:

∠BAZ + ∠CBX + ∠ACY = ∠ABC + ∠ACB + ∠BAC + ∠ACB + ∠BAC + ∠ABC

⇒ ∠BAZ + ∠CBX + ∠ACY = 2(∠ABC + ∠ACB + ∠BAC)

According to the angle sum property of triangles, we have:

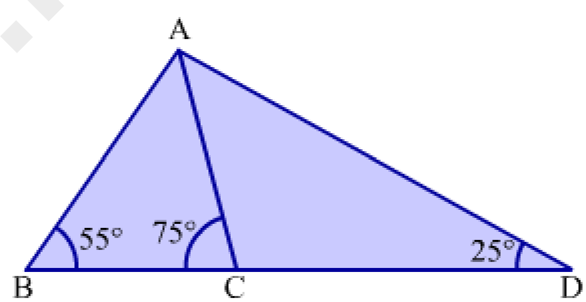
∠ABC + ∠ACB + ∠BAC = 180°

∴ ∠BAZ + ∠CBX + ∠ACY = 2 × 180° = 360°

Thus, the sum of the three exterior angles is 360°.

Example 2:

Show that AC is the bisector of ∠BAD in the given figure.



Solution:

On applying the angle sum property in ΔABC, we get:

∠BAC + ∠ABC + ∠ACB = 180°

⇒ ∠BAC + 55° + 75° = 180°

⇒ ∠BAC = 180° − 130°

⇒ ∠BAC = 50° … (1)

Now, by using the exterior angle property, we get:

∠ACB = ∠ADC + ∠CAD

⇒ 75° = 25° + ∠CAD

⇒ ∠CAD = 75° − 25°

⇒ ∠CAD = 50°

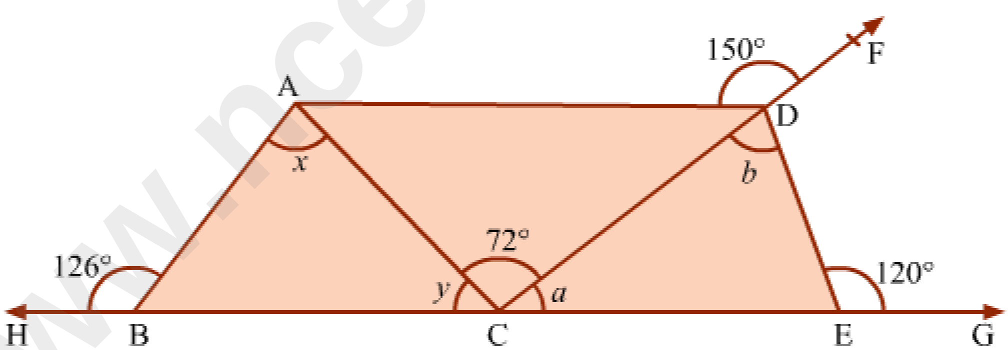
We know that ∠BAC + ∠CAD = ∠BAD. We have found ∠BAC = ∠CAD = 50°.

Thus, AC is the bisector of ∠BAD.

Hard

Example 1:

If AD||BE in the given figure, then find the values of *a*, *b*, *x* and *y*.



Solution:

From the figure, we have:

∠ADC + ∠ADF = 180° (Linear pair of angles)

⇒ ∠ADC + 150° = 180°

⇒ ∠ADC = 30°

Consider the parallel lines AD and BE and the transversal CF.

∠ADF = ∠DCB (Corresponding angles)

⇒ 150° = 72° + y

⇒ y = 78° … (1)

Now, y + 72° + a = 180° (As they form line BCE)

⇒ 78° + 72° + a = 180° (Using equation 1)

⇒ a = 180° − 150°

⇒ a = 30° … (2)

Consider ΔCDE.

∠DEG = a + b (Exterior angle property)

⇒ 120° = 30° + b (Using equation 2)

⇒ b = 90°

Now, consider ΔABC.

∠ABH = x + y (Exterior angle property)

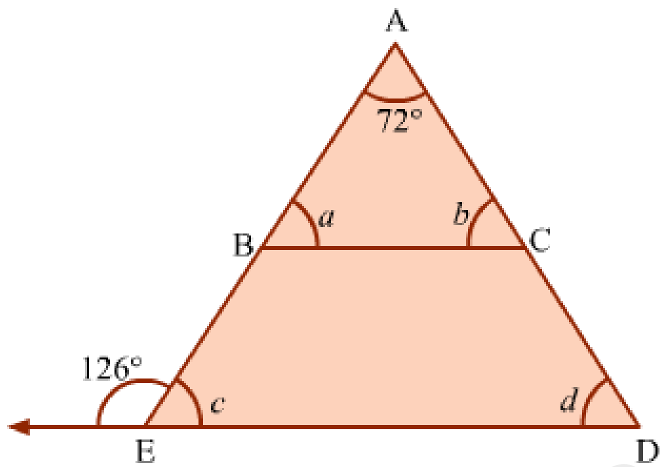
⇒ 126° = x + 78°

⇒ x = 48°

Hence, a = 30°, b = 90°, x = 48° and y = 78°.

Example 2:

ΔABC is placed atop trapezium EBCD in the given figure. Find the values of *a*, *b*, *c* and *d*.



Solution:

The exterior angle at E forms a linear pair with *c*.

∴ $126° + c = 180°$

⇒ $c = 180° - 126°$

⇒ $c = 54°$

On using the exterior angle property in ΔAED, we get:

$126° = 72° + d$

⇒ $d = 126° - 72°$

⇒ $d = 54°$

Since EBCD is a trapezium, BC is parallel to ED. Also, BE and CD are transversals lying on the two parallel lines.

So, we have:

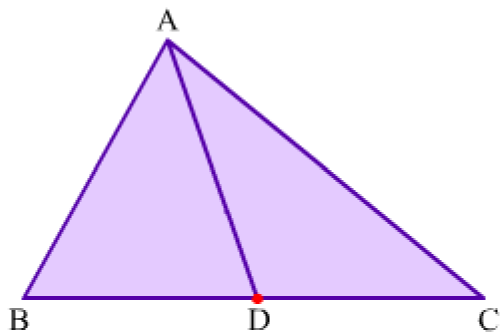
$a = c = 54°$ (Pair of corresponding angles)

$b = d = 54°$ (Pair of corresponding angles)

Thus, $a = b = c = d = 54°$.

Medians of Triangles

Let us consider the following triangle ABC.



In the given figure, A is the vertex of the triangle ABC and $\overline{BC}$ is the side opposite to vertex A. A line segment $\overline{AD}$ is drawn joining the point A and the point D, where D is the mid-point of $\overline{BC}$.

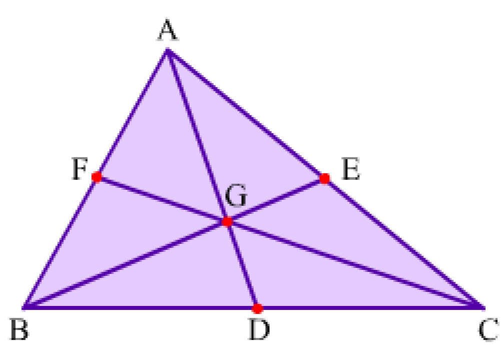
Then, we say that $\overline{AD}$ **is the median of ΔABC.**

A median can be defined as follows.

"The line segment joining any vertex of a triangle to the mid-point of its opposite side is called the median of the triangle."

Now we know what a median is, can we tell how many medians can be drawn inside a triangle?

In a triangle, there are three vertices. Therefore, ***a triangle can have three medians***, as shown in the following figure.



Here, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are the three medians of ΔABC.

The medians of a triangle always lie inside the triangle.

From the figure, it can be observed that the medians $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ intersect each other at a common point G.

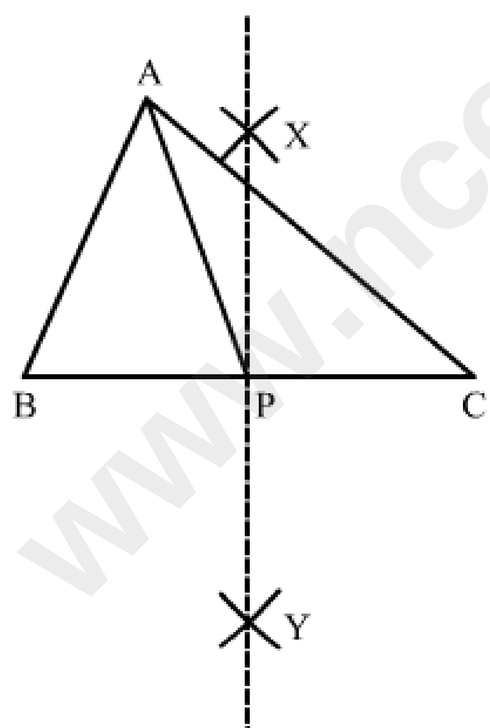
"The point of intersection of the medians is called the **centroid** of the triangle."

Thus, medians of a triangle are concurrent.

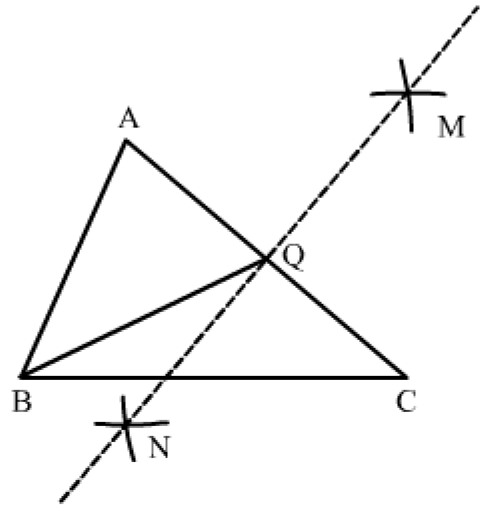
The point where medians intersect each other is known as the point of concurrence.

In the above given figure, G is the point of concurrence.

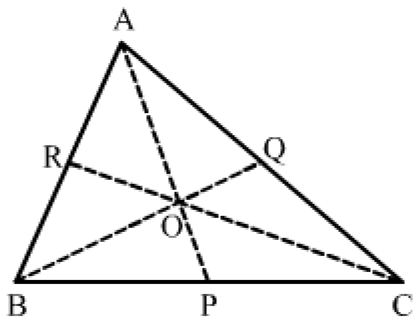
Construction of Median of Triangle
1. Draw a △△ABC.
2. With B and C as centres and radius more than half of BC, draw two arcs intersecting at points X and Y. Join XY thus meeting the line BC at point P.



3. With A and C as centres and radius more than half of AC, draw two arcs intersecting at points M and N. Join MN thus meeting the line AC at point Q.



4. Similarly, draw the perpendicular bisector of line AB meeting AB at point R.
5. Join AP, BQ and CR. Let the meeting point be O.

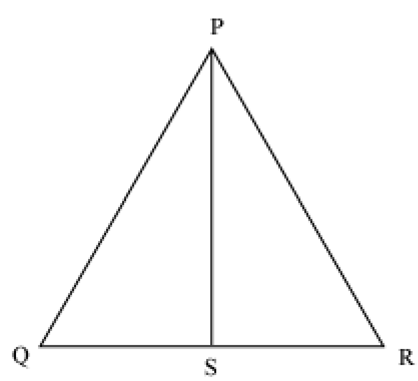


Point O is the centroid of △△ABC and AP, BQ and CR are the medians of sides BC, AC and AB respectively.

Now, let us look at an example.

Example 1:

In the triangle PQR, PS is a median and the length of $\overline{SR}$ = 6.5 cm. Find the length of $\overline{QR}$.



Solution:

Here, PS is the median to the side $\overline{QR}$ and we know that the median connects vertex to the midpoint of other side. Therefore, S is the mid-point of QR.

Therefore, $\overline{QR}$ = 2$\overline{SR}$

= 2 × 6.5 cm

= 13 cm

Altitudes of Triangles

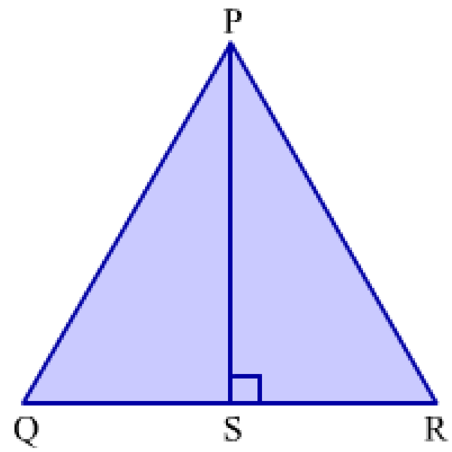
The students of class VII were being taken to a tour to Corbett National Park. They stayed there in a tent. The entrance in the tent was of a triangular shape as shown in the following figure.

Now can you tell what the height of the tent is?

As we can see in the above figure, the height of the tent is the length of the vertical pole which is standing in the centre of the tent.

Similarly, in any triangle, we can draw a perpendicular which represents its height. The perpendicular representing the **height** of a triangle is called the **altitude** of the triangle.

Look at the triangle PQR below.

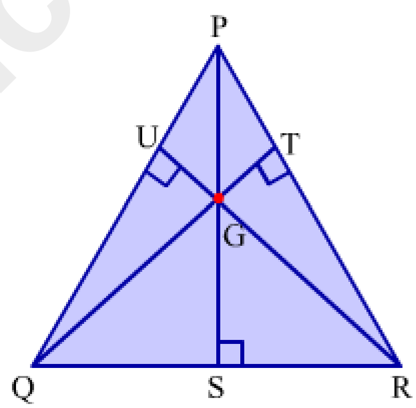


Here, P is a vertex of ΔPQR and $\overline{QR}$ is the opposite side of the vertex P. $\overline{PS}$ is a perpendicular drawn from P to $\overline{QR}$. Line segment $\overline{PS}$ is called the height or altitude of the triangle.

An altitude can be defined as follows.

"An altitude of a triangle is the perpendicular drawn from a vertex to the opposite side of the triangle."

Note: ***A triangle can have three altitudes.***



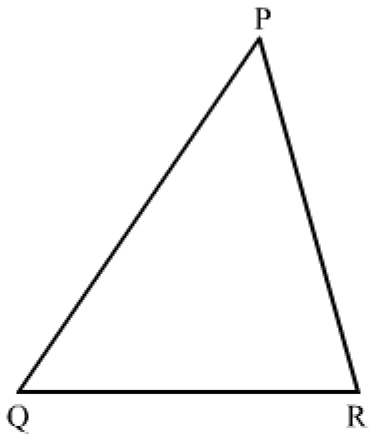
In the above figure, $\overline{PS}$, $\overline{QT}$, and $\overline{RU}$ are the three altitudes of ΔPQR.

"The point of intersection of the altitudes is called the **orthocentre** of the triangle."

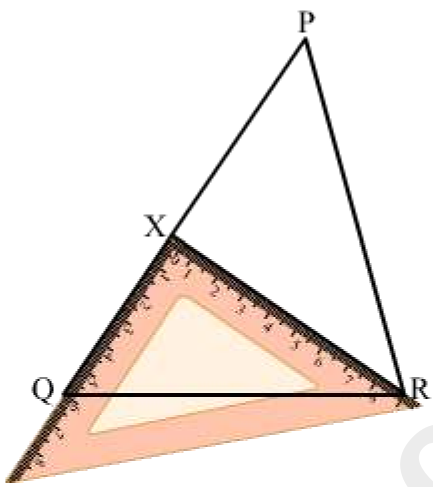
Construction of Altitudes of a triangle

I. Using set-square

1. Draw a △△PQR.

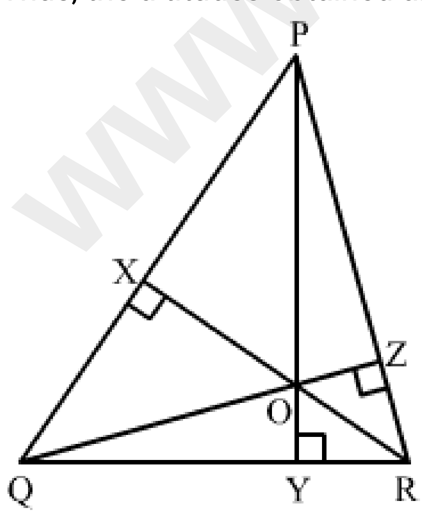


2. From point R, draw a perpendicular on side PQ. Where the perpendicular meets the side PQ, name it as point X. XR is the altitude formed on side PQ.



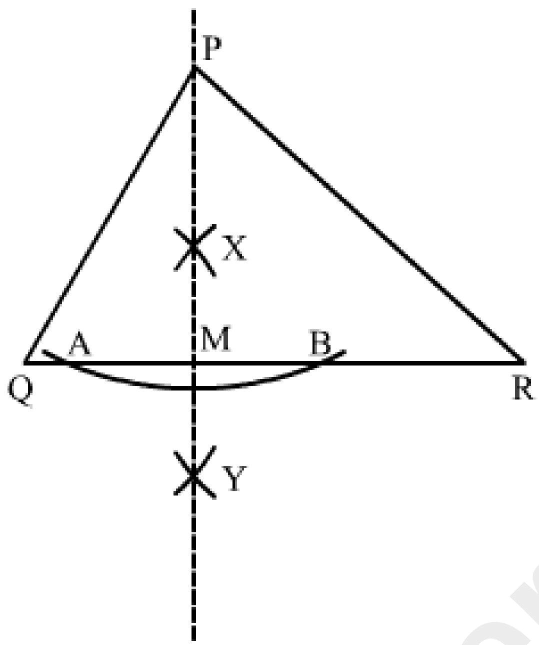
3. In a similar way construct the altitude from point P to side QR and from side Q to line PR.

Thus, the altitudes obtained are XR, QZ and PY.

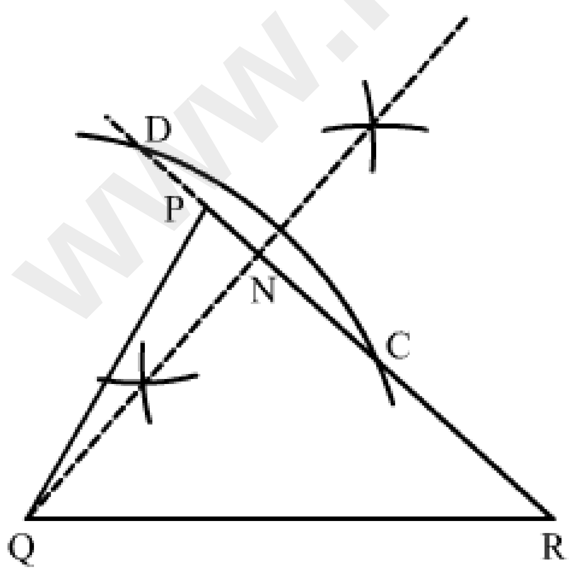


II. Using Compasses

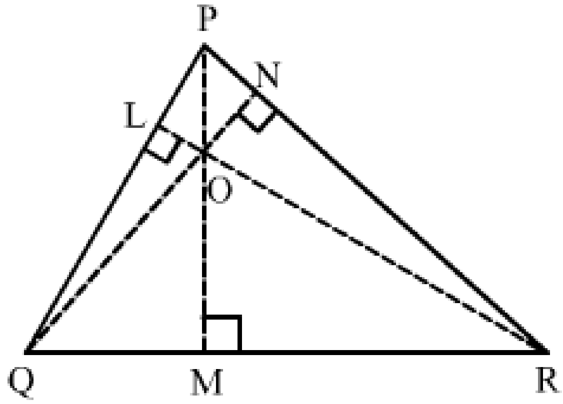
1. Draw a △PQR.
2. With P as centre, draw an arc on line QR cutting it at points A and B.
3. With A and B as centres, draw two intersecting arcs at points X and Y. Draw a line joining XY cutting the line QR at point M. Join PM.



4. With Q as centre draw an arc on side RP extended to cut it at points C and D. With C and D as centres, draw two intersecting arcs. Let this line intersect PR at point N. Join QN.



5. Similarly, draw altitude from point R on PQ cutting the line PQ at point L.
6. Join PM, RL and QN and name the meeting point of these three altitudes as O.



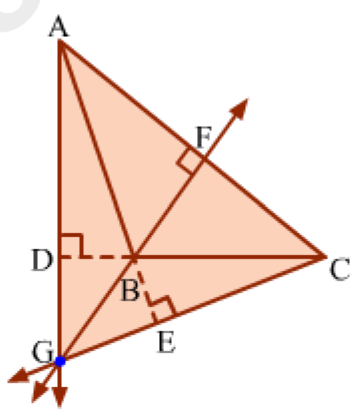
O is called the orthocentre of the △△PQR.

Remember

The altitudes of a triangle may not always lie inside it.

In an obtuse-angled triangle, the altitude drawn from the vertex of an acute angle lies outside the triangle. In this case, we have to extend the opposite side of the vertex from which the altitude is drawn.

For example,



In the above figure, ΔABC is an obtuse-angled triangle where ∠ABC is an obtuse angle. $\overline{AD}$ is the altitude of ΔABC drawn from the vertex A to extended side $\overline{BC}$. Similarly, $\overline{CE}$ is the altitude drawn from the vertex C to extended side $\overline{AB}$. And, $\overline{BF}$ is the altitude drawn from B to $\overline{CA}$.

Now, observe the altitudes drawn in the triangles ΔPQR and ΔABC.

It can be seen that altitudes in each triangle intersect each other at a common point.

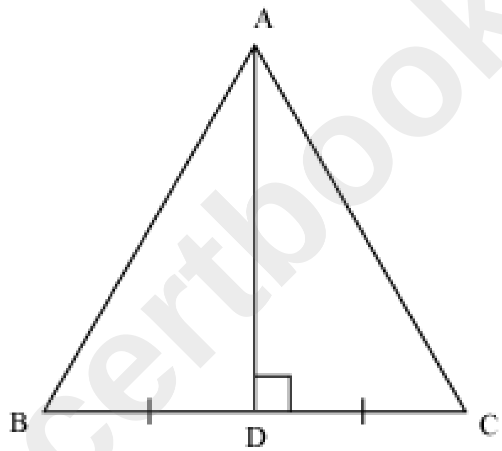
Thus, altitudes of a triangle are concurrent.

In ΔPQR and ΔABC, G is the point of concurrence.

Let us look at another example now.

Example:

In triangle ABC, $\overline{AD}$ is perpendicular to $\overline{BC}$ such that $\overline{BD} = \overline{CD}$. Are the median and the altitude drawn from A to $\overline{BC}$ same?



Solution:

Here,

$\overline{AD} \perp \overline{BC}$

Therefore, $\overline{AD}$ is an altitude of ΔABC drawn from the vertex A to $\overline{BC}$.

Also, $\overline{BD} = \overline{CD}$

Therefore, $\overline{AD}$ is a median of ΔABC drawn from the vertex A to $\overline{BC}$.

Thus, the altitude and the median drawn from A to $\overline{BC}$ are the same.

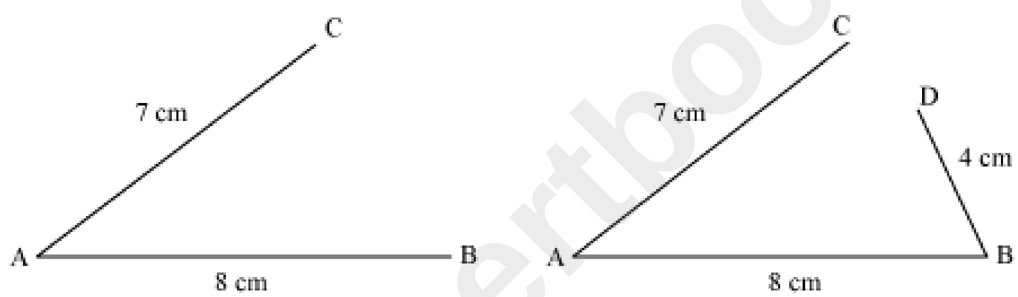
Construction of a Triangle when the Lengths of its Sides Are Given

Suppose if someone asks us to draw a triangle. The first question that strikes us is that what are the lengths of the sides of the triangle which is to be drawn?

Therefore, if the three sides of a triangle are given to us, then can we draw the triangle?

If we try to draw the triangle only with the help of a ruler, then it is not possible to draw it. With the help of a ruler, we can draw two sides of the triangle very easily. However, when we try to draw the third side, it may or may not intersect the third side.

Let us assume that we are asked to draw a triangle and the sides of the triangle are 8 cm, 7 cm, and 4 cm. Firstly, we draw the two sides of the triangle, which are 8 cm and 7 cm and then the third side of length 4 cm as shown in the following figure.



In these figures, we can see that a triangle is not formed.

Thus, we cannot draw a triangle only with the help of a ruler, but by using the ruler and compass.

Now, let us see the construction of a triangle using ruler and compass.

Before constructing a triangle, we should check whether the triangle is possible with the given sides or not.

In a triangle, the sum of any two sides must be greater than the third side.

For example: Can we draw a triangle with sides of length 6 cm, 9 cm, and 2 cm?

Here, 6 cm + 2 cm = 8 cm < 9 cm

i.e., the sum of the lengths of two sides is less than the length of the third side.

Therefore, we cannot draw a triangle with sides of given lengths.

Let us solve some examples based on the construction of triangles.

Example 1:

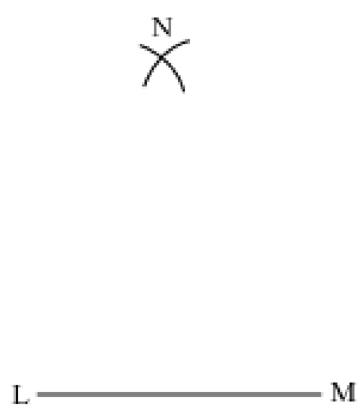
Construct an isosceles triangle such that the two equal sides are of lengths 9 cm each and the unequal side is of length 4 cm.

Solution:

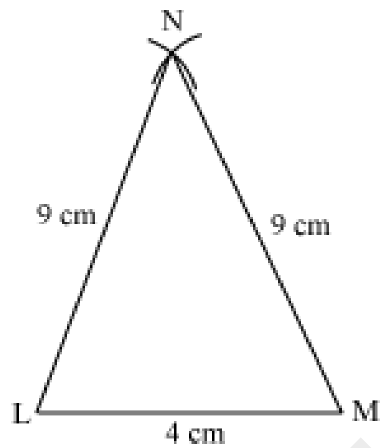
Firstly, we draw a line-segment $\overline{LM}$ of length 4 cm using a ruler.

Then using compass, we draw an arc of radius 9 cm taking L as the centre.

Again, taking M as the centre, we draw another arc of radius 9 cm. Now, both the arcs intersect each other at a point N.



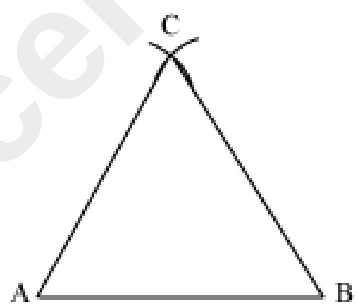
Now, we join the line segments $\overline{LN}$ and $\overline{MN}$.



Hence, ΔLMN is the required isosceles triangle with sides of given lengths.

Example 2:

A line segment $\overline{AB}$ is drawn. Then, two arcs with radius equal to the length of $\overline{AB}$ are drawn taking A and B as centres. The arcs intersect at C as shown in the given figure.



What type of a triangle is ΔABC?

Solution:

ΔABC is an equilateral triangle. As the two arcs have been drawn with radius equal to the length of $\overline{AB}$, therefore, $\overline{AC}$ and $\overline{BC}$ both are equal to $\overline{AB}$ i.e., all the three sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ of ΔABC are equal.

Example 3:

Construct an equilateral triangle of side 4.9 cm.

Solution:

We know that to construct a triangle, we require the measure of the lengths of all its three sides.

Now, here, we are required to construct an equilateral triangle of side 4.9 cm.

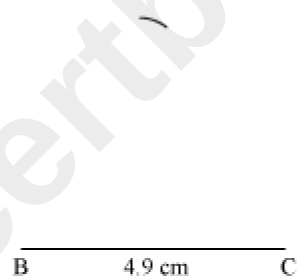
To construct the required triangle, we will use a property of an equilateral triangle.

We know that all sides of an equilateral triangle are of equal length. So, we have to construct a triangle ABC with AB = BC = CA = 4.9 cm.

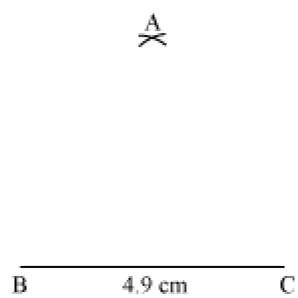
The steps of construction are as follows:

1. Draw a line segment BC of length 4.9 cm.

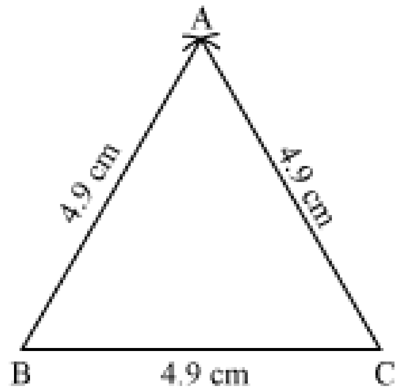
2. Taking point B as centre draw an arc of 4.9 cm radius.



2. Taking point C as centre draw an arc of 4.9 cm radius to meet the previous arc at the point A.



2. Join A to B and A to C.

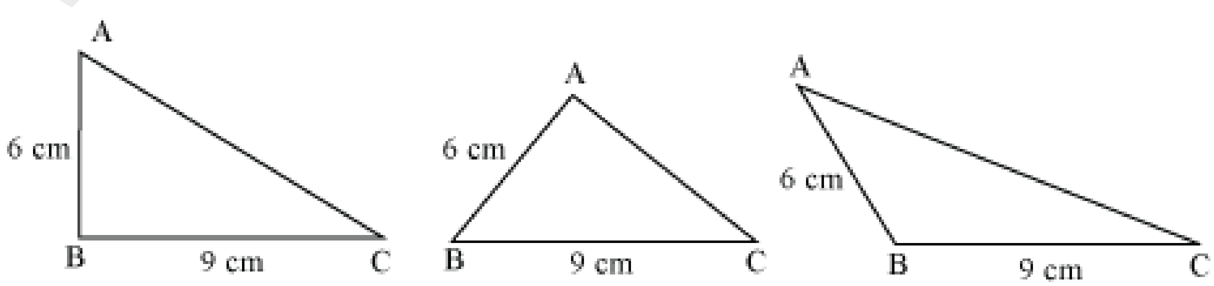


ABC is the required equilateral triangle.

Construction of a Triangle when the Lengths of Two Sides and Angle Between Them Are Given

Abhijit constructed ΔABC. He told Ravi that the lengths of two sides of ΔABC are $\overline{AB}$ = 6 cm and $\overline{BC}$ = 9 cm. Then, he asked Ravi to construct a triangle with the same dimensions.

Ravi started constructing the triangle and observed that more than one triangle could be constructed using the same dimensions as shown in the following figures.



He asked Abhijit to tell something else about the triangle also. Therefore, Abhijit told him the measure of the angle between the two known sides.

Now, can Ravi construct a unique triangle based on the information given by Abhijit?

Yes, Ravi can construct a unique triangle because with the given information, one and only one triangle can be constructed.

The point to remember here is that

> **'If the lengths of any two sides and the measure of the angle between them are given, then a unique triangle can be constructed".**

Let us look at some more examples.

Example 1:

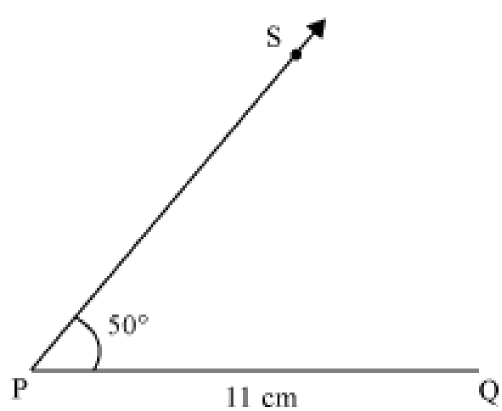
Construct a triangle PQR in which $\overline{PQ}$ = 11 cm, $\overline{PR}$ = 9 cm, and ∠QPR = 50°.

Solution:

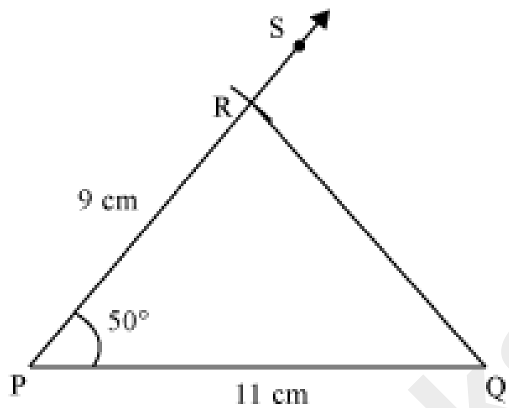
First, we draw a line segment $\overline{PQ}$ of length 11 cm using a ruler.

This is one of the sides of the triangle.

Now, we draw a ray $\overrightarrow{PS}$ from point P making an angle of measure 50° with the line segment $\overline{PQ}$.



Next, we draw an arc of radius 9 cm taking point P as the centre, which cuts $\overrightarrow{PS}$ at a point R. Then, we join the points Q and R to obtain the line segment $\overrightarrow{QR}$.



Thus, ΔPQR is the required triangle.

Example 2:

Construct a triangle ABC, where $\overline{AB}$ = 6 cm, $\overline{AC}$ = 2 $\overline{AB}$, and ∠BAC = 110°.

Solution:

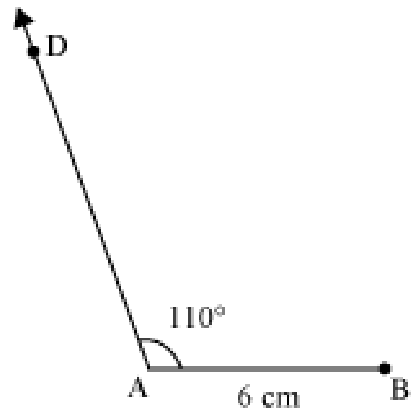
Given, ∠BAC = 110°

$\overline{AB}$ = 6 cm

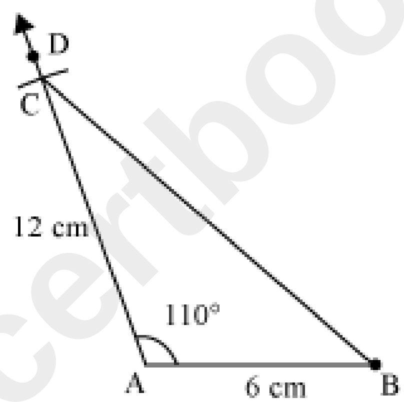
$\overline{AC}$ = 2 $\overline{AB}$ = 2 × 6 cm = 12 cm

First, we draw a line segment $\overline{AB}$ of length 6 cm.

Then, we draw a line segment $\overrightarrow{AD}$ making an angle of measure 110° with $\overline{AB}$ such that ∠BAD = 110°.



Now, using compass, we draw an arc of radius 12 cm taking A as the centre, which cuts $\overrightarrow{AD}$ at a point C. Then, we join points B and C.



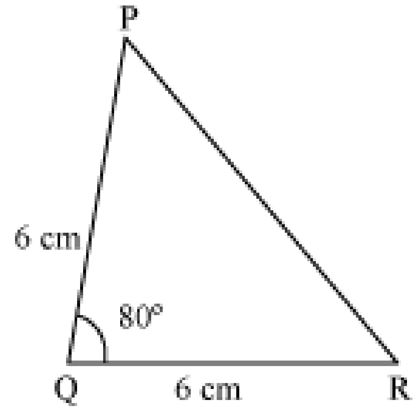
Thus, ΔABC so obtained is the required triangle.

Example 3:

Construct an isosceles triangle in which the length of each of its equal sides is 6 cm and the angle between them is 80°.

Solution:

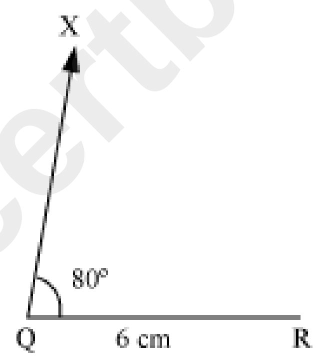
We have to construct an isosceles triangle PQR with PQ = QR = 6 cm. A rough sketch of the required triangle may be drawn as follows:



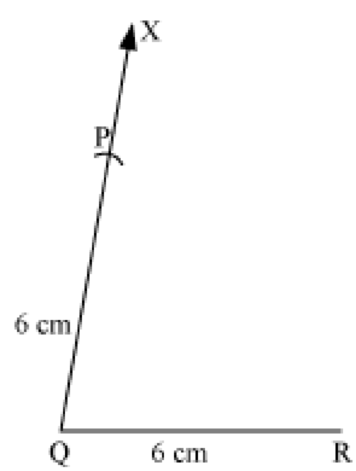
The steps of construction are as follows:

1. Draw the line segment QR of length 6 cm.

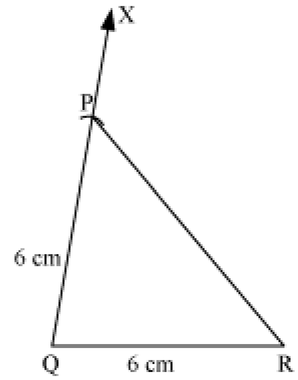
2. At point Q, draw a ray QX making an angle 80° with QR.



3. Taking Q as centre, draw an arc of 6 cm radius. It intersects QX at the point P.



iv. Join P to R to obtain the required triangle PQR.



Example 4:

Construct a triangle ABC, given that AB = 5 cm, BC = 6.5 cm and $\angle B = 60°$ **.**

Solution:

The steps of construction are as follows:

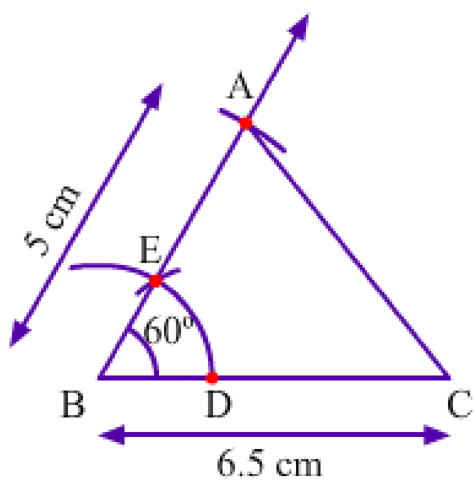
1. Draw a line segment BC of length 6.5 cm.

2. At B, using a compass draw an angle ∠EBC = 60 º.

3. With B as centre and radius 5 cm, draw an arc intersecting BE at A.

4. Join AC.

Thus, we get the required triangle ABC as shown below.



Construction of a Triangle when Two Angles and the Length of Side Between Them Are Given

Riya had studied in a book that if the measure of two angles of a triangle and the length of included side are given, then a unique triangle can be constructed based on this information.

She wants to try and see if this is really true or not? So, she tries to construct a triangle ABC such that two angles ∠C = 80°, ∠B = 40° and one side $\overline{BC} = 6$ cm are given.

Now, let us see how she draws the triangle.

Thus, to construct a triangle when the measure of two angles and the length of the included side are given, first we draw the side whose length is given and then we draw two rays making given angles with this side.

Let us solve some examples using this method.

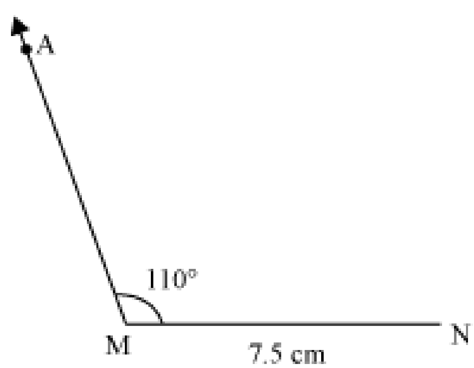
Example 1:

Construct a triangle such that the measures of two of its angles are 30° and 110° and the length of the side included between these two angles is 7.5 cm.

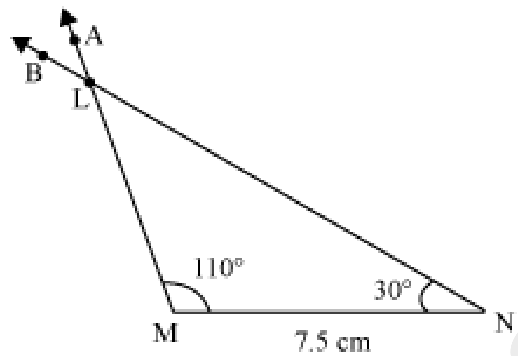
Solution:

First we draw a line segment $\overline{MN}$ of length 7.5 cm using a ruler.

Next we draw a ray $\overrightarrow{MA}$ from point M making an angle of measure 110° with the line segment $\overline{MN}$.



Now, we draw another ray $\overrightarrow{NB}$ from the point N, making an angle of 30° with $\overline{MN}$. Let it intersect the previously drawn ray $\overrightarrow{MA}$ at point L.



Thus, ΔLMN is the required triangle with the given measures.

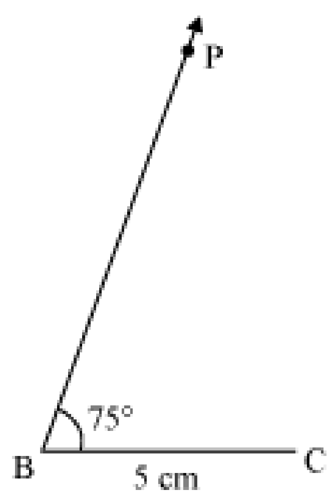
Example 2:

Construct an isosceles triangle such that the length of its unequal side is 5 cm and each of the two angles opposite to the equal sides is of measure 75°.

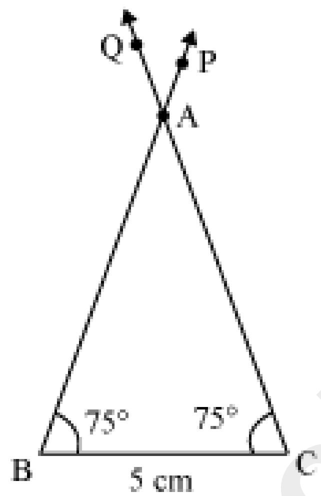
Solution:

First we draw a line segment $\overline{BC}$ of length 5 cm using a ruler.

Now, we draw a ray $\overrightarrow{BP}$ from point B making an angle of measure 75° with $\overline{BC}$.



Again, we draw another ray $\overrightarrow{CQ}$ from point C making an angle of measure 75° with $\overline{BC}$. Let it intersect the previously drawn ray $\overrightarrow{BP}$ at point A.



Thus, ΔABC is the required isosceles triangle with the given measures.