ISC 2025 EXAMINATION

Sample Question Paper - 2

Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

This Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions*

EITHER *from* ***Section B OR Section C.***

Section A*: Internal choice has been provided in* ***two questions of two marks each, two questions of four marks each and two questions of six marks each.***

Section B: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

Section C: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

1. **In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.** **[15]**

(a) If A is a skew-symmetric matrix of order n, and C is a column matrix of order n $\times 1$, then CAC is **[1]**

a) An identity matrix of order n b) A unit matrix of order one

c) A unit matrix of order Zero d) A zero matrix of order one

(b) $\int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right) dx$ is equal to **[1]**

a) $\frac{\pi}{5}$ b) 0

c) 1 d) – 1

(c) If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$ then $\cot^{-1} x + \cot^{-1} y$ equals **[1]**

a) $\frac{3\pi}{5}$ b) $\frac{\pi}{5}$

c) $\frac{2\pi}{5}$ d) π

(d) Find the particular solution for $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0;\ y = 2$ when x = 1 **[1]**

a) $y = \frac{2x}{1-\log|x|}\ (x \neq 0, x \neq e)$ b) $y = \frac{3x}{1-\log|x|}\ (x \neq 0, x \neq e)$

c) $y = \frac{2x}{1+\log|x|}\ (x \neq 0, x \neq e)$ d) $y = \frac{5x}{1+\log|x|}\ (x \neq 0, x \neq e)$

(e) If P(A) = $\frac{4}{5}$, and P(A $\cap$ B) = $\frac{7}{10}$ then P(B|A) is equal to **[1]**

a) $\frac{7}{8}$

b) $\frac{17}{20}$

c) $\frac{1}{10}$

d) $\frac{1}{8}$

(f) Let A = {1, 2, 3, ...n} and B = {a, b}. Then the number of surjections from A into B is **[1]**

a) $2^n + 1$

b) $^nP_2$

c) $2^n - 2$

d) $2^n - 1$

(g) If $y = e^{1/x}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{-e^{1/x}}{x^2}$

b) $e^{1/x} \log x$

c) $\frac{1}{x} \cdot e^{(1/x-1)}$

d) $2e^{1/x} \log x$

(h) If y = $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{-2}{(1-x^2)}$

b) $\frac{-2}{\sqrt{1-x^2}}$

c) $\frac{-2}{(1+x^2)}$

d) $\frac{2}{\sqrt{1-x^2}}$

(i) Which of the following is not correct? **[1]**

a) $|kA| = k^3|A|$, where A = $[a_{ij}]_{3\times3}$

b) If A is a skew-symmetric matrix of odd order, then |A| = 0

c) $\begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix} = \begin{vmatrix} a & c \\ e & g \end{vmatrix} + \begin{vmatrix} b & d \\ f & h \end{vmatrix}$

d) $|A| = |A^T|$, where A = $[a_{ij}]_{3\times3}$

(j) **Assertion (A):** $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ **[1]**

Reason (R): $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

(k) Let f : R $\rightarrow$ R : $f(x) = x^{-2} + 1$. Find : $f^{-1}\{10\}$ **[1]**

(l) Solve for x given that $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ **[1]**

(m) Prove that the greatest integer function f: $\mathbb{R} \rightarrow \mathbb{R}$, given by f(x) = [x], is neither one-one nor onto. **[1]**

(n) If P(A) = 0.8, P (B) = 0.5 and P(B|A) = 0.4, find P(A|B) **[1]**

(o) Given two independent events A and B such that P (A) = 0.3, P (B) = 0.6. Find P(A and B). **[1]**

2. Show that the function f(x) = $\begin{cases} 1+x, & \text{if } x \leq 2 \\ 5-x, & \text{if } x > 2 \end{cases}$ is not differentiable at x = 2. **[2]**

OR

Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

3. Evaluate the Integral: $\int \left(\frac{1-\tan x}{x+\log\cos x}\right) dx$ **[2]**

4. Find the absolute maximum value and the absolute minimum value of the function $f(x) = (\frac{1}{2} - x)^2 + x^3$ in the given interval [-2, 25]. **[2]**

5. Evaluate: **[2]**

$\int_{-1}^{5}(|x| + |x + 1| + |x - 5|)\, dx$

OR

Evaluate: $\int \sin^3 x \cos x\, dx$

6. If $f(x) = \frac{2x}{(1+x^2)}$ then show that $f(\tan\theta) = \sin 2\theta$ **[2]**

7. Prove that $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\} = \frac{2b}{a}$. **[4]**

8. Evaluate: $\int \frac{1}{2e^{2x}+3e^{x}+1}dx$ **[4]**

9. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, find $\frac{d^2y}{dx^2}$. Also, find its value at $\theta = \frac{\pi}{6}$ **[4]**

OR

Find $\frac{dy}{dx}$ when $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$

10. **Read the text carefully and answer the questions:** **[4]**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(a) How is Bayes' theorem different from conditional probability?

(b) Write the rule of Total Probability.

(c) What is the probability that the shell fired from exactly one of them hit the plane?

(d) If it is know that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

OR

Read the text carefully and answer the questions: **[4]**

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let: E_1: represent the event when many workers were not present for the job;

E_2: represent the event when all workers were present; and

E: represent completing the construction work on time.

(a) What is the probability that all the workers are present for the job?

(b) What is the probability that construction will be completed on time?

(c) What is the probability that many workers are not present given that the construction work is completed on time?

(d) What is the probability that all workers were present given that the construction job was completed on time?

11. **Read the text carefully and answer the questions:** **[6]**

A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below:

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
I	10,000	2,000	18,000
II	6,000	20,000	8,000

If the unit Sale price of Pencil, Eraser and Sharpener are ₹2.50, ₹1.50 and ₹1.00 respectively,Based on the information given above, answer the following questions:

(a) What is the total revenue collected from Market-I?

(b) What is the total revenue collected from Market-II?

(c) What is the gross profit from both markets considering the unit costs of the three commodities as ₹2.00, ₹1.00, and 50 paise respectively?

12. Find the equation of a curve passing through the point (1, 1). If the tangent drawn at any point P(x, y) on the curve meets the co-ordinate axes at A and B such that P is the mid-point of AB. **[6]**

OR

Find the particular solution of the following differential equation : $(x^2 + xy)dy = (x^2 + y^2)dx$, given that y=0,when x=1.

13. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius 10 cm is a square of side $10\sqrt{2}$ cm. **[6]**

OR

Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$

14. **Read the text carefully and answer the questions:** **[6]**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(a) Find the probability that Ajay gets Grade A in all subjects.

(b) Find the probability that he gets Grade A in no subjects.

(c) Find the probability that he gets Grade A in two subjects.

(d) Find the probability that he gets Grade A in at least one subject.

SECTION B - 15 MARKS

15. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is: **[1]**

a) $-\frac{17}{3}$ b) 3

c) -3 d) $\frac{17}{3}$

(b) Find the distance between the planes 2x - y + 2z = 5 and 5x - 2.5y + 5z = 20. **[1]**

(c) Find the value of p for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel. **[1]**

(d) Find the coordinates of the foot of the perpendicular drawn from the origin to 2x + 3y + 4z – 12 = 0 **[1]**

a) $\left(\frac{27}{29}, \frac{36}{29}, \frac{48}{29}\right)$ b) $\left(\frac{24}{29}, \frac{39}{29}, \frac{48}{29}\right)$

c) $\left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29}\right)$ d) $\left(\frac{24}{29}, \frac{36}{29}, \frac{49}{29}\right)$

(e) Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i} - 3\hat{j} + 6\hat{k}$. **[1]**

16. Find a unit vector in the direction of the vector $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$. **[2]**

OR

Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$, $\vec{a}$ and $\vec{b}$

17. Show that the distance d from point P to the line l having equation $\vec{r} = \vec{a} + \lambda\vec{b}$ is given by d = $\frac{|\vec{b}\times\overrightarrow{PQ}|}{|\vec{b}|}$ where Q is any point on the line l. **[4]**

OR

Show that the lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ intersect and find their point of intersection.

18. Find the area under the given curves and given lines: y = x^2, x = 1, x = 2 and x - axis. **[4]**

SECTION C - 15 MARKS

19. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) If C(x) and R(x) are respectively Cost function and Revenue function, then profit function P(x) is given by **[1]**

a) P(x) = R(x)·C(x) b) P(x) = R(x)

c) P(x) = C(x) + R(x) d) P(x) = R(x) - C(x)

(b) The value of objective function Z = 2x + 3y at comer point (3, 2) is **[1]**

a) 9 b) 5

c) 15 d) 12

(c) Find the coefficient of correlation from the regression lines: x - 2y + 3 = 0 and 4x - 5y + 1 = 0 **[1]**

(d) The total cost and total revenue functions of a commodity are given by C(x) = x + 40 and R(x) = 10x - $0.2x^2$. Find the break-even point. **[1]**

(e) The demand function for a certain commodity is given by p = 1000 - 15x - x, 0 < x < 25. What is the price per unit and the total revenue from the sale of 2 units? **[1]**

20. A monopolist has a demand function x = 106 - 2p and the average cost function AC = 5 + $\frac{x}{50}$, where p is the price per unit output and x is the number of units of output. If the total revenue is R = px, determine the most profitable output and the maximum profit. **[2]**

OR

The total cost function is given by C = x + $2x^3$ - $3.5x^2$, find the marginal average cost function (MAC). Also, find the points where the MAC curve cuts the x-axis and y-axis.

21. The following table shows the mean and standard deviation of the marks of Mathematics and Physics scored by the students in a school: **[4]**

	Mathematics	**Physics**
Mean	84	81
Standard Deviation	7	4

The correlation coefficient between the given marks is 0.86. Estimate the likely marks in Physics if the marks in Mathematics are 92.

22. Two tailors A and B, earn Rs 300 and Rs 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP. **[4]**

OR

A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

Solution

SECTION A - 65 MARKS

1. In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(i) **(d)** A zero matrix of order one

Explanation: {

By definition of skew-symmetric matrix.

(ii) **(b)** 0

Explanation: {

$f(x) = \int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right) dx$

$f(-x) = \int_{-1/2}^{1/2} \cos(-x) \log\left(\frac{1-x}{1+x}\right) dx = -f(x)$

Hence, f(x) is odd function.

(iii) **(b)** $\frac{\pi}{5}$

Explanation: {

We know that,

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

We have,

$\tan^{-1} x + \tan^{-1} y = 4\pi/5$ … (1)

Let, $\cot^{-1} x + \cot^{-1} y = k$ … (2)

Adding (1) and (2)

$\tan^{-1} x + \tan^{-1} y + \cot^{-1} x + \cot^{-1} y = \frac{4\pi}{5} + k$ ......(3)

Now, $\tan^{-1} A + \cot^{-1} A = \frac{\pi}{2}$ for all real numbers.

So, $(\tan^{-1} x + \cot^{-1} x) + (\tan^{-1} y + \cot^{-1} y) = \pi$ … (4)

From (3) and (4), we get,

$\frac{4\pi}{5} + k = \pi$

$\Rightarrow k = \pi - \frac{4\pi}{5}$

$\Rightarrow k = \frac{\pi}{5}$

(iv) **(a)** $y = \frac{2x}{1-\log|x|}$ $(x \neq 0, x \neq e)$

Explanation: {

Let y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Question becomes $v + x\frac{dv}{dx} = \frac{2v+v^2}{2}$

$x\frac{dv}{dx} = \frac{2v+v^2}{2} - v$

$x\frac{dv}{dx} = \frac{2v+v^2-2v}{2}$

$2\int \frac{dv}{v^2} = \int \frac{dx}{x}$

$\frac{-2}{v} = logx + c$

When x=1 y=2 we get

$\frac{-2x}{y} = logx + c$

$\frac{-2}{2} = log1 + c \implies c = -1$

$\frac{-2x}{y} = logx - 1$

$y = \frac{2x}{1-log|x|}$

(v) **(a)** $\frac{7}{8}$

Explanation: {

$\because P(A)=\frac{4}{5}, P(A\cap B)=\frac{7}{10}$

$\therefore P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{7/10}{4/5}=\frac{7}{8}$

(vi) **(c)** $2^n - 2$

Explanation: {

Given, A = (1, 2, 3,..., n] and B = {a, b}

Since, the number of surjections from A to B = total number of functions from A to B - number of functions from A to B

whose, images are proper subset of B and total number of functions from a set with p number of elements into a set with q number of elements = q^p.

$\therefore$ Number of surjections from A into B = $2^n - 2$

(vii) **(a)** $\frac{-e^{1/x}}{x^2}$

Explanation: {

Here $y=e^{\frac{1}{x}}$

Taking log both sides, we get

$\log_e y=\frac{1}{x}$ (Since $\log_a b^c = c\log_a b$)

Differentiating with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx}=-\frac{1}{x^2}$ or $\frac{dy}{dx}=-\frac{1}{x^2}\times y$

Therefore, $\frac{dy}{dx}=-\frac{1}{x^2}\times e^{\frac{1}{x}}$

(viii) **(b)** $\frac{-2}{\sqrt{1-x^2}}$

Explanation: {

$\Rightarrow y=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$\Rightarrow \sec y=\frac{1}{2x^2-1}$

$\Rightarrow$ cos y = $2x^2 - 1$

$\Rightarrow$ y = $\cos^{-1}(2x^2 - 1)$

Put x = cos θ ,we get

$\Rightarrow$ y = $\cos^{-1}(2\cos^2\theta - 1)$

$\Rightarrow$ y = $\cos^{-1}(\cos 2\theta)$

$\Rightarrow$ y = 2θ

But $\theta = \cos^{-1} x$, we get

$\Rightarrow \frac{dy}{dx}=\frac{d(\cos^{-1}x)}{dx}$

$\Rightarrow \frac{dy}{dx}=2\cdot\frac{d(\cos^{-1}x)}{dx}$

$\Rightarrow \frac{dy}{dx}=2\cdot\left(\frac{-1}{\sqrt{1-x^2}}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{-2}{\sqrt{1-x^2}}$

(ix) **(c)** $\begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix}=\begin{vmatrix} a & c \\ e & g \end{vmatrix}+\begin{vmatrix} b & d \\ f & h \end{vmatrix}$

Explanation: {

For adding the determinants, we need to find the value of the determinants and add them.We cannot apply the method applicable for matrix addition.

(x) **(d)** A is false but R is true.

Explanation: {

Assertion: $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}=\begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}=\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}=\begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}=\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$

Hence, $\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix} \neq \begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$

Reason: Here, $\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}$

$= \begin{bmatrix}-1+0+6 & 1-2+9 & 0+2+12\\0+0+0 & 0+(-1)+0 & 0+1+0\\-1+0+0 & 1-1+0 & 0+1+0\end{bmatrix}$

$= \begin{bmatrix}5 & 8 & 14\\0 & -1 & 1\\-1 & 0 & 1\end{bmatrix}$

and $\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$

$= \begin{bmatrix}-1+0+0 & -2+1+0 & -3+0+0\\0+0+1 & 0-1+1 & 0+0+0\\2+0+4 & 4+3+4 & 6+0+0\end{bmatrix}$

$= \begin{bmatrix}-1 & -1 & -3\\1 & 0 & 0\\6 & 11 & 6\end{bmatrix}$

Hence, $\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix} \neq \begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$

(xi) We have, $f(x) = x^2 + 1$

Let $f^{-1}(10) = x$. Then,we have,

$f(x) = 10 \Rightarrow x^2 + 1 = 10 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$.

$\therefore f^{-1}\{10\} = \{-3, 3\}$

(xii) $\begin{bmatrix}2x-3y\\x+y\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix}$

$\Rightarrow 2x - 3y = 1$

$\Rightarrow x + y = 3$

$\Rightarrow x = 3 - y$

$\Rightarrow 2(3 - y) - 3y = 1$

$\Rightarrow -5y = -5$

$\Rightarrow y = 1$

$\Rightarrow x = 3 - 1$

$\Rightarrow x = 2$

(xiii) **For not one-one:**

$1.1, 1.2 \in R$(domain)

now, $1.1 \neq 1.2$ but $f(1.1) = f(1.2) = 1 \Rightarrow f$ is not one-one.

For not onto:

Let $\frac{1}{2} \in R$ (co-domain), but $[x] = \frac{1}{2}$ is not possible for x in domain.

so, f is not onto.

(xiv) Given: $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$

By definition of conditional probability $P(B|A) = \frac{P(A\cap B)}{P(A)}$

$\Rightarrow P(A\cap B) = P(B|A)\ P(A) = 0.32$

$Now,\ P(A|B) = \frac{0.32}{0.5} = 0.64$

$\Rightarrow P(A|B) = 0.64$

(xv) $P(A \text{ and } B) = P(A).P(B) = 0.3 \times 0.6 = 0.18$

2. RHD at x = 2 = $Rf'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\left[\frac{5-(2+h)-3}{h}\right]$

$= \lim_{h\to 0} \frac{-h}{h} = \lim_{h\to 0}(-1) = -1$

And, LHD at x=2 = $Lf'(2) = \lim_{h\to 0} \frac{f(2-h)-f(2)}{-h}$

$= \lim_{h\to 0}\left[\frac{1+(2-h)-3}{-h}\right] = \lim_{h\to 0}\frac{-h}{-h} = \lim_{h\to 0} 1 = 1$

Thus, $Rf'(2) \neq Lf'(2)$.

Hence, f(x) is not differentiable at x = 2.

OR

It is given that $ay^2 = x^3$

Now, differentiating both sides with respect to x, we get

$2ay.\frac{dy}{dx} = 3x^2$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$

Then, the slope of the tangent to the given curve at (am^2, am^3) is

$\left.\frac{dy}{dx}\right]_{(am^2, am^2)} = \frac{3(am^2)^2}{2a(am^3)} = \frac{3a^2m^4}{2a^2m^3} = \frac{3m}{2}$

Then, slope of normal at $(am^2, am^3) = \frac{-1}{\text{Slope of the tangent at }(am^2, am^2)} = \frac{-2}{3m}$

Therefore, equation of the normal at (am^2, am^3) is given by:

$y - am^3 = \frac{-2}{3m}(x - am^2)$

$\Rightarrow 3my - 3am^4 = -2x + 2am^2$

$\Rightarrow 2x + 3my - am^2(2 + 3m^2) = 0$

3. Let $I = \int\left(\frac{1-\tan x}{x+\log(\cos x)}\right)dx$

Since $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int\left(\frac{1-\tan x}{x+\log(\cos x)}\right)dx$ .... (i)

Let x + log (cos x) = t

$\Rightarrow 1 + \frac{1\cdot(-\sin x)}{\cos x} = \frac{dt}{dx}$

$\Rightarrow 1 - \tan x = \frac{dt}{dx}$

$\Rightarrow$(1 - tan x) dx = dt

Putting this value in equation (i), we get

$I = \int \frac{dt}{t}$

I = log |t| + c

I = log |x + log (cos x)| + c

4. We have,

$f(x) = \left(\frac{1}{2} - x\right)^2 + x^3$, where $x \in [-2, 25]$.

$\Rightarrow f'(x) = -2(\frac{1}{2} - x) + 3x^2 = -1 + 2x + 3x^2$

At the points of local maximum and local minimum, we must have

$f'(x) = 0 \Rightarrow 3x^2 + 2x - 1 = 0 \Rightarrow (3x - 1)(x + 1) = 0 \Rightarrow x = \frac{1}{3}, -1$

The values of f(x) at these points and also at the end - points of the interval are computed as given below.

$f(-2) = \left(\frac{1}{2} + 2\right)^2 + (-2)^3 = \frac{25}{4} - 8 = -\frac{7}{8}$, $f\left(\frac{1}{3}\right) = \left(\frac{1}{2} - \frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 = \frac{1}{36} + \frac{1}{27} = \frac{7}{108}$,

$f(-1) = \left(\frac{1}{2} + 1\right)^2 + (-1)^3 = \frac{5}{4}$ and, $f(2.5) = \left(\frac{1}{2} - 2.5\right)^2 + (2.5)^2 = \frac{157}{8}$

Of these values, the maximum value of f(x) is $\frac{157}{8}$ and the minimum value is $-\frac{7}{8}$

Thus, the absolute maximum = $\frac{157}{8}$ and, the absolute minimum = $-\frac{7}{8}$.

5. If $x \in [-1, 0] \Rightarrow f(x) = -x + x + 1 - x + 5 = 6 - x$

If $x \in [0, 5] \Rightarrow f(x) = x + x + 1 - x + 5 = x + 6$

$\therefore \int_{-1}^{5}(|x| + |x+1| + |x-5|)\,dx = \int_{-1}^{0}(6 - x)dx + \int_{0}^{5}(x + 6)dx$

$= \left[\frac{(6-x)^2}{-2}\right]_{-1}^{0} + \left[\frac{(x+6)^2}{2}\right]_{0}^{5}$

$= \frac{13}{2} + \frac{85}{2} = 49$

OR

Let $I = \int \sin^3 x \cos x\, dx$

Now let sin x - t. Then, d (sin x) = dt $\Rightarrow$ cos x dx - dt $\Rightarrow dx = \frac{dt}{\cos x}$

Put sin x = t and $dx = \frac{dt}{\cos x}$, we get

I = $\int$ sin^3 x cos x dx = $\int$ t^3 cos x $\times \frac{dt}{\cos x} = \int t^3 dt = \frac{t^4}{4} + C = \frac{\sin^4 x}{4} + C$

6. Here we are given that, f(x) = $\frac{2x}{(1+x^2)}$

Need to prove: f(tan θ) = sin 2θ

f(tan θ) = $\frac{2\tan\theta}{1+\tan^2\theta}$

$\Rightarrow$ f(tan θ) = $\frac{2\,tan\theta}{sec^2\theta}$ [as 1 + tan^2 θ = sec^2 θ]

$\Rightarrow$ f(tan θ) = 2$\frac{\sin\theta}{\cos\theta}$cos^2 θ [as tan θ = $\frac{\sin\theta}{\cos\theta}$ and sec θ = $\frac{1}{\cos\theta}$]

$\Rightarrow$ f(tan θ) = 2 sin θ cos θ = sin 2θ [Proved]

7. Here we need to prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$

Let LHS $= \tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right)$

Put $\cos^{-1}\frac{a}{b} = \theta \Rightarrow \cos\theta = \frac{a}{b}$

$\text{LHS} = \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$= \frac{\tan\frac{\pi}{4} + \tan\frac{\theta}{2}}{1 - \tan\frac{\pi}{4}\tan\frac{\theta}{2}} + \frac{\tan\frac{\pi}{4} - \tan\frac{\theta}{2}}{1 + \tan\frac{\pi}{4}\tan\frac{\theta}{2}}$

$= \frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} + \frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{2}{2}}$

$= \frac{\left(1+\tan\frac{\theta}{2}\right)^2 + \left(1-\tan\frac{\theta}{2}\right)^2}{\left(1-\tan\frac{\theta}{2}\right)\left(1+\tan\frac{\theta}{2}\right)}$

$= 2\left(\frac{1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right) = \frac{2}{\cos\theta} = \frac{2}{a/b} = \frac{2b}{a}$

$=$ RHS

Hence proved.

8. We have,

$$\int \frac{1}{2e^{2x}+3e^x+1}dx = \int \frac{1}{\frac{2}{e^{-2x}+\frac{3}{e^{-x}}+1}}dx = \int \frac{e^{-2x}}{2+3e^{-x}+e^{-2x}}dx$$

Let e^{-x} = t.

$\Rightarrow$ -e^{-x} dx = dt

$\Rightarrow dx = -\frac{dt}{e^{-x}}$

$\therefore \quad I = \int \frac{-t\,dt}{2+3t+t^2} = -\int \frac{t}{t^2+3t+2}dt$

Let t = λ (2t + 3) + μ

Comparing the coefficients of like powers of t, we get

2λ = 1, 3λ + μ = 0 $\Rightarrow$ λ = $\frac{1}{2}$, and μ = $\frac{-3}{2}$

$\therefore\ I = -\int \frac{\lambda(2t+3)+\mu}{t^2+3t+2}dt$

$\Rightarrow$ I = $-\lambda\int \frac{2t+3}{t^2+3t+2}dt - \mu\int \frac{1}{t^2+3t+2}dt$

By using the respective values of λ and μ

$\Rightarrow$ I = $-\frac{1}{2}\int \frac{2t+3}{t^2+3t+2}dt + \frac{3}{2}\int \frac{1}{(t+3/2)^2-(1/2)^2}dt$

$\Rightarrow$ I = $-\frac{1}{2}\log\left|t^2+3t+2\right| + \frac{3}{2} \times \frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{t+\frac{3}{2}-\frac{1}{2}}{t+\frac{3}{2}+\frac{1}{2}}\right| + C$

$\Rightarrow$ I = $-\frac{1}{2}\log\left|e^{-2x}+3e^{-x}+2\right| + \frac{3}{2}\log\left|\frac{e^{-x}+1}{e^{-x}+2}\right| + C$

9. According to the question, x = a cos^3 θ and y = a sin^3 θ

We are required to find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$

Therefore,on differentiating both sides of x and y w.r.t θ ,we get,

$\frac{dx}{d\theta} = 3a\cos^2\theta\frac{d}{d\theta}(\cos\theta)$

$= 3a\cos^2\theta \cdot (-\sin\theta)$

$= -3a\cos^2\theta \cdot \sin\theta$

and $\frac{dy}{d\theta} = 3a\sin^2\theta\frac{d}{d\theta}(\sin\theta)$

$= 3a\sin^2\theta \cdot (\cos\theta) = 3a\sin^2\theta \cdot \cos\theta$

Now, $\frac{dy}{dx} = \left(\frac{dy/d\theta}{dx/d\theta}\right)$

$= \frac{3a\sin^2\theta\cdot\cos\theta}{-3a\cos^2\theta\cdot\sin\theta} = -\tan\theta$

Again, On differentiating both sides w.r.t x, we get,

$\frac{d^2y}{dx^2} = \frac{d}{dx}(-\tan\theta) = -\frac{d}{d\theta}(\tan\theta)\frac{d\theta}{dx}$

$= -\sec^2\theta \cdot \frac{d\theta}{dx}$

$= -\sec^2\theta \cdot \left(\frac{-1}{3a\cos^2\theta\cdot\sin\theta}\right) = \left(\frac{1}{3a\cos^4\theta\cdot\sin\theta}\right)$

$\therefore$ At $\theta = \frac{\pi}{6}$

$\left(\frac{d^2y}{dx^2}\right)_{at\theta=\frac{\pi}{6}} = \frac{1}{3a\left(\cos\frac{\pi}{6}\right)^4\left(\sin\frac{\pi}{6}\right)}$

$= \frac{1}{3a\left(\frac{\sqrt{3}}{2}\right)^4\left(\frac{1}{2}\right)}$

$= \frac{1}{3a\left(\frac{9}{16}\right)\left(\frac{1}{2}\right)} = \frac{32}{27a}$

OR

We have, y = $(\sin x)^{\cos x} + (\cos x)^{\sin x}$

$\Rightarrow y = e^{\log(\sin x)^{\cos x}} + e^{\log(\cos x)^{\sin x}}$

$\Rightarrow$ y = $e^{\cos x \log \sin x} + e^{\sin x \log \cos x}$

Differentiating with respect to x,

$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cos x\log\sin x}\right) + \frac{d}{dx}\left(e^{\sin x\log\cos x}\right)$

$= e^{\cos x\log\sin x}\frac{d}{dx}(\cos x\log\sin x) + e^{\sin x\log\cos x}\frac{d}{dx}(\sin x\log\cos x)$

$= e^{\log(\sin x)^{\cos x}}\left[\cos x\frac{d}{dx}\log\sin x + \log\sin x\frac{d}{dx}(\cos x)\right] + e^{\log(\cos x)^{\sin x}}\left[\sin x\frac{d}{dx}\log\cos x + \log\cos x\frac{d}{dx}(\sin x)\right]$

$= (\sin x)^{\cos x}\left[\cos x\frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log\sin x \times (-\sin x)\right] + (\cos x)^{\sin x}\left[\sin x\frac{1}{\cos x}\frac{d}{dx}(\cos x) + \log\cos x \times (\cos x)\right]$

= $(\sin x)^{\cos x}$[cot x cos x - sin x log sin x] + $(\cos x)^{\sin x}$[tan x(-sin x) + cos x log cos x]

= $(\sin x)^{\cos x}$[cot x cos x - sin x log sin x] + $(\cos x)^{\sin x}$[cos x log cos x - sin x tan x]

10. **Read the text carefully and answer the questions:**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) Bayes' theorem defines the probability of an event based on the prior knowledge of the conditions related to the event whereas in case of the condition probability, we find the reverse probabilities using Bayes' theorem.

(ii) Consider on event E which occurs via two different events A and B. The probability of E is given by the value of total probability as:

P(E) = P(A ∩ E) + P(B ∩ E)

P(E) = P(A) P$\left(\frac{E}{A}\right)$ + P(B)$\left(\frac{E}{B}\right)$

(iii) Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

E_1 = PQ, E_2 = $\bar{P}\bar{Q}$, E_3 = $\bar{P}$Q, E_4 = P$\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

$P(E_1) = 0.3 \times 0.2 = 0.06$,

$P(E_2) = 0.7 \times 0.8 = 0.56$,

$P(E_3) = 0.7 \times 0.2 = 0.14,$

$P(E_4) = 0.3 \times 0.8 = 0.24$

$P\left(\frac{E}{E_1}\right) = 0, P\left(\frac{E}{E_2}\right) + 0, P\left(\frac{E}{E_3}\right) = 1, P\left(\frac{E}{E_4}\right) = 1$

$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3).$

$P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)$

$= 0.14 + 0.24 = 0.38$

(iv)By Bayes' Theorem,

$$P\left(\frac{E_3}{E}\right) = \frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)}$$

$= \frac{0.14}{0.38} = \frac{7}{19}$

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses E_1 and E_2 are actually eliminated as $P\left(\frac{E}{E_1}\right) = P\left(\frac{E}{E_2}\right) = 0$

Alternative way of writing the solution:

i. P(Shell fired from exactly one of them hits the plane)
   = P[(Shell from A hits the plane and Shell from B does not hit the plane) or (Shell from A does not hit the plane and Shell from B hits the plane)]
   $= 0.3 \times 0.8 + 0.7 \times 0.2 = 0.38$

ii. $\frac{\text{P(Shell fired from B hit the plane} \cap \text{Exactly one of them hit the plane)}}{\text{P(Exactly one of them hit the plane)}}$
   $= \frac{P\text{ (Shell from only } B \text{ hit the plane)}}{P\text{ (Exactly one of them hit the plane)}}$
   $= \frac{0.14}{0.38} = \frac{7}{19}$

OR

Read the text carefully and answer the questions:

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let: E_1: represent the event when many workers were not present for the job;

E_2: represent the event when all workers were present; and

E: represent completing the construction work on time.

(i) $P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$

(ii) $P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$

$= 0.65 \times 0.35 + 0.35 \times 0.8$

$= 0.35 \times 1.45$

$= 0.51$

(iii) $P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \cdot P\left(\frac{E}{E_1}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)} = \frac{0.65 \times 0.35}{0.51} = 0.45$

(iv) $P\left(\frac{E_2}{E}\right) = \frac{P(E_2) \cdot P\left(\frac{E}{E_2}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)} = \frac{0.35 \times 0.8}{0.51} = 0.55$

11. **Read the text carefully and answer the questions:**

A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below:

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
I	10,000	2,000	18,000
II	6,000	20,000	8,000

If the unit Sale price of Pencil, Eraser and Sharpener are ₹2.50, ₹1.50 and ₹1.00 respectively,Based on the information given above, answer the following questions:

(i) Let A be the 2 $\times$ 3 matrix representing the annual sales of products in two markets.

$\therefore$ A = $\begin{bmatrix} x & y & z \\ 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}$

Let B be the column matrix representing the sale price of each unit of products x, y, z.

$\therefore$ B = $\begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$

Now, revenue = sale price $\times$ number of items sold

= $\begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$

= $\begin{bmatrix} 25000 + 3000 + 18000 \\ 15000 + 30000 + 8000 \end{bmatrix}$ = $\begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$

Therefore, the revenue collected from Market I = ₹ 46000

(ii) Let A be the 2 $\times$ 3 matrix representing the annual sales of products in two markets.

$\therefore$ A = $\begin{bmatrix} x & y & z \\ 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}$

Let B be the column matrix representing the sale price of each unit of products x, y, z.

$\therefore$ B = $\begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$

Now, revenue = sale price $\times$ number of items sold

= $\begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$

= $\begin{bmatrix} 25000 + 3000 + 18000 \\ 15000 + 30000 + 8000 \end{bmatrix}$ = $\begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$

The revenue collected from Market II = ₹ 53000.

(iii) Let C be the column matrix representing cost price of each unit of products x, y, z.

Then, C = $\begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$

$\therefore$ Total cost in each market is given by

$$AC = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

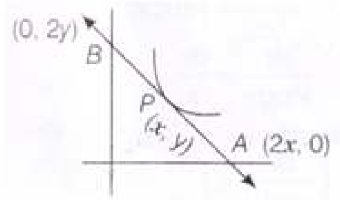
$$= \begin{bmatrix} 20000 + 2000 + 9000 \\ 12000 + 20000 + 4000 \end{bmatrix} = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$$

Now, Profit matrix = Revenue matrix - Cost matrix

$$= \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} - \begin{bmatrix} 31000 \\ 36000 \end{bmatrix} = \begin{bmatrix} 15000 \\ 17000 \end{bmatrix}$$

Therefore, the gross profit from both the markets = ₹ 15000 + ₹ 17000 = ₹ 32000

12. The below figure obtained by the given information



Let the coordinate of the point P is (x, y). It is given that, P is mid-point of AB.

So, the coordinates of points A and B are (2x, 0) and (0, 2y) respectively.

$\therefore$ Slope of $AB = \frac{0-2y}{2x-0} = -\frac{y}{x}$

Since, the segment AB is a tangent to the curve at P.

$\therefore \frac{dy}{dx} = -\frac{y}{x}$

$\Rightarrow \frac{dy}{y} = -\frac{dx}{x}$

On integrating both sides, we get

$\log y = -\log x + \log c$

$\log y = \log \frac{C}{x}$ ...(i)

Since, the given curve passes through (1, 1).

$\therefore \log 1 = \log \frac{C}{1}$

$\Rightarrow 0 = \log C$

$\Rightarrow C = 1$

$\therefore \log y = \log \frac{1}{x}$

$\Rightarrow y = \frac{1}{x}$

$\Rightarrow xy = 1$

OR

Given, $(x^2 + xy)dy = (x^2 + y^2)dx$

$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{x^2+y^2}{x^2+xy}\right)$ ....(i)

This is a homogeneous differential equation.

On putting y = vx $\Rightarrow \frac{dy}{dx} = v \cdot 1 + x\frac{dv}{dx}$ in Eq. (i), we get,

$v + x\frac{dv}{dx} = \left(\frac{x^2+v^2x^2}{x^2+x\cdot xv}\right)$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v^2}{1+v} - v = \frac{1+v^2-v-v^2}{1+v} = \frac{1-v}{1+v}$

$\therefore \left(\frac{1+v}{1-v}\right) dv = \frac{1}{x}dx$

On integrating both sides, we get

$\int \left(\frac{1+v}{1-v}\right) dv = \int \frac{1}{x}dx$

$\Rightarrow \quad \int \left[-1 + \frac{2}{1-v}\right] dv = \log|x| + \log C$

$\Rightarrow$ -v - 2 log (1 - v) = log | x | + log C

$\Rightarrow$ -v = 2log(1 - v) + log | x | + log C

$\Rightarrow$ -v = log$(1 - v)^2$ + log {C |x|} [$\because$ log m + log n = log mn]

$\Rightarrow$ -v = log{C|x|$(1-v)^2$}

$\Rightarrow \quad C|x|\left(1 - \frac{y}{x}\right)^2 = e^{-y/x} \left[\because v = \frac{y}{x}\right]$' ...(ii)

On putting x = 1 and y = 0 in Eq. (ii), we get

C·1(1 - 0) = e^0

$\Rightarrow$ C = 1

Thus, the required solution is

$|x|\left(1-\frac{y}{x}\right)^2=e^{-y/x}$

$\Rightarrow \quad (x-y)^2=|x|e^{-y/x}$

13. AB = 2x; BC = 2y

In $\triangle$ABC,

$(2x)^2 + (2y)^2 = (20)^2$

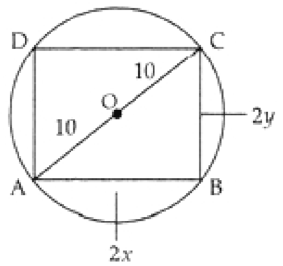
$\Rightarrow 4x^2 + 4y^2 = 400$

$\Rightarrow x^2 + y^2 = 100$ ...(i)

P = 4x + 4y = 4x + $4 \cdot \sqrt{100-x^2}$

$\therefore \frac{dP}{dx}=4-\frac{4x}{\sqrt{100-x^2}}=0$ [from (i)]

$\Rightarrow$ x = $5\sqrt{2}$ cm



$\frac{d^2P}{dx^2}=-4\left\{\frac{\sqrt{100-x^2}-\frac{x(-x)}{\sqrt{100-x^2}}}{100-x^2}\right\}$

$=\frac{-4\times100}{(100-x^2)^{\frac{3}{2}}}<0$

Hence, perimeter is maximum, when x = $5\sqrt{2}$

$\therefore y=5\sqrt{2}$ [from (i)]

$\Rightarrow$ x = y

$\therefore$ ABCD is square of side $10\sqrt{2}$ cm

OR

$V=\frac{1}{3}\pi R^2H$

$=\frac{1}{3}\pi R^2.(r+x)$

$=\frac{1}{3}\pi.(r^2-x^2)(r+x)\ \left[\because R^2=r^2-x^2\right]$

$\frac{dv}{dx}=\frac{1}{3}\pi\left[(r^2-x^2)(1)+(r+x)(0-2x)\right]$

$\frac{dv}{dx}=\frac{1}{3}\pi\left[r^2-x^2-2rx-2x^2\right]$

$=\frac{1}{3}\pi\left[r^2-2rx-3x^2\right]$

$=\frac{1}{3}\pi\left[r^2-3rx+rx-3x^2\right]$

$=\frac{1}{3}\pi\left[r(r-3x)+x(r-3x)\right]$

$=\frac{1}{3}\pi\left[(r-3x)(r+x)\right]$

For critical points let $\frac{dv}{dx}=0$

r = 3x

$\frac{r}{3}=x$

$\frac{d^2v}{dx^2}=\frac{1}{3}\pi\left[0-2r-6x\right]$

$\left.\frac{d^2v}{dx^2}\right]_{x=\frac{r}{3}}=\frac{1}{3}\pi\left[-2r-6\times\frac{r}{3}\right]$

$=\frac{1}{3}\pi\left[-4r\right]$

= - tive maximum

Altitude = r + x

$=\frac{r}{3}+r$

$=\frac{4r}{3}$

14. **Read the text carefully and answer the questions:**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) P(Grade A in Maths) = P(M) = 0.2
P(Grade A in Physics) = P(P) = 0.3
P(Grade A in Chemistry) = P(C) = 0.5
P(Grade A in all subjects) = $P(M \cap P \cap C) = P(M) \cdot P(P) \cdot P(C)$
P(Grade A in all subjects) = $0.2 \times 0.3 \times 0.5 = 0.03$

(ii) P(Grade A in Maths) = P(M) = 0.2
P(Grade A in Physics) = P(P) = 0.3
P(Grade A in Chemistry) = P(C) = 0.5
P(Grade A in no subjects) = $P(\bar{M} \cap \bar{P} \cap \bar{C}) = P(\bar{M}) \cdot P(\bar{P}) \cdot P(\bar{C})$
P(Grade A in no subjects) = $0.8 \times 0.7 \times 0.5 = 0.280$

(iii) P(Grade A in Maths) = P(M) = 0.2
P(Grade A in Physics) = P(P) = 0.3
P(Grade A in Chemistry) = P(C) = 0.5
P(Grade A in 2 subjects) = $P(M \cap P \cap \bar{C}) + P(P \cap C \cap \bar{M}) + P(M \cap C \cap \bar{P})$
P(Grade A in 2 subjects) = $0.2 \times 0.3 \times 0.5 + 0.3 \times 0.5 \times 0.8 + 0.2 \times 0.5 \times 0.7 = 0.03 + 0.12 + 0.07 = 0.22$
P(Grade A in 2 subjects) = 0.22

(iv) P(Grade A in Maths) = P(M) = 0.2
P(Grade A in Physics) = P(P) = 0.3
P(Grade A in Chemistry) = P(C) = 0.5
P(Grade A in atleast one subject) = 1 − P(grade A in no subject) = $1 - P(\bar{M} \cap \bar{P} \cap \bar{C})$
P(Grade A in atleast one subjects) = 1 − 0.280 = 0.72

SECTION B - 15 MARKS

15. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) **(b)** 3
Explanation: {
3

(ii) We need to find the distance between the given two planes,2x - y + 2z = 5 and 5x - 2.5y + 5z = 20.
Now, 2x - y + 2z = 5
2x - y + 2z - 5 = 0....(i)
=5[x - 0. 5y + z] = 20 (simplifying it)
$\Rightarrow \quad x - \frac{1}{2}y + z = 4$
and 5x - 2.5y + 5z = 20
= 2x - y + 2z - 8 = 0.....(ii)
Clearly, planes (i) and (ii) are parallel.
∴Distance between two parallel planes,

$$d = \left|\frac{d_2 - d_1}{\sqrt{a^2+b^2+c^2}}\right| = \left|\frac{-8-(-5)}{\sqrt{\left(2^2+(-1)^2+2^2\right.}}\right|$$

[∵d_2 = -8, d_1= -5, a = 2, b = -1 and c = 2]

$$= \left|\frac{-8+5}{\sqrt{4+1+4}}\right| = \left|\frac{-3}{\sqrt{9}}\right| = |-1| = 1$$

(iii) $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are two parallel vectors, so their direction ratios will be proportional.

$$\therefore \quad \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3} \Rightarrow \frac{2}{-2p} = \frac{3}{1}$$

$\Rightarrow -6p = 2 \Rightarrow p = \frac{2}{-6} \Rightarrow p = -\frac{1}{3}$

(iv) **(c)** $\left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29}\right)$

Explanation: {

Direction ratio's of the line are (2, 3, 4).

Therefore, the equation of the line is:

$\frac{x-0}{2} = \frac{y-0}{3} = \frac{z-0}{4} = \lambda$

Thus, the coordinates of any point P on the above line are P (2λ, 3λ, 4λ).

But , this point P also lies on the given plane: $2(2\lambda) + 3(3\lambda) + 4(4\lambda) - 12 = 0$.

$\Rightarrow 29\lambda = 12 \Rightarrow \lambda = \frac{12}{29}$

Therefore, the coordinates of the foot of perpendicular are given by: $\left(2 \times \frac{12}{29}, 3 \times \frac{12}{29}, 4 \times \frac{12}{29}\right)$

(v) Here $\vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$, then

$|\vec{n}| = \sqrt{4 + 9 + 36} = \sqrt{49}$

= 7 units

$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

So, the required equation is

$\vec{r} \cdot \left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = 5$

16. We have, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \Rightarrow |\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$.

Unit vector in the direction of $\vec{a}$ is given by

$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{(\hat{i}+2\hat{j}+3\hat{k})}{\sqrt{14}} = \left(\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}\right)$.

OR

L.H.S $= \left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right)$

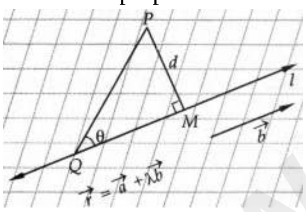
$= \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}$

$= 0 + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - 0 \,.[\vec{a} \times \vec{a} = \vec{b} \times \vec{b} = 0\,]$

$= \vec{a} \times \vec{b} + \vec{a} \times \vec{b}\ [\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}]$

$= 2(\vec{a} \times \vec{b})$

17. Let PM be perpendicular from P to line l and Q be point on it such that PQ makes an angle θ with l.



In right triangle PMQ, we have

$\sin\theta = \frac{PM}{PQ}$

$\Rightarrow \sin\theta = \frac{d}{PQ}$

$\Rightarrow$ d = PQ sin θ [by cross multiplication]

$\Rightarrow d|\vec{b}| = |\overrightarrow{PQ}||\vec{b}|\sin\theta$ [Multiplying both sides by $|\vec{b}|$]

$\Rightarrow d|\vec{b}| = |\vec{b} \times \overrightarrow{PQ}|$

[$\because \vec{b}$ is parallel to line /. So, the angle between $\vec{b}$ and $\overrightarrow{PQ}$ is also θ]

$\Rightarrow$ d = $\frac{|\vec{b} \times \overrightarrow{PQ}|}{|\vec{b}|}$

OR

The coordinates of any point on the first line are given by

$\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3} = \lambda$

$\Rightarrow$ x = λ

y = 2λ + 2

z = 3λ - 3

The coordinates of a general point on the first line is

$(\lambda, 2\lambda + 2, 3\lambda - 3)$

And the coordinates of any point on the second line are given by

$\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}=\mu$

$\Rightarrow$ x = 2μ + 2

y = 3μ + 6

z = 4μ + 3

The coordinates of a general point on the second line are

$(2\mu+2, 3\mu+6, 4\mu+3)$

If the lines intersect, then they have a common point. So, for some values of λ and μ we must have

$\lambda=2\mu+2, 2\lambda+2=3\mu+6, 3\lambda-3=4\mu+3$

$\Rightarrow \lambda-2\mu=2$ .....(i)

$2\lambda-3\mu=4$ ....(ii)

$3\lambda-4\mu=6$ ....(iii)

Substituting λ = 2 and μ = 0 in (iii), we get

LHS = $3\lambda-4\mu$

= 3(2) - 4(0)

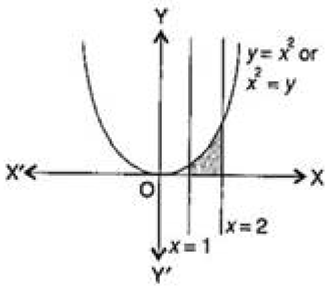
= 6

= RHS

Therefore $\lambda=2$ and μ = 0 satisfy the third equation, the required given lines intersect at (2, 6, 3).

18. Equation of the curve (parabola) is

y = x^2 ...(i)



Required area bounded by curve (i), vertical line x = 1, x = 2 and x - axis

$=\left|\int_1^2 y dx\right|$

$=\left|\int_1^2 x^2 dx\right|$

$=\left(\frac{x^3}{3}\right)_1^2$

$=\frac{8}{3}-\frac{1}{3}=\frac{7}{3}$ sq units

SECTION C - 15 MARKS

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) **(d)** P(x) = R(x) - C(x)

Explanation: {

P(x) = R(x) - C(x)

(ii) **(d)** 12

Explanation: {

Z = 2x + 3y

Z (3, 2) = 2 $\times$ 3 + 3 $\times$ 2

= 6 + 6 = 12

(iii) Let the line of regression of y on x be

x - 2y + 3 = 0

$\Rightarrow$ 2y = x + 3

$\Rightarrow$ y = $\frac{x}{2}+\frac{3}{2}$

$\therefore$ b_{xy} = $\frac{1}{2}$

Let the line of regression of x on y be

$\Rightarrow$ 4x - 5y + 1 = 0

$\Rightarrow$ 4x = 5y - 1

$\therefore$ x = $\frac{5}{4}$x - $\frac{1}{4}$

b_{xy} = $\frac{5}{4}$

$\therefore r^2 = b_{yx} \times b_{xy}$

= $\left(\frac{1}{2}\right)\left(\frac{5}{4}\right) = \frac{5}{8} < 1$

Hence, our assumption of regression equation is correct.

$\therefore$ r = $\sqrt{\frac{5}{8}}$ = 0.79

(iv)The break-even points are given by C(x) = R(x)

Now, C(x) = R(x)

$\Rightarrow$ x + 40 =10x - $0.2x^2$

$\Rightarrow$ $0.2x^2$ - 9x + 40 = 0

$\Rightarrow$ $2x^2$ - 90x + 400 = 0

$\Rightarrow$ x^2 - 45x + 200 = 0

$\Rightarrow$ (x - 40) (x - 5) = 0

$\Rightarrow$ x = 5, 40

Hence, the break-even points are x = 5 and x = 40

(v) Let R(x) be the revenue function. Then,

R(x) = px

$\Rightarrow$ R(x) = 1000x - $15x^2$ - x^3

When x = 2, we get

p = 1000 - 15 $\times$ 2 - 2^2 = 966 and,

R = 2000 - 15 $\times$ 4 - 8 = 1932

20. x = 106 - 2p

p = $\frac{1}{2}(106 - x)$

Revenue, R = px

= $\frac{1}{2}(106 - x)x$

= 53x - $\frac{x^2}{2}$

AC = 5 + $\frac{x}{50}$

C = AC(x)

= $\left(5 + \frac{x}{50}\right)x$

= 5 + $\frac{x^2}{50}$

P = Revenue - cost

$\frac{dP}{dx} = 48 - \frac{13(2x)}{25}$

$\frac{dP}{dx}$ = 0

48 - $\frac{13(2x)}{25}$ = 0

48 = $\frac{13\times 2x}{25}$

x = $\frac{13\times 25}{13\times 2}$ = 46.1538

$\frac{d^2P}{dx^2} = 0 - \frac{(13)^2}{25}$, negative since $\frac{d^2P}{dx^2}$ is negative

Profit is maximum at x = 46 units

Profit = 48x - $\frac{13}{25}x^2$

When x = 46

Profit = 48 $\times$ 46 - $\frac{13}{25}$ $\times$ 46 $\times$ 46

= 2208 - $\frac{27508}{25}$

= 2208 - 1100.32

= Rs 1107.68

OR

Given, total cost function,

C(x) = x + $2x^3$ - $3.5x^2$

Here, $AC = \frac{C}{x}$

= $\frac{1}{x}$ (x + $2x^3$ - $3.5x^2$)

= 1 + $2x^2$ - 3.5x

= $2x^2$ - 3.5x + 1

We know that, marginal average cost (MAC) is

MAC = $\frac{d}{dx}(AC)$

= $\frac{d}{dx}\left(2x^2 - 3.5x + 1\right)$

⇒ MAC = 4x - 3.5

Since, MAC is linear, it is a straight line.

⇒ It will cut x-axis at (0.875, 0) and y-axis at (0, -3.5)

21. Given, $\bar{x}$ = 84, $\bar{y}$ = 81

σ_x = 7, σ_y = 4

and r = 0.86

$\therefore b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$

= 0.86 × $\frac{4}{7}$ = 0.49

∴ Regression equation of y on x

y - $\bar{y}$ = b_{yx}(x $\bar{x}$)

⇒ y - 81 = 0.49(x - 84)

⇒ y - 81 = 0.49x - 41.16

⇒ y = 0.49x + 39.84

Putting x = 92,

y = 45.08 + 39.84 = 84.92

Hence, the likely marks in Physics are 84.92.

22. According to the given situation , the given data can be tabularised as following

	Tailor A	**Tailor B**	**Minimum Total No.**
No. of Shirts	6	10	60
No. of Trousers	4	4	32
Wage	Rs 300/day	Rs 400/day	

Let tailor A and tailor B work for x days and y days, respectively.Given that the minimum number of shirts that can be stitched per day is 60. The inequality representing the information is given as

∴ 6x + 10y $\geq$ 60 ⇒(shirt constraint) (dividing by 2 we get)

3x + 5y $\geq$ 30

Given that the minimum number of trousers that can be stitched per day is 32.

∴ 4x + 4y $\geq$ 32 ⇒ (trouser constraint) (dividing throughout by 4 we get

x + y = 8

∴ x $\geq$ 0, y $\geq$ 0 (non negative constarint which restricts the feasible region in the first quadrant only , since it is real world situation and the variables cannot take negative values.

Let z be the objective function representing the total labour cost. Hence the equation for the cost function is given as z = 300x + 400y

So, the given L P.P. is designed as

z = 300x + 400y

x $\geq$ 0, y $\geq$ 0, 3x + 5y $\geq$ 30 and x + y $\geq$ 8

OR

Let x and y be the number of packets of food P and Q respectively. Obviously $x \geq 0, y \geq 0$. Mathematical formulation of the given problem is as follows: Minimise Z = 6x + 3y (vitamin A)

subject to the constraints

$12x + 3y \geq 240$ (constraint on calcium), i.e. $4x + y \geq 80$ ......(i)

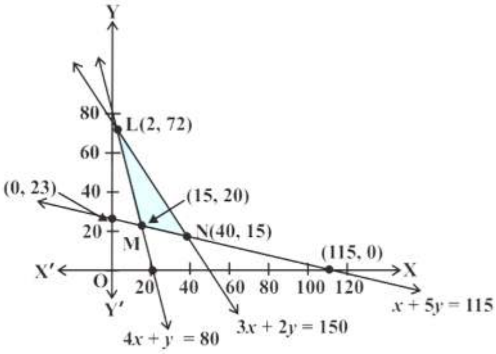
$4x + 20y \geq 460$ (constraint on iron), i.e. $x + 5y \geq 115$ ......(ii)

$6x + 4y \leq 300$ (constraint on cholesterol), i.e. $3x + 2y \leq 150$ ......(iii)

$x \geq 0, y \geq 0$ ......(iv)

Let us graph the inequalities (i) to (iv).

The feasible region (shaded) determined by the constraints (i) to (iv) is shown in Figure and note that it is bounded.



The coordinates of the corner points L, M and N are (2, 72), (15, 20) and (40, 15) respectively. Let us evaluate Z at these points:

Corner Point	Z = 6x + 3y
(2, 72)	228
(15, 20)	**150** Minimum
(40, 15)	285

From the table, we find that Z is minimum at the point (15, 20). Hence, the amount of vitamin A under the constraints given in the problem will be minimum, if 15 packets of food P and 20 packets of food Q are used in the special diet. The minimum amount of vitamin A will be 150 units.