ISC 2025 EXAMINATION

Sample Question Paper - 1

Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

This Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions*

EITHER** from **Section B OR Section C.

Section A**: Internal choice has been provided in **two questions of two marks each, two questions of four marks each and two questions of six marks each.

Section B:** Internal choice has been provided in **one question of two marks and one question of four marks.

Section C:** Internal choice has been provided in **one question of two marks and one question of four marks.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

1. **In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.** **[15]**

(a) If $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}$ = 0, then the value of x is **[1]**

a) $\pm 6\sqrt{5}$ b) $5\sqrt{5}$

c) $\pm 4\sqrt{3}$ d) $\pm 3\sqrt{5}$

(b) $\int_0^\infty \frac{1}{1+e^x} dx$ equals **[1]**

a) - log 2 b) log 2

c) log 2 - 1 d) log 4 - 1

(c) $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$ is equal to **[1]**

a) $\frac{3\pi}{4}$ b) $-\frac{5\pi}{4}$

c) $\frac{5\pi}{4}$ d) $-\frac{\pi}{4}$

(d) The solution of $x\frac{dy}{dx}$ + y = e^x is: **[1]**

a) $x = \frac{e^y}{y} + \frac{k}{y}$ b) $y = \frac{e^x}{x} + \frac{k}{x}$

c) y = xe^x + k d) y = xe^x + cx

(e) A bag X contains 2 white and 3 black balls and another bag Y contains 4 white and 2 black balls. One **[1]** bag is selected at random and a ball is drawn from it. Then, the probability of the chosen ball to be white is

a) $\frac{2}{15}$

b) $\frac{7}{15}$

c) $\frac{8}{15}$

d) $\frac{14}{15}$

(f) A relation R defined on a set of human beings as $R = \{(x, y) : x \text{ is 5 cm shorter than } y\}$ is **[1]**

a) reflexive only

b) symmetric and transitive

c) neither transitive, nor symmetric, nor reflexive

d) reflexive and transitive

(g) If $y^2 = ax^2 + b$, then $\frac{d^2y}{dx^2}$ is equal to **[1]**

a) $\frac{ab}{x^2}$

b) $\frac{ab}{y^2}$

c) $\frac{ab}{y^3}$

d) $\frac{ab}{x^3}$

(h) The function f(x) = $\begin{cases} 1, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$ is not continuous at **[1]**

a) x = 0

b) x = -1

c) x = 2

d) x = 1

(i) If A is an invertible matrix of order 3, then which of the following information is NOT true? **[1]**

a) $(AB)^{-1} = B^{-1} A^{-1}$, where $B = [b_{ij}]_{3 \times 3}$ and $|B| \neq 0$

b) $(A^{-1})^{-1} = A$

c) $|\text{adj } A| = |A|^2$

d) If BA = CA, then $B \neq C$, where B and C are square matrices of order 3

(j) **Assertion (A):** If $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$, then x = 2, y = 2, z = -5 and w = 4. **[1]**

Reason (R): Two matrices are equal, if their orders are same and their corresponding elements are equal.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

(k) For the set A = {1, 2, 3}, define a relation R in the set A as follows: **[1]**

R = {(1, 1), (2, 2), (3, 3), (1, 3)}.

Write the ordered pairs to be added to R to make it the smallest equivalence relation.

(l) Construct a 2 × 2 matrix A = $[a_{ij}]$ whose element $a_{ij} = \frac{|-3i+j|}{2}$. **[1]**

(m) Give an example of a function which is neither one-one nor onto. **[1]**

(n) Let E_1 and E_2 be two independent events such that $P(E_1) = P_1$ and $P(E_2) = P_2$. Describe in words the events whose probability is: P_1P_2 **[1]**

(o) Let E_1 and E_2 be two independent events such that $P(E_1) = P_1$ and $P(E_2) = P_2$. Describe in words of the events whose probability is: $(1 - P_1)P_2$ **[1]**

2. Differentiate w.r.t. x: $\sin\sqrt{x} + \cos^2\sqrt{x}$ **[2]**

OR

Show that f(x) = sin x is increasing on (0, $\frac{\pi}{2}$) and decreasing on ($\frac{\pi}{2}$, π) and neither increasing nor decreasing in (0, π).

3. Evaluate: $\int \frac{2x+3}{(x-1)^2} dx$ **[2]**

4. Show that the tangents to the curve y = $2x^3$ - 4 at the points where x = 2 and x = -2 are parallel. **[2]**

5. Integrate the function: $\frac{x \cos^{-1} x}{\sqrt{1-x^2}}$ **[2]**

OR

Evaluate the definite integral: $\int_0^{\infty} \frac{1}{a^2+b^2x^2} dx$

6. Let R be a relation on the set A of ordered pairs of positive integers defined by **[2]**

(x, y) R (u, v) if and only if xv = yu.

Show that R is an equivalence relation.

7. Find the value of $\tan^{-1}\left(\tan \frac{5\pi}{6}\right) + \cos^{-1}\left(\cos \frac{13\pi}{6}\right)$ **[4]**

8. Evaluate: $\int \frac{1}{\cos x(5-4\sin x)} dx$ **[4]**

9. If $y = e^{\tan^{-1} x}$, prove that $(1 + x^2) y_2 + (2x - 1) y_1 = 0$ **[4]**

OR

Differentiate $\sin^{-1} \sqrt{1-x^2}$ with respect to $\cos^{-1}x$, if x ∈ (-1, 0)

10. **Read the text carefully and answer the questions:** **[4]**

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.

(a) Teacher ask Govind, what is the probability that tickets are drawn by Abhishek, shows a prime number on one ticket and a multiple of 4 on other ticket?

(b) Teacher ask Girish, what is the probability that tickets drawn by Ankit, shows an even number on first ticket and an odd number on second ticket?

(c) Teacher asks Abhishek, what is the probability that tickets drawn by Vinod, shows a multiple of 4 on one ticket and a multiple 5 on other ticket?

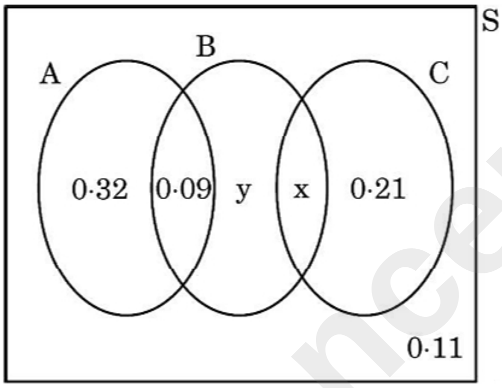
(d) Teacher asks Vinod, what is the probability that both tickets drawn by Girish shows odd number?

OR

Read the text carefully and answer the questions: **[4]**

There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.



(a) Find the value of x.

(b) Find the value of y.

(c) Find $P\left(\frac{C}{B}\right)$.

(d) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

11. **Read the text carefully and answer the questions:** **[6]**

If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are two matrices, then $A \pm B$ is of order $m \times n$ and is defined as $(A \pm B)_{ij} = a_{ij} \pm b_{ij}$, where i = 1, 2, ..., m and j = 1, 2, ..., n

If $A = [a_{ij}]_{m \times n}$ and $B = [a_{ij}]_{m \times p}$ are two matrices, then AB is of order $m \times p$ and is defined as $(AB)_{ik} = \sum_{r=1}^{n} a_{ir}\, b_{rk} = a_{1k}b_{1k} + a_{i2}b_{2k} + ... + a_{in}b_{nk}$

Consider $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ and $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(a) What is the product of matrices A and B?

(b) What are the values of 'a' and 'c' in matrix D such that the equation CD - AB = 0 holds?

(c) What are the values of 'b' and 'd' in matrix D such that the equation CD - AB = 0 holds?

12. Find the general solution of $\frac{dy}{dx}$ - 3y = sin 2x **[6]**

OR

In a bank, principal increases continuously at the rate of 5% per year. An amount of ₹ 1000 is deposited with this bank, how much will it worth after 10 years $\left(e^{0.5} = 1.648\right)$.

13. Two sides of a triangle have lengths a and b and the angle between them is θ. What value of θ will maximize the area of the triangle? **[6]**

OR

Find the dimensions of the rectangle of area 96 cm^2 whose perimeter is the least. Also, find the perimeter of the rectangle.

14. **Read the text carefully and answer the questions:** **[6]**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

(a) Find the probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics?

(b) Find the probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics?

(c) Find the probability that the selected student has passed in Mathematics, if it is known that he has failed in Economics?

(d) Find the probability that the selected student has passed in Economics, if it is known that he has failed in Mathematics?

SECTION B - 15 MARKS

15. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) The vectors $\lambda\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} + \lambda\hat{j} - \hat{k}$, and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if **[1]**

a) $\lambda = -2$ b) $\lambda = 0$

c) $\lambda = 1$ d) $\lambda = -1$

(b) The equations of a line are 5x - 3 = 15y + 7 = 3 - 10z. Write the direction cosines of the line and find the coordinates of a point through which it passes. **[1]**

(c) If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$. **[1]**

(d) A variable plane moves so that the sum of the reciprocals of its intercepts on the coordinate axes is (1/2). Then, the plane passes through the point. **[1]**

a) (0, 0, 0) b) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

c) (1, 1, 1) d) (2, 2, 2)

(e) Determine the direction cosine of the normal to the plane z = 3 and the distance from the origin. **[1]**

16. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be given as $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, and $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then show that **[2]**

$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

OR

Find the value of λ for which $\vec{a}$ and $\vec{b}$ are perpendicular, where $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k})$.

17. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $r.\left(2\hat{i} - \hat{j} + \hat{k}\right) + 3 = 0$ **[4]**

OR

Find the angle between the two planes 2x + y - 2z = 5 and 3x - 6y - 2z = 7 using vector method.

18. Draw the rough sketch of $y^2 + 1$ = x, x $\leq$ 2. Find the area enclosed by the curve and the line x = 2 **[4]**

SECTION C - 15 MARKS

19. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) Which condition is true if AC is increases with x and AC curve is rising? **[1]**

a) AC = MC
b) MC > AC
c) MC < AC
d) MC = AC

(b) The optimal value of the objective function is attained at the points: **[1]**

a) given by intersection of inequalities with the axes only.
b) given by the corner points of the feasible reason.
c) given by intersection of inequalities with the x-axis only.
d) given by intersection of inequalities with the y-axis only.

(c) Two regression lines are represented by 4x + 10y = 9 and 6x + 3y = 4. Find the line of regression of y on x. **[1]**

(d) If the total cost function for a manufacturer is given by C = $\frac{5x^2}{\sqrt{x^2+3}} + 5000$, find the marginal cost function. **[1]**

(e) The total variable cost of manufacturing x units in a firm is ₹$\left(3x + \frac{x^5}{25}\right)$. Show that the average variable cost increases with output x. **[1]**

20. A company produces a commodity with ₹ 24,000 as fixed cost. The variable cost estimated to be 25% of the total revenue received on selling the product, is at the rate of 8 per unit. Find the break-even point. **[2]**

OR

In the production unit of a firm it is found that the total number of units (x) produced is dependent upon the number of workers (n) and is obtained by the relation x = 25n $(n^3 + 36)^{-1/2}$, the demand function of the product is p = $\frac{250}{x+15}$. Determine the marginal revenue when n = 4.

21. For the given lines of regression, 3x - 2y = 5 and x - 4y = 7, find **[4]**

i. regression coefficients b_{yx} and b_{xy}

ii. coefficient of correlation r(x, y).

22. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹ 50 each for type A and ₹ 60 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit? Formulate the above LPP and solve it graphically and also find the maximum profit. **[4]**

OR

Minimize Z = 3x + 2y subject to the constraints:

$x + y \geq 8$

$3x + 5y \leq 15$

$x \geq 0,\ y \geq 0$

Solution

SECTION A - 65 MARKS

1. In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(i) **(c)** $\pm 4\sqrt{3}$

Explanation: {

Given, $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

⇒ x × 1 + (-5) × 0 + (-1) × 2 x × 0 + (-5) × 2 + (-1) × 0

$x \times 2 + (-5) \times 1 + (-1) \times 3] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

⇒ [(x - 2) × x + (-10) × 4 + (2x - 8) × 1] = 0

⇒ x^2 - 2x - 40 + 2x - 8 = 0

⇒ x^2 = 48

$\Rightarrow x = \pm\sqrt{48} = \pm 4\sqrt{3}$

(ii) **(b)** log 2

Explanation: {

log 2

We have

$I = \int_0^\infty \frac{1}{1+e^x} dx$

Putting $e^x = t$

⇒ $e^x dx = dt$

$\Rightarrow dx = \frac{dt}{t}$

When $x \to 0; t \to 1$

and $x \to \infty; t \to \infty$

$\therefore I = \int_1^\infty \frac{1}{t(1+t)} dt$

$= \int_1^\infty \frac{1}{t+t^2} dt$

$= \int_1^\infty \frac{1}{\left(t+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dt$

$= \frac{1}{2\times\frac{1}{2}} \left[\log\left|\frac{t+\frac{1}{2}-\frac{1}{2}}{t+\frac{1}{2}+\frac{1}{2}}\right|\right]_1^\infty$

$= \left[\log\left|\frac{t}{t+1}\right|\right]_1^\infty$

$= \left[\log\left|\frac{\frac{t}{t}}{\frac{t}{t}+\frac{1}{t}}\right|\right]_1^\infty$

$= \left[\log\left|\frac{1}{1+\frac{1}{t}}\right|\right]_1^\infty$

$= \log\frac{1}{1+0} - \log\frac{1}{1+1}$

$= \log(1) - \log\left(\frac{1}{2}\right)$

= 0 - (- log 2)

= log 2

(iii) **(a)** $\frac{3\pi}{4}$

Explanation: {

We know that principle value branch of $\cos^{-1}$ is $[0,\pi]$

and $\frac{3\pi}{4} \notin [0,\pi]$ but $\left(2\pi - \frac{5\pi}{4}\right) \in [0,\pi]$

$\therefore \cos^{-1}\left(\cos\frac{5\pi}{4}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{4}\right)\right) = \cos^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right) = \frac{3\pi}{4}$

(iv) **(b)** $y = \frac{e^x}{x} + \frac{k}{x}$

Explanation: {

We have, $x\frac{dy}{dx} + y = e^x$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$

This is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

P = $\frac{1}{x}$ and Q = $\frac{e^x}{x}$

$\therefore$ IF $= e^{\int \frac{1}{x}dx} = e^{(\log x)} = x$

So, the general solution is:

$y \cdot x = \int \frac{e^x}{x} x dx$

$\Rightarrow$ y·x = $\int e^x dx$

$\Rightarrow$ y ·x = e^x + k

$\Rightarrow y = \frac{e^x}{x} + \frac{k}{x}$

(v) **(c)** $\frac{8}{15}$

Explanation: {

A white ball can be drawn in two mutually exclusive ways:

i. Selecting bag X and then drawing a white ball from it.
ii. Selecting bag Y and then drawing a white ball from it.

Let E_1, E_2 and A be three events as defined below:

E_1 = Selecting bag X

E_2 = Selecting bag Y

A = Drawing a white ball

We know that one bag is selected randomly.

$\therefore$ P(E_1) = $\frac{1}{2}$

P(E_2) = $\frac{1}{2}$

$P\left(\frac{A}{E_1}\right) = \frac{2}{5}$

$P\left(\frac{A}{E_2}\right) = \frac{4}{6} = \frac{2}{3}$

Using the law of total probability, we get

P(A) = $\frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{2}{3}$

= $\frac{1}{5} + \frac{1}{3}$

= $\frac{3+5}{15} = \frac{8}{15}$

(vi) **(c)** neither transitive, nor symmetric, nor reflexive

Explanation: {

The relation R defined on a set of human beings as

$R = \{(x,y) : x$ is 5 cm shorter than y}

is neither transitive, nor symmetric, nor reflexive.

(vii) **(c)** $\frac{ab}{y^3}$

Explanation: {

$y^2 = ax^2 + b \Rightarrow 2y\frac{dy}{dx} = 2ax \Rightarrow y\frac{dy}{dx} = ax$

$\Rightarrow \frac{dy}{dx} = \frac{ax}{y} \Rightarrow \frac{d^2y}{dx^2} = \frac{ya - ax\frac{dy}{dx}}{y^2}$

$= \frac{ya - ax\frac{ax}{y}}{y^2} = \frac{a(y^2 - ax^2)}{y^3} = \frac{ab}{y^3}$

(viii) **(a)** x = 0

Explanation: {

Given, function f(x) = $\begin{cases} 1, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$

At x = 0,

Value of function, f(0) = 2

And $\lim_{x\to 0}$ f(x) = 1

$\because \lim_{x\to 0}$ f(x) $\neq$ f(0)

or $\lim_{x\to a}$ f(x) $\neq$ value of function

$\therefore$ Function f(x) is not continuous at x = 0.

(ix) **(d)** If BA = CA, then B $\neq$ C, where B and C are square matrices of order 3

Explanation: {

BA = CA

$\Rightarrow$ $BAA^{-1} = CAA^{-1}$

$\Rightarrow$ BI = CI

$\Rightarrow$ B = C

(x) **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: {

We have, $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$

On comparing both the matrices, we get

z + 5 = 0 $\Rightarrow$ z = -5

4 = w $\Rightarrow$ w = 4

x + y = 4 and xy = 4 $\Rightarrow$ y = $\frac{4}{x}$

$\therefore x + \frac{4}{x} = 4 \Rightarrow x^2 + 4 = 4x$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow$ x = $\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(4)}}{2(1)}$

$\Rightarrow x = \frac{4\pm\sqrt{16-16}}{2} = \frac{4}{2} = 2$

$\therefore$ y = 4 - x = 4 - 2 = 2

(xi)(3, 1) is the single ordered pair which needs to be added to R to make it the smallest equivalence relation.

(xii) Here, A = $[a_{ij}]_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ ....(i)

$a_{ij} = \frac{|-3i+j|}{2}$

Thus, $a_{11} = \frac{|-3\times 1+1|}{2} = \frac{|-3+1|}{2} = 1$

$a_{12} = \frac{|-3\times 1+2|}{2} = \frac{|-3+2|}{2} = \frac{1}{2}$

$a_{21} = \frac{|-3\times 2+1|}{2} = \frac{|-6+1|}{2} = \frac{5}{2}$

$a_{22} = \frac{|-3\times 2+2|}{2} = \frac{|-6+2|}{2} = 2$

Thus, A = $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$

(xiii) let function f:R$\to R$ by defined by f(x)=x^2

Now f(x) = x^2

for one one:

let $f(x_1) = (x_1)^2$, $f(x_2) = (x_2)^2 \in R(co-domain)$

if $f(x_1) = f(x_2)$

$=(x_1)^2 = (x_2)^2$

$= x_1 = x_2$ or $x_1 = -x_2$

Since x_1 does not have a unique image it is not one - one

For onto

f(x) = y

such that y ∈ R

x^2 = y

= x = $\pm\sqrt{y}$

If y is negative under root of a negative number is not real

Hence, f(x) is not onto.

∴ f(x) is neither onto nor one - one

(xiv) Here, P_1P_2 = $P(E_1)$. $P(E_2)$ = P ($E_1 \cap E_2$)

So, E_1 and E_2 occur simultaneously.

(xv) We know that P(A) + P(A') = 1

Since the events are independent

∴ P ($E_1' \cap E_2$) =$P(E_1')\cdot P(E_2)$

So, E_1 does not occur but E_2 occurs.

2. Let $y=\sin\sqrt{x}+(\cos\sqrt{x})^2$

$\therefore \frac{dy}{dx}=\frac{d}{dx}\sin\left(x^{1/2}\right)+\frac{d}{dx}\left[\cos\left(x^{1/2}\right)\right]^2$

$=\cos x^{1/2}\frac{d}{dx}x^{1/2}+2\cos\left(x^{1/2}\right)\frac{d}{dx}\left[\cos\left(x^{1/2}\right)\right]$

$=\cos\left(x^{1/2}\right)\frac{1}{2}x^{-1/2}+2\cdot\cos\left(x^{1/2}\right)\cdot\left[-\sin\left(x^{1/2}\right)\cdot\frac{d}{dx}x^{1/2}\right]$

$=\cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}\left[-2\cos\left(x^{1/2}\right)\right]\cdot\sin x^{1/2}\cdot\frac{1}{2\sqrt{x}}$

$=\frac{1}{2\sqrt{x}}[\cos(\sqrt{x})-\sin(2\sqrt{x})]$

OR

Given: f(x) = sin x

Theorem: Let f be a differentiable real function defined on an open interval (a,b), then

i. If f'(x) > 0 for all x $\in(a,b)$, then f(x) is increasing on (a, b)
ii. If f'(x) < 0 for all x $\in(a,b)$, then f(x) is decreasing on (a, b)

For the value of x obtained in (ii) f(x) is increasing and for remaining points in its domain it is decreasing.

Here we have,

f(x) = sin x

$\Rightarrow f(x)=\frac{d}{dx}(\sin x)$

⇒ f'(x) = cosx

Taking different region from 0 to 2π

let x $\in\left(0,\frac{\pi}{2}\right)$

⇒ cos(x) > 0

⇒ f'(x) > 0

Thus, f(x) is increasing in $\left(0,\frac{\pi}{2}\right)$

Let x $\in\left(\frac{\pi}{2},\pi\right)$

⇒ cos(x) < 0

⇒ f'(x) < 0

Thus f(x) is decreasing in $\left(\frac{\pi}{2},\pi\right)$

Therefore, from the above condition we find that

⇒ f(x) is increasing in $\left(0,\frac{\pi}{2}\right)$ and decreasing in $\left(\frac{\pi}{2},\pi\right)$

Hence, f(x) is neither increasing nor decreasing in (0,π)

3. Let I = $\int\frac{2x+3}{(x-1)^2}dx$. Then,

$I=\int\frac{2x+2-2+3}{(x-1)^2}\times dx$

$=\int\frac{2x-2+5}{(x-1)^2}\times dx$

$=2\int\frac{(x-1)}{(x-1)^2}\times dx+5\int\frac{1}{(x-1)^2}dx$

$=2\int\frac{1}{x-1}\times dx+5\int(x-1)^{-2}\times dx$

$=2\log|x-1|+5\times\frac{(x-1)^{-1}}{-1}+c$

$= 2\log|x-1| - \frac{5}{x-1} + c$

$\therefore\ I = 2\log|x-1| - \frac{5}{x-1} + c$

4. Slope = m: $\frac{dy}{dx}$ = $6x^2$

m at (x =2) = 24

m at (x = -2) = 24

we know that if the slope of curve at two different point is equal then straight lines are parallel at that points, so straight lines are parallel.

5. Let $I = \frac{x\cos^{-1}x}{\sqrt{1-x^2}}$

$I = -\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1}x dx$

Now, integrating by parts, we get,

$I = \frac{-1}{2}\left[\cos^{-1}x \int \frac{-2x}{\sqrt{1-x^2}}dx - \int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int \frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$

$= -\frac{1}{2}\left[\cos^{-1}x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}dx\right]$

$= -\frac{1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x + \int 2dx\right]$

$= -\frac{1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x + 2x\right] + C$

$= -\left[\sqrt{1-x^2}\cos^{-1}x + x\right] + C$

OR

Let I = $\int_0^\infty \frac{1}{a^2+b^2x^2}dx$. Then we have

$I = \frac{1}{a^2}\int_0^\infty \frac{1}{1+\frac{b^2x^2}{a^2}}dx$

$\Rightarrow I = \frac{1}{a^2}\int_0^\infty \frac{1}{1+\left(\frac{bx}{a}\right)^2}dx$

$\Rightarrow I = \frac{a}{ba^2}\left[\tan^{-1}\left(\frac{bx}{a}\right)\right]_0^\infty$

$\Rightarrow I = \frac{1}{ab}\left(\tan^{-1}\infty - \tan^{-1}0\right)$

$\Rightarrow I = \frac{\pi}{2ab}$

6. Clearly, (x, y) R (x, y), $\forall$ (x, y) $\in$ A, since xy = yx.

This shows that R is reflexive.

Further, (x, y) R (u, v)

$\Rightarrow$ xv = yu

$\Rightarrow$ uy = vx

$\Rightarrow$ (u, v) R (x, y).

This shows that R is symmetric.

Similarly, (x, y) R (u, v) and (u, v) R (a, b) $\Rightarrow$ xv = yu and ub = va

$\Rightarrow xv\frac{a}{u} = yu\frac{a}{u} \Rightarrow xv\frac{b}{v} = yu\frac{a}{u} \Rightarrow$ xb = ya

$\Rightarrow$ (x, y) R (a, b).

Thus, R is transitive. Thus, R is an equivalence relation.

7. We know that, $\tan^{-1}(\tan x) = x$; $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\cos^{-1}(\cos x) = x$, $x \in [0,\ \pi]$

$\therefore \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right] + \cos^{-1}\left[\cos\left(\pi + \frac{7\pi}{6}\right)\right]$

$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cos^{-1}\left(-\cos\frac{7\pi}{6}\right)$ $[\cos(\pi+\theta) = -\cos\theta]$

$= -\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \pi - \left[\cos^{-1}\cos\left(\frac{7\pi}{6}\right)\right]$ ....{$\because$ $\tan^{-1}(-x) = -\tan^{-1}x$, x $\in$ R and $\cos^{-1}(-x) = \pi - \cos^{-1}x$, x $\in$ [-1, 1]}

$= -\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \pi - \cos^{-1}\left[\cos\left(\pi + \frac{\pi}{6}\right)\right]$

$= -\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \pi - \left[\cos^{-1}\left(-\cos\frac{\pi}{6}\right)\right]$ $[\because \cos(\pi-\theta) = -\cos\theta]$

$= -\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \pi - \pi + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$ $[\because \cos^{-1}(-x) = \pi - \cos^{-1}x]$

$= -\frac{\pi}{6} + 0 + \frac{\pi}{6} = 0$

Note : Remember that, $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) \neq \frac{5\pi}{6}$ and $\cos^{-1}\left(\cos\frac{13\pi}{6}\right) \neq \frac{13\pi}{6}$

Since, $\frac{5\pi}{6} \notin \left(-\frac{\pi}{2},\ \frac{\pi}{2}\right)$ and $\frac{13\pi}{6} \notin [0,\ \pi]$

8. Let, I = $\int \frac{dx}{\cos x(5-4\sin x)}$

Multiplying and dividing by cos x

$\Rightarrow$ I = $\int \frac{\cos x dx}{\cos^2 x(5-4\sin x)}$

$\Rightarrow$ I = $\int \frac{\cos x dx}{(1-\sin^2 x)(5-4\sin x)}$

Let, sin x = t, cos x dx = dt

$\because$ I = $\int \frac{dt}{(1-t^2)(5-4t)}$

Now, by using partial fractions

$\frac{1}{(1-t^2)(5-4t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{5-4t}$

$\Rightarrow$ 1 = A(1 + t) (5 - 4t) + B(1 - t) (5 - 4t) + C(1 - t^2)

For t = 1,A = $\frac{1}{2}$

For t = -1, B =$\frac{1}{18}$

For t = $\frac{5}{4}$,c = -$\frac{16}{9}$

$\because$ I = $\frac{1}{2}\int \frac{dt}{1-t} + \frac{1}{18}\int \frac{dt}{1+t}$ - $\frac{16}{9}\int \frac{dt}{5-4t}$

$\Rightarrow$ I = $-\frac{1}{2}\log|1-\mathrm{t}| + \frac{1}{18}\log|1+t| + \frac{4}{9}\log|5-4t| + c$

So, I = $-\frac{1}{2}\log|1-\sin x| + \frac{1}{18}\log|1+\sin x| + \frac{4}{9}\log|5-4\sin x| + c$

9. Note: y_2 represents second order derivative i.e., $\frac{d^2y}{dx^2}$ and y_1 = dy/dx

Given,

$y = e^{\tan^{-1} x}$ ...(i)

To prove: (1 + x^2) y_2 + (2x - 1) y_1 = 0

To find the above we will do the double differentiation of the given function

As, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

So, lets first find dy/dx

$\frac{dy}{dx} = \frac{d}{dx}e^{\tan^{-1} x}$

Using chain rule we will differentiate the above expression

Let t = $\tan^{-1}$ x $\Rightarrow$ $\frac{dt}{dx} = \frac{1}{1+x^2}$

$\left[\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}\right]$

And y = e^t

$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$

$\frac{dy}{dx} = e^t\frac{1}{1+x^2} = \frac{e^{\tan^{-1} x}}{1+x^2}$ ...(ii)

Again differentiating with respect to x applying product rule:

$\frac{d^2y}{dx^2} = e^{\tan^{-1} x}\frac{d}{dx}\left(\frac{1}{1+x^2}\right) + \frac{1}{1+x^2}\frac{d}{dx}e^{\tan^{-1} x}$

Using chain rule we will differentiate the above expression-

$\frac{d^2y}{dx^2} = \left(\frac{e^{\tan^{-1} x}}{(1+x^2)^2}\right) - \frac{2xe^{\tan^{-1} x}}{(1+x^2)^2}$ [using equation (ii); $\frac{d}{dx}(x^n) = nx^{n-1}$ & $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$]

$(1+x^2)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x}}{1+x^2} - \frac{2xe^{\tan^{-1} x}}{1+x^2}$

$(1+x^2)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x}}{1+x^2}(1-2x)$

Using equation (ii)

$(1+x^2)\frac{d^2y}{dx^2} = \frac{dy}{dx}(1-2x)$

$\therefore$ (1 + x^2) y_2 + (2x - 1) y_1 = 0

Hence proved

OR

Let $\mathbf{u} = \sin^{-1}\sqrt{1-x^2}$

Put x = cos θ

$\Rightarrow u=\sin^{-1}\sqrt{1-\cos^2\theta}$

$\Rightarrow$u = $\sin^{-1}(\sin\theta)$ ...(i)

And, v = $\cos^{-1}x$ ....(ii)

Now, x $\in$ (-1, 0)

$\Rightarrow$ $\cos\theta \in$ (-1, 0)

$\Rightarrow \theta \in \left(\frac{\pi}{2}, \pi\right)$

So, from equation (i),

$u=\pi-\theta\left[\text{ since }, \sin^{-1}(\sin\theta)=\pi-\theta\, if \theta \in\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\right]$

$\Rightarrow u=\pi-\cos^{-1}x[\text{ since, } x=\cos\theta]$

Differentiating it with respect to x,

$\frac{du}{dx}=0-\frac{-1}{\sqrt{1-x^2}}$

$\Rightarrow \frac{du}{dx}=\frac{1}{\sqrt{1-x^2}}$ ...(iii)

from equation (ii),

v = $\cot^{-1}x$

Differentiating it with respect to x,

$\frac{dv}{dx}=\frac{-1}{\sqrt{1-x^2}}$ ...(iv)

Dividing equation (iii) by (iv),

$\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{1}{\sqrt{1-x^2}}\times\frac{\sqrt{1-x^2}}{-1}$

$\therefore \frac{du}{dx}=-1$

10. **Read the text carefully and answer the questions:**

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.

(i) Required probability = P(one ticket with prime number and other ticket with a multiple of 4)

$=2\left(\frac{15}{50}\times\frac{12}{49}\right)=\frac{36}{245}$

(ii) P(First ticket shows an even number and second ticket shows an odd number)

$=\frac{25}{50}\times\frac{25}{49}=\frac{25}{98}$

(iii)Required probability = P(one number is a multiple of 4 and other is a multiple of 5)

= P(multiple of 5 on first ticket and multiple of 4 on second ticket) + P(multiple of 4 on first ticket and multiple of 5 on second ticket)

$=\frac{10}{50}\times\frac{12}{49}+\frac{12}{50}\times\frac{10}{49}$

$=\frac{12}{245}+\frac{12}{245}$

$=\frac{25}{245}$

$=\frac{5}{49}$

(iv)Probability that both tickets drawn by Girish shows odd number

$=\frac{25}{50}\times\frac{24}{49}$

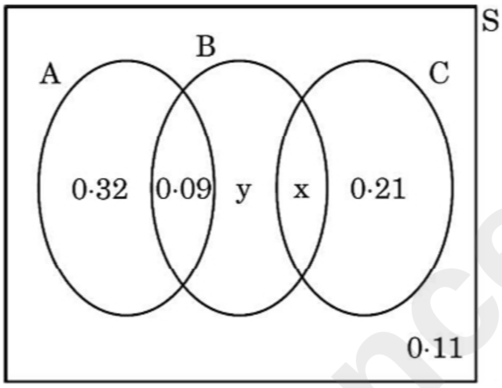
$=\frac{12}{49}$

OR

Read the text carefully and answer the questions:

There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.



(i) $x + 0.21 = 0.44 \Rightarrow x = 0.23$

(ii) $0.41 + y + 0.44 + 0.11 = 1 \Rightarrow y = 0.04$

(iii) $P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)}$

$P(B) = 0.09 + 0.04 + 0.23 = 0.36$

$P\left(\frac{C}{B}\right) = \frac{0.23}{0.36} = \frac{23}{36}$

(iv) P(A or B but not C)

$= 0.32 + 0.09 + 0.04$

$= 0.45$

11. **Read the text carefully and answer the questions:**

If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ are two matrices, then $A \pm B$ is of order $m \times n$ and is defined as $(A \pm B)_{ij} = a_{ij} \pm b_{ij}$, where i = 1, 2, ..., m and j = 1, 2, ..., n

If $A = [a_{ij}]_{m \times n}$ and $B = [a_{ij}]_{m \times p}$ are two matrices, then AB is of order $m \times p$ and is defined as $(AB)_{ik} = \sum_{r=1}^{n} a_{ir}\, b_{rk} = a_{1k}b_{1k} + a_{i2}b_{2k} + \ldots + a_{in}b_{nk}$

Consider $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ and $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(i) $AB = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$

$= \begin{bmatrix} 10 - 7 & 4 - 4 \\ 15 + 28 & 6 + 16 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$

(ii) We have, CD - AB = 0

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+5c-3 & 2b+5d \\ 3a+8c-43 & 3b+8d-22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

By equality of matrices, we get 2a + 5c - 3 = 0 ...(i)

3a + 8c - 43 = 0 ...(ii)

2b + 5d = 0 ...(iii)

3b + 8d - 22 = 0 ...(iv)

Solving (i) and (ii), we get a = -191, c = 77

(iii)We have, CD - AB = 0

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+5c-3 & 2b+5d \\ 3a+8c-43 & 3b+8d-22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

By equality of matrices, we get 2a + 5c - 3 = 0 ...(i)

3a + 8c - 43 = 0 ...(ii)

2b + 5d = 0 ...(iii)

3b + 8d - 22 = 0 ...(iv)

Solving (iii) and (iv), we get b = -110, d = 44

12. Given equation is

$\frac{dy}{dx}$ - 3y = sin 2x

Compare $\frac{dy}{dx}$ - 3y = sin 2x with $\frac{dy}{dx} + Py = Q$

we get P = -3 and Q = sin2x

This is linear differential equation where P and Q are functions of x

Now, IF = $e^{\int Pdx}$

$\Rightarrow$ IF = $e^{\int(-3)dx}$

$\Rightarrow$ IF = e^{-3x}

The solution of linear differential equation is given by y(IF) = $\int \mathrm{Q(IF)d}x + \mathrm{c}$

$\Rightarrow$ ye^{-3x} = $\int e^{-3x}\sin 2xdx$ ......(i)

Let I = $\int e^{-3x}\sin 2xdx$

$= \mathrm{e}^{-3x}\int \sin 2\mathrm{x} - \int[(\mathrm{e}^{-3x})'\int \sin 2\mathrm{x}]dx$

$= -\mathrm{e}^{-3x}\frac{\cos 2x}{2} + \int 3\mathrm{e}^{-3\times\frac{\cos 2x}{2}} + \mathrm{c}$

Again, applying the above stated rule in $\int 3e^{-3x}\frac{\cos 2x}{2}$ we get

$= -\mathrm{e}^{-3x}\frac{\cos 2x}{2} - \frac{3}{2}\left[\mathrm{e}^{-3x}\frac{\sin 2x}{2} + \int 3\mathrm{e}^{-3x}\frac{\sin 2x}{2}\right] + \mathrm{c}$

So,

$\int e^{-3x}\sin 2xdx = -\mathrm{e}^{-3x}\frac{\cos 2x}{2} - \frac{3}{2}\mathrm{e}^{-3x}\frac{\sin 2x}{2} - \frac{9}{4}\int \mathrm{e}^{-3x}\sin 2\mathrm{x} + \mathrm{c}$

$\int e^{-3x}\sin 2xdx + \frac{9}{4}\int e^{-3x}\sin 2x = -e^{-3x}\frac{\cos 2x}{2} - \frac{3}{2}e^{-3x}\frac{\sin 2x}{2} + c$

$\frac{13}{4}\int e^{-3x}\sin 2xdx = -e^{-3x}\frac{\cos 2x}{2} - \frac{3}{2}e^{-3x}\frac{\sin 2x}{2} + c$

$\frac{13}{4} \times 4\int e^{-3x}\sin 2xdx$ = $-e^{-3x}$ 2 cos 2x - 3 e^{-3x} sin 2x + c

13 $\int e^{-3x}\sin 2xdx$ = $-e^{-3x}$ (2 cos 2x + 3 sin 2x) + c

$\int e^{-3x}\sin 2xdx = \frac{-\mathrm{e}^{-3x}(2\cos 2x+3\sin 2x)}{13} + \mathrm{c}$

Put this value in (i) to get,

$ye^{-3x} = \int e^{-3x}\sin 2xdx$

$\mathrm{ye}^{-3x} = \frac{-\mathrm{e}^{-3x}(2\cos 2x+3\sin 2x)}{13} + \mathrm{c}$

$\Rightarrow \mathrm{y} = -\frac{1}{13}(3\sin 2\mathrm{x} + 2\cos 2\mathrm{x}) + \mathrm{ce}^{3x}$

OR

Let P be the principal (amount) at the end of t years.

According to the given condition, rate of increase of principal per year = 5% (of principal)

$\Rightarrow \frac{dP}{dt} = \frac{5}{100} \times P$

$\Rightarrow \frac{dP}{dt} = \frac{P}{20}$

$\Rightarrow \frac{dP}{P} = \frac{dt}{20}$ [Separating variables]

Integrating both sides,

log P = $\frac{1}{20}$ t + c ...(i)

[Since P being principal > 0, hence $\log|P| = \log P$]

Now initial principal = ₹ 1000 (given), i.e., when t = 0 then P = 1000

Therefore, putting t = 0, P = 1000 in eq. (i), log 1000 = c

Putting $\log 1000 = c$ in eq. (i), log P = $\frac{1}{20}$ t + log 1000

$\Rightarrow$ log P - log 1000 = $\frac{1}{20}$t

$\Rightarrow$log $\frac{P}{1000} = \frac{1}{20}t$ ...(ii)

Now putting t = 10 years (given)

$\log \frac{P}{100} = \frac{1}{20} \times 10 = \frac{1}{2} = 0.5$

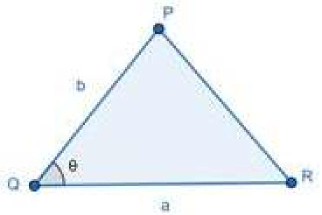
$\Rightarrow \frac{P}{1000} = e^{0.5}$ [$\because$ If x = t, then x = e^t

P = 1000 $\times$ 1.648 = ₹ 1648

13. Given, The length two sides of a triangle are ‘a’ and ‘b’

Angle between the sides ‘a’ and ‘b’ is θ.

Then,the area of the triangle is maximum.



Let us consider,

The area of the $\triangle$PQR is given be

$A = \frac{1}{2}ab\sin\theta$ ... (i)

For finding the maximum/ minimum of given function, we can find it by differentiating it with θ and then equating it to zero. This is because if the function A (θ) has a maximum/minimum at a point c then A’(c) = 0.

Differentiating the equation (i) with respect to θ,we get

$\frac{dA}{d\theta} = \frac{d}{d\theta}\left[\frac{1}{2}ab\sin\theta\right]$

$\frac{dA}{d\theta} = \frac{1}{2}ab\cos\theta$ ... (ii)

[Since $\frac{d}{dx}(\sin\theta) = \cos\theta$]

To find the critical point, we need to equate equation (ii) to zero.we get

$\frac{dA}{d\theta} = \frac{1}{2}ab\cos\theta = 0$

Cos θ = 0

$\theta = \frac{\pi}{2}$

Now to check if this critical point will determine the maximum area, we need to check with second differential which needs to be negative.

Consider differentiating the equation (ii) with θ,we get

$\frac{d^2A}{d\theta^2} = \frac{d}{d\theta}\left[\frac{1}{2}ab\cos\theta\right]$

$\frac{d^2A}{d\theta^2} = -\frac{1}{2}ab\sin\theta$ ... (ii)

[Since $\frac{d}{dx}(\cos\theta) = -\sin\theta$]

Now let us find the value of

$\frac{d^2A}{d\theta^2}_{\theta=\frac{\pi}{2}} = -\frac{1}{2}$ ab $\sin\left(\frac{\pi}{2}\right) = -\frac{1}{2}ab$

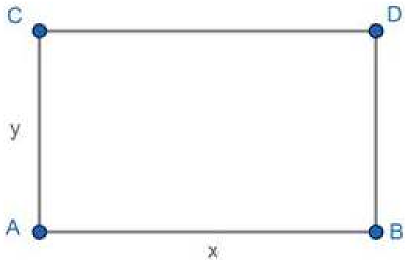
As $\frac{d^2A}{d\theta^2}_{\theta=\frac{\pi}{2}} = -\frac{1}{2}ab < 0$, therefore, function A is maximum at $\theta = \frac{\pi}{2}$

Therefore, the area of the triangle is maximum when $\theta = \frac{\pi}{2}$

OR

Given, Area of the rectangle is 93 cm^2. The perimeter of the rectangle is also fixed.

Let us consider,



x and y be the lengths of the base and height of the rectangle.

Area of the rectangle = A = x × y = 96 cm^2

Perimeter of the rectangle = P = 2 (x + y)

As, x $\times$ y = 96

$y = \frac{96}{x}$ .... (i)

Consider the perimeter function,

P = 2 (x + y)

Now substituting equation(1) in P,

$P = 2\left(x + \frac{96}{x}\right)$ ....(ii)

For finding the maximum/ minimum of given function, we can find it by differentiating it with x and then equating it to zero.

This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating the equation (ii) with respect to x:

$\frac{dP}{dx} = \frac{d}{dx}\left[2\left(x + \frac{96}{x}\right)\right]$

$\frac{dP}{dx} = \frac{d}{dx}(2x) + 2\frac{d}{dx}\left(\frac{96}{x}\right)$

$\frac{dP}{dx} = 2(1) + 2\left(\frac{96}{x^2}\right)(-1)$

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right)$ = $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$\frac{dP}{dx} = 2 - \left(\frac{192}{x^2}\right)$ ... (iii)

To find the critical point, we need to equate equation (iii) to zero.

$\frac{dP}{dx} = 2 - \left(\frac{192}{x^2}\right) = 0$

$2 = \left(\frac{192}{x^2}\right)$

$x^2 = \left(\frac{192}{2}\right) = 96$

$x = \sqrt{96}$

$x = \pm 4\sqrt{6}$

As the length and breadth of a rectangle cannot be negative, hence $x = 4\sqrt{6}$

Now to check if this critical point will determine the least perimeter, we need to check with the second differential which needs to be positive.

Consider differentiating the equation (iii) with x:

$\frac{d^2P}{dx^2} = \frac{d}{dx}\left[2 - \left(\frac{192}{x^2}\right)\right]$

$\frac{d^2P}{dx^2} = \frac{d}{dx}(2) - \frac{d}{dx}\left(\frac{192}{x^2}\right)$

$\frac{d^2P}{dx^2} = 0 - (-2)\left(\frac{192}{x^3}\right)$

[Since $\frac{d}{dx}(\text{ constant }) = 0$ and $\frac{d}{dx}\left(\frac{1}{x^n}\right)$ = $\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$]

$\frac{d^2P}{dx^2} = \left(\frac{2\times 192}{x^3}\right)$ ... (iv)

Now, consider the value of $\left(\frac{d^2P}{dx^2}\right)_{x=4\sqrt{6}}$

$\frac{d^2P}{dx^2} = \left(\frac{2\times 192}{(4\sqrt{6})^3}\right)$

$= \left(\frac{2\times 192}{4\sqrt{6}\times 4\sqrt{6}\times 4\sqrt{6}}\right)$

$= \left(\frac{2\times 192}{4\sqrt{6}\times 4\sqrt{6}\times 4\sqrt{6}}\right) = \frac{1}{\sqrt{6}}$

As $\left(\frac{d^2P}{dx^2}\right)_{x=4\sqrt{6}} = \frac{1}{\sqrt{6}} > 0$, so the function P is minimum at $x = 4\sqrt{6}$.

Now substituting $x = 4\sqrt{6}$ in equation (i):

$y = \frac{96}{4\sqrt{6}}$

$y = \frac{96\sqrt{6}}{4\times 6}$

[By rationalizing he numerator and denominator with $\sqrt{6}$]

y = $4\sqrt{6}$

Hence, area of the rectangle with sides of a rectangle with x = $4\sqrt{6}$ and y = $4\sqrt{6}$ is 96cm^2 and has the least perimeter.

Now the perimeter of the rectangle is

$P = 2(4\sqrt{6} + 4\sqrt{6})$ = $2(8\sqrt{6}) = 16\sqrt{6}cms$

The least perimeter is $16\sqrt{6}cms$.

14. **Read the text carefully and answer the questions:**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

(i) Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Economics if it is known that he has failed in Mathematics.

Required probability = $P(\frac{E}{M})$

= $\frac{P(E\cap M)}{P(M)} = \frac{\frac{1}{4}}{\frac{7}{20}}$ = $\frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$

(ii) Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Mathematics if it is known that he has failed in Economics.

Required probability = P(M/E)

$= \frac{P(M\cap E)}{P(E)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

(iii)Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Mathematics if it is known that he has failed in Economics

Required probability = P(M'/E)

$\Rightarrow P\left(M'/E\right) = \frac{P(M'\cap E)}{P(E)}$

= $\frac{P(E)-P(E\cap M)}{P(E)}$

= $\frac{\frac{1}{2}-\frac{1}{4}}{\frac{1}{2}}$

$\Rightarrow P\left(M'/E\right) = \frac{1}{2}$

(iv)Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Economics if it is known that he has failed in Mathematics

Required probability = P(E'/M)

$\Rightarrow P\left(E'/M\right) = \frac{P(E'\cap M)}{P(M)}$

$= \frac{P(M)-P(E\cap M)}{P(M)}$

$= \frac{\frac{7}{20}-\frac{1}{4}}{\frac{7}{20}}$

$\Rightarrow P(E'/M) = \frac{2}{7}$

SECTION B - 15 MARKS

15. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) **(a)** $\lambda = -2$

Explanation: {

Given that, $\lambda\hat{\imath} + \hat{j} + 2\hat{k}$, $\hat{\imath} + \lambda\hat{j} - \hat{k}$, and $2\hat{\imath} - \hat{j} + \lambda\hat{k}$ are coplanar.

Let $\vec{a} = \lambda\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + \lambda\hat{k}$

Now, $\vec{a}, \vec{b}, \vec{c}$ are coplanar

If, $\begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0$

$\Rightarrow \lambda\left(\lambda^2 - 1\right) - 1(\lambda + 2) + 2(-1 - 2\lambda) = 0$

$\Rightarrow \lambda^3 - \lambda - \lambda - 2 - 2 - 4\lambda = 0$

$\Rightarrow \lambda^3 - 6\lambda - 4 = 0$

$\Rightarrow (\lambda + 2)\left(\lambda^2 - 2\lambda - 2\right) = 0$

$\Rightarrow \lambda = -2$ and $\lambda = \frac{2\pm\sqrt{(-2)^2-4\times1\times-2}}{2} = \frac{2\pm\sqrt{12}}{2}$

$\Rightarrow \lambda = -2$ and $\lambda = 1 \pm \sqrt{3}$.

(ii) The equation of the given line is

$\frac{x-\frac{3}{5}}{\frac{1}{5}} = \frac{y+\frac{7}{15}}{\frac{1}{15}} = \frac{z-\frac{3}{10}}{\frac{-1}{10}}$

Its direction ratios are.

$\left\langle \frac{1}{5}, \frac{1}{15}, -\frac{1}{10} \right\rangle$ or ⟨6, 2, -3⟩

Direction cosines are $\left\langle \pm\frac{6}{7}, \pm\frac{2}{7}, \mp\frac{3}{7} \right\rangle$

The point through which it passes is $\left(\frac{3}{5}, \frac{-7}{15}, \frac{3}{10}\right)$

(iii) Let θ be the angle between $\vec{a}$ and $\vec{b}$.

$\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ (Given)

$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{9 + 4 + 36} = 7$

We know that ,

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$\Rightarrow 7 = (2)(7)\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

(iv) **(d)** (2, 2, 2)

Explanation: {

Let the required equation of the plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Then $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$

$\Rightarrow \frac{2}{a} + \frac{2}{b} + \frac{2}{c} = 1$

It means that the plane passes through the point (2, 2, 2).

(v) Here,it is given the equition of plane is z = 3

Here direction ratios of normal to the plane is 0,0,1

Now $\sqrt{0^2 + 0^2 + 1^2} = 1$

∴ Direction cosines are 0,0,1 And distance from the origin P = 3.

16. Given that

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

We have to show, $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

Solving for left hand side,

$\vec{b}+\vec{c}=(b_1+c_1)\,\hat{i}+(b_2+c_2)\,\hat{j}+(b_3+c_3)\,\hat{k}$

$\vec{a}\times(\vec{b}+\vec{c})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\end{vmatrix}$

$\Rightarrow \vec{a}\times(\vec{b}+\vec{c})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}+\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ c_1 & c_2 & c_3\end{vmatrix}$ (property of determinant)

$=(\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})$ = R.H.S.

$\Rightarrow \overrightarrow{a}\times(\vec{b}+\vec{c})=(\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})$

Hence proved.

OR

Here,it is given$\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$

$\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$

since these two vectors are perpendicular, their dot product is zero.

$\Rightarrow \vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta=|\vec{a}||\vec{b}|\cos\frac{\pi}{2}=0$

$\Rightarrow \vec{a}\cdot\vec{b}=(2\hat{i}+\lambda\hat{j}+\hat{k})\cdot(\hat{i}-2\hat{j}+3\hat{k})=0$

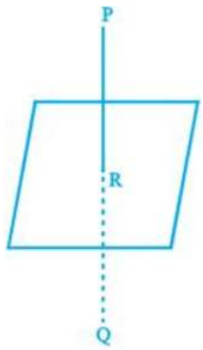
$\Rightarrow \vec{a}\cdot\vec{b}=(2\times1)+(\lambda\times-2)+(1\times3)=0$

$\Rightarrow \vec{a}\cdot\vec{b}=2-2\lambda+3=0$

$\Rightarrow 5=2\lambda$

$\Rightarrow \lambda=\frac{5}{2}$

17. Let the given point be $P\left(\hat{i}+3\hat{j}+4\hat{k}\right)$ and Q be the image of P in the plane $\hat{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)+3=0$ as shown in the Fig.



Then PQ is the normal to the plane. Since PQ passes through P and is normal to the given plane, so the equation of PQ is given by $\vec{r}=\left(\hat{i}+3\hat{j}+4\hat{k}\right)+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$

Since Q lies on the line PQ, the position vector of Q can be expressed as $\left(\hat{i}+3\hat{j}+4\hat{k}\right)+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$ i.e.,

$(1+2\lambda)\,i+(3-\lambda)\,\hat{j}+4\,(4+\lambda)\,\hat{k}$

Since R is the mid point of PQ, the position vector of R is $\frac{\left[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(4+\lambda)\hat{k}\right]+\left[\hat{i}+3\hat{j}+4\hat{k}\right]}{2}$

i.e., $(\lambda+1)\,\hat{i}+\left(3-\frac{\lambda}{2}\right)\hat{j}+\left(4+\frac{\lambda}{2}\right)\hat{k}$

Again, since R lies on the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)+3=0$, we have

$\left\{(\lambda-1)\,\hat{i}+\left(3-\frac{\lambda}{2}\right)\hat{j}+\left(4+\frac{\lambda}{2}\right)\hat{k}\right\}.\left(2\hat{i}-\hat{j}-\hat{k}\right)+3=0$

$\Rightarrow \lambda=-2$

Hence, the position vector of Q is $\left(\hat{i}+3\hat{j}+4\hat{k}\right)-2\left(2\hat{i}-\hat{j}+\hat{k}\right)$, i.e., $-3\hat{i}+5\hat{j}+2\hat{k}$.

OR

$\vec{N}_1=2\hat{i}+\hat{j}-2\hat{k}$

$\vec{N}_2=3\hat{i}-6\hat{j}-2\hat{k}$

$\cos\theta=\left|\frac{\overline{N}_1\cdot\overline{N}_2}{|\overline{N}_1||\overline{N}_2|}\right|$

$=\left|\frac{(2\hat{i}+\hat{j}-2\hat{k})\cdot(3\hat{i}-6\hat{j}-2\hat{k})}{\sqrt{4+1+4}\sqrt{9+36+4}}\right|$

$=\frac{4}{21}$

$\theta=\cos^{-1}\left(\frac{4}{21}\right)$

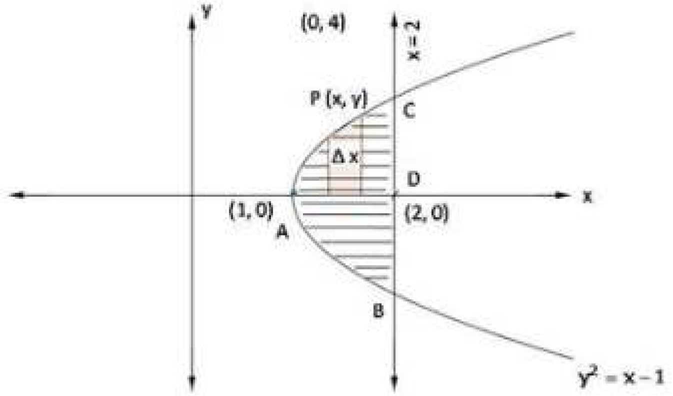
18. We have to find area enclosed by the given curve

$y^2 = x - 1$ ...(i)

and x = 2 ...(ii)

Equation (i) is a parabola with vertex at (1, 0) and axis as x-axis,. Equation (ii) represents a line parallel to y-axis passing through (2, 0).

A rough sketch of curves is as below in the Figure.



Shaded region shows the required area, We slice it in appropriation rectangle with its Width $\triangle x$ and length = y - 0 = y

Area of the rectangle $= y\triangle x$

This rectangle can slide from x = 1 to x = 2, so

the Required area = Region AB CA= twice area of region AOCA

= 2 (Region AOCA)

$= 2\int_1^2 y dx$

$= 2\int_1^2 \sqrt{x-1} dx$

$= 2\left(\frac{2}{3}(x-1)\sqrt{x-1}\right)_1^2$

$= \frac{4}{3}[((2-1)\sqrt{2-1}) - ((1-1)\sqrt{1-1})]$

$= \frac{4}{3}(1-0)$

Required area $= \frac{4}{3}$ square units

SECTION C - 15 MARKS

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) **(b)** MC > AC

Explanation: {

For MC > AC ⇒ AC increases with x and AC curve is rising.

(ii) **(b)** given by the corner points of the feasible reason.

Explanation: {

Objective function attain its optimal value on the corner points of the feasible reason.

(iii) Assume that 4x + 10y = 9 is the line of regression of y on x.

∴ 10y = -4x + 9

$y = -\frac{4}{10}x + \frac{9}{10}$

$\Rightarrow b_{yx} = -\frac{2}{5}$

Now, 6x + 3y = 4 is the line of regression of x on y.

∴ 6x = -3y + 4

$\Rightarrow x = -\frac{3}{6}y + \frac{4}{6}$

$\Rightarrow b_{xy} = -\frac{1}{2}$

Here, $b_{xy} \times b_{yx} = \left(-\frac{2}{5}\right) \times \left(-\frac{1}{2}\right)$

$= \frac{1}{5} < 1$

Which is true. Hence our assumption is correct and line of regression of y on x is 3x + 10y = 9.

(iv) Given cost function $C(x) = \frac{5x^2}{\sqrt{(x^2+3)}} + 5000$

Marginal cost function is given by C′(x)

$$C'(x) = \frac{\left(\sqrt{x^2+3}\right)(10x) - \frac{5x^2}{2\sqrt{x^2+3}} \times (2x+10)}{\left(\sqrt{x^2+3}\right)^2}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$C'(x) = \frac{2(x^2+3)(10x) - 5x^2(2x)}{2\left(\sqrt{x^2+3}\right)^3} = \frac{5x\left(x^2+6\right)}{\left(x^2+3\right)^{\frac{3}{2}}}$$

(v) Given TVC = $3x + \frac{x^5}{25}$

$\therefore$ AVC = $\frac{\text{TVC}}{x} = \frac{3x + \frac{x^5}{25}}{x} = 3 + \frac{x^4}{25}$

$\Rightarrow \frac{d}{dx}$(AVC) = $\frac{4x^3}{25}$

$\therefore \frac{d}{dx}$(AVC) > 0 when x > 0

So, the average variable cost increases with output x.

20. Given fixed cost = ₹ 24000

Let number of units be: x

Selling price per unit P(X) = ₹ 8

$\therefore$ Revenue function R(X) = 8x

Cost function = C(x) = 24000 + 25% of 8x

= 24000 + 2x

For breakeven points: R(x) = C(x)

8x = 24000 + 2x

6x = 24000

x = 4000 units

OR

We have, p = $\frac{250}{x+15}$

Let R be the total revenue when x units are sold. Then,

R = px $\Rightarrow$ R = $\frac{250x}{x+15}$

$\Rightarrow \frac{dR}{dx} = \frac{250(x+15) - 250x}{(x+15)^2}$

$\Rightarrow$ MR = $\frac{3750}{(x+15)^2}$ ... (i)

We have, x = $\frac{25n}{\sqrt{n^3+36}}$

Therefore, for n = 4, we obtain x = $\frac{25 \times 4}{\sqrt{64+36}}$ = 10

When n = 4 we obtain x = 10

Therefore, putting x = 10 in (i), we get

MR = $\frac{3750}{(10+15)^2}$ = 6

21. i. Let the line of regression of x on y be

3x - 2y = 5

and the line of regression of y on x be

x - 4y = 7

Written the first equation in the form

x = $\frac{2}{3}$y + $\frac{5}{3}$

we get, $b_{xy} = \frac{2}{3}$

Written the second equation in the form

y = $\frac{1}{4}$x - $\frac{7}{4}$

we get, $b_{yx} = \frac{1}{4}$

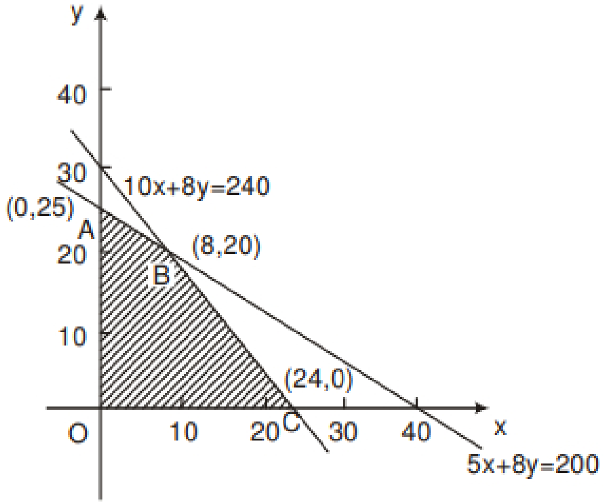
ii. Now, $r^2 = b_{yx} \times b_{xy}$

$= \frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$

$\therefore r = \frac{1}{\sqrt{6}}$

Since r, b_{yx} and b_{xy} all have same sign.

22. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise P = 50x + 60y

such that 5x + 8y $\leq$ 200

10x + 8y $\leq$ 240

x, y $\geq$ 0

P(at A) = ₹ 1500

P(at B) = ₹ (400 + 1200) = ₹ 1600

P(at C) = ₹ (1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20.

OR

Linear constraints

$x + y \geq 8$

$3x + 5y \leq 15$

$x \geq 0, y \geq 0$

Objective function is $\min(Z) = 3x + 2y$

Reducing the all inequations into equations and finding their point of intersections, i.e.,

$x + y = 8$ ... (i)

$3x + 5y = 15$ ... (ii)

$x = 0, y = 0$ ... (iii)

Equations	Point of Intersection
(i) and (ii)	$x = \frac{25}{2}$ and $y = \frac{-9}{2}$
	$\Rightarrow \left(\frac{25}{2}, \frac{-9}{2}\right)$
(i) and (iii)	when x = 0 ⇒ y = 8 ⇒ (0, 8)
	when y = 0 ⇒ x = 8 ⇒ (8, 0)
(ii) and (iii)	when x = 0 ⇒ y = 3 ⇒ (0, 3)
	when y = 0 ⇒ x = 5 ⇒ (5, 0)

For feasible region.

For $x + y \geq 8$ let $x = 0, y = 0$

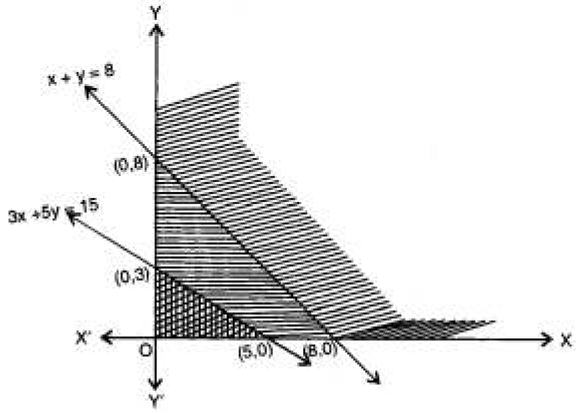
$\Rightarrow 0 \geq 8$ i.e., Not true

$\Rightarrow$ The shaded region will be away from origin

Again, for $3x + 5y \leq 15$, let $x = 0, y = 0$

$\Rightarrow 0 \leq 15$ i.e. true

We have, no negative restriction, $x \geq 0, y \geq 0$ indicates that the shaded region will exist in first quadrant only.



The problem will not have any feasible region. Therefore there will be no feasible solution.