ISC 2026 EXAMINATION
Sample Question Paper - 5
Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

This Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions* ***EITHER*** *from* ***Section B OR Section C.***

Section A*: Internal choice has been provided in* ***two questions of two marks each, two questions of four marks each and two questions of six marks each.***

Section B: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

Section C: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

1. **In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.** **[15]**

(a) If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then A^T - B^T is _______. **[1]**

a) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -2 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 3 & 4 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

c) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ d) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -1 \end{bmatrix}$

(b) $\int \frac{1}{\cos x+\sqrt{3}\sin x}dx$ is equal to **[1]**

a) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{3}\right)+C$ b) $\log\tan\left(\frac{x}{2}-\frac{\pi}{3}\right)+C$

c) $\log\tan\left(\frac{\pi}{3}+\frac{x}{2}\right)+C$ d) $2\log\tan\left(\frac{\pi}{3}+\frac{x}{4}\right)+C$

(c) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$ is equal to **[1]**

a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$

c) $\frac{-\pi}{3}$ d) $-\frac{2\pi}{3}$

(d) Solution of $(x+1)\frac{dy}{dx}=2xy$ is **[1]**

a) $log|y| = 2(x - log|1+x|) + c$ b) $log|y| = 2(x + log|1-x|) + c$

c) $\log y = \{x + \log|x|\} + C$ d) $\log y = \{x - \log|x|\} + C$

(e) A problem in Statistics is given to three students A, B and C whose chances of solving it independently are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively. The probability that the problem will be solved, is **[1]**

a) $\frac{1}{12}$ b) $\frac{11}{12}$

c) $\frac{1}{2}$ d) $\frac{3}{4}$

(f) Let R be any relation in the set A of human beings in a town at a particular time. If R = { [x, y) : x is exactly 7 cm taller than y], then R is **[1]**

a) an equivalence relation b) reflexive

c) not symmetric d) symmetric but not transitive

(g) If $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, then $\frac{dy}{dx}$ is equal to **[1]**

a) $y^2 + 1$ b) $1 + y^2$

c) $1 - y^2$ d) $y^2 - 1$

(h) If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to **[1]**

a) 5 y b) -25 y

c) 15 y d) 25 y

(i) If A and B are invertible matrices, then which of the following is not correct? **[1]**

a) adj. A = $|A|.A^{-1}$ b) $(A + B)^{-1} = B^{-1} + A^{-1}$

c) $(AB)^{-1} = B^{-1}A^{-1}$ d) $\det(A)^{-1} = [\det(A)]^{-1}$

(j) **Assertion (A):** If A = $\begin{bmatrix} 2 & x-3 & x-2 \\ 3 & -2 & -1 \\ 4 & -1 & -5 \end{bmatrix}$ is a symmetric matrix, then x = 6. **[1]**

Reason (R): If A is a symmetric matrix, then A = A'.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

(k) Let f and g be two real function defined by f(x) = $\frac{1}{x+4}$ and g(x) = $(x + 4)^3$ find the $\frac{1}{f}$. **[1]**

(l) Find the matrix X such that 2A - B + X = 0, where A = $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$ **[1]**

(m) Show that the function f : N $\rightarrow$ N : f(x) = x^3 is one - one into **[1]**

(n) Let A and B be the events such that P(A) = $\frac{7}{13}$, P(B) = $\frac{9}{13}$ and P(A∩B) = $\frac{4}{13}$ find P(B/A). **[1]**

(o) A die is thrown. If E is the event **the number appearing is a multiple of 3** and F be the event **the number appearing is even** then find whether E and F are independent? **[1]**

2. If $f'(x) = \sqrt{2x^2 - 1}$ and y = f(x^2), then find $\frac{dy}{dx}$ at x = 1. **[2]**

OR

Find the interval in which the function $f(x) = x^8 + 6x^2$ is increasing or decreasing.

3. Evaluate $\int_0^{\frac{\pi}{2}}$ (2 log cos x - log sin 2x) dx **[2]**

4. Show that f(x) = $\log_a$ x, 0 < a < 1 is a decreasing function for all x> 0. **[2]**

5. Evaluate: $\int \frac{1}{(x-1)(x+1)(x+2)} dx$ **[2]**

OR

Evaluate the definite integral: $\int_0^{\pi/4} x^2 \sin x dx$

6. Let A be the set of all triangles in a plane. Show that the relation $R = \{(\Delta_1, \Delta_2) : \Delta_1 \sim \Delta_2\}$ is an equivalence relation on A. **[2]**

7. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. **[4]**

8. Evaluate: $\int \frac{dx}{x(x-2)(x-4)}$. **[4]**

9. Find $\frac{dy}{dx}$, if $y = \tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$. **[4]**

OR

Find $\frac{dy}{dx}$ when sin x sin 2x sin 3x sin 4x

10. **Read the text carefully and answer the questions:** **[4]**

To hire a marketing manager, it's important to find a way to properly assess candidates who can bring radical changes and has leadership experience.

Ajay, Ramesh and Ravi attend the interview for the post of a marketing manager. Ajay, Ramesh and Ravi chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8, and 0.5. If the change does take place.

(a) Find the probability that it is due to the appointment of Ajay (A).

(b) Find the probability that it is due to the appointment of Ramesh (B).

(c) Find the probability that it is due to the appointment of Ravi (C).

(d) Find the probability that it is due to the appointment of Ramesh or Ravi.

OR

Read the text carefully and answer the questions: **[4]**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(a) What is the probability that the shell fired from exactly one of them hit the plane?

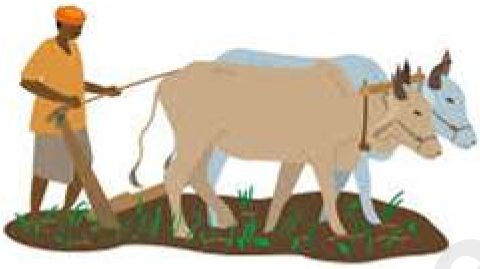
(b) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

(c) What is the probability that the shell was fired from A?

(d) How many hypotheses are possible before the trial, with the guns operating independently? Write the conditions of these hypotheses.

11. **Read the text carefully and answer the questions:** **[6]**

Two farmers Ankit and Girish cultivate only three varieties of pulses namely Urad, Massor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.



September sales (in ₹):

A = $\begin{pmatrix} Urad & Masoor & Mung \\ 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$

October sales (in ₹):

A = $\begin{pmatrix} Urad & Masoor & Mung \\ 5000 & 10000 & 6000 \\ 20000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$

(a) Find the combined sales of Masoor in September and October, for farmer Girish.

(b) Find the combined sales of Urad in September and October, for farmer Ankit.

(c) Find a decrease in sales from September to October.

12. Solve the following differential equation: $e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)+\left(1+e^{\frac{y}{x}}\right)\frac{dx}{dy}$ = 0, when x = 0, y = 1 **[6]**

OR

Solve the differential equation: $\frac{dy}{dx} - 3y\cot x$ = sin 2x, given y = 2 when $x = \frac{\pi}{2}$.

13. Show that the radius of a closed right circular cylinder of given surface area and maximum volume is equal to half of its height. **[6]**

OR

Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius 'a' is a square of side $\sqrt{2}a$.

14. **Read the text carefully and answer the questions:** **[6]**

Family photography is all about capturing groups of people that have family ties. These range from the small group, such as parents and their children. New-born photography also falls under this umbrella. Mr Ramesh, His wife Mrs Saroj, their daughter Sonu and son Ashish line up at random for a family photograph, as shown in figure.

(a) Find the probability that daughter is at one end, given that father and mother are in the middle.

(b) Find the probability that mother is at right end, given that son and daughter are together.

(c) Find the probability that father and mother are in the middle, given that son is at right end.

(d) Find the probability that father and son are standing together, given that mother and daughter are standing together.

SECTION B - 15 MARKS

15. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) The vector $\vec{b} = 3\hat{i} + 4\hat{k}$ is to be written as the sum of a vector $\vec{\alpha}$ parallel to $\vec{a} = \hat{i} + \hat{j}$ and a vector $\vec{\beta}$ perpendicular to $\vec{a}$. Then $\vec{\alpha} =$ **[1]**

a) $\frac{2}{3}(i + j)$ b) $\frac{3}{2}(i + j)$

c) $\frac{1}{2}(i + j)$ d) $\frac{1}{3}(i + j)$

(b) Find the direction cosines of x, y and z-axis. **[1]**

(c) Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$. **[1]**

(d) The distance between the planes; 2x + 2y - z + 2 = 0 and 4x + 4y - 2z + 5 = 0 is **[1]**

a) $\frac{1}{2}$ b) $\frac{1}{6}$

c) $\frac{1}{8}$ d) $\frac{1}{4}$

(e) Find the distance of the point (2, 1, -1) from the plane x - 2y + 4z = 9. **[1]**

16. Find $|\vec{a} \cdot \vec{b}$ if $|\vec{a}|$ = 2, $|\vec{b}|$ = 5 and $|\vec{a} \times \vec{b}|$ = 8. **[2]**

OR

If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a} + \vec{b}|$ = 13 and $|\vec{a}|$ = 5, find the value of $|\vec{b}|$.

17. Find the shortest distance between the pairs of lines whose vector equations are: **[4]**

$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$ and $\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$

OR

Reduce the equation 2x - 3y - 6z = 14 to the normal form and hence find the length of perpendicular from the origin to the plane. Also, find the direction cosines of the normal to the plane.

18. Find the area of the region lying in the first quadrant and enclosed by the x-axis, the line y = x and the circle $x^2 + y^2 = 32$. **[4]**

SECTION C - 15 MARKS

19. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) Law of variable proportion explain three stages of production. In the first stage of production: **[1]**

a) AP falls b) MP is zero

c) MP rises d) Both MP and AP rise

(b) The corner points of the feasible region for a Linear Programming problem are P(0, 5), Q(1, 5), R(4, 2)
and S(12, 0). The minimum value of the objective function Z = 2x + 5y is at the point. **[1]**

a) R b) Q

c) S d) P

(c) If $\bar{x}$ = 18, $\bar{y}$ = 100, σ_x = 14, σ_y = 20 and correlation coefficient r_{xy} = 0.8, find the regression equation of y on x. **[1]**

(d) The demand function is x = $\frac{24-2p}{3}$, where x is the number of units demanded and p is the 3 price per unit. Find the revenue function R in terms of p. **[1]**

(e) If P = $\frac{100}{q+2}$ - 4 represents the demand function for a product where p is the price per unit when q units are sold, find the marginal revenue. **[1]**

20. Given the total cost function for x units of a commodity as C(x) = $\frac{1}{3}x^3$ + x^2 - 8x + 5, find: **[2]**

i. the marginal cost function
ii. average cost function
iii. slope of average cost function

OR

A company produced a commodity with ₹ 20,000 fixed costs. The variable cost is estimated to 35% of the total revenue. When it is sold at a rate of ₹ 6 per unit. Find the total revenue, total cost and profit function of the revenue when it is sold.

21. Find the line of regression of y on x from the following table: **[4]**

x	1	2	3	4	5
y	7	6	5	4	3

Hence, estimate the value of y when x = 6.

22. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹ 50 each for type A and ₹ 60 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit? Formulate the above LPP and solve it graphically and also find the maximum profit. **[4]**

OR

Maximise Z = 3x + 4y, subject to the constraints: $x + y \leqslant 1, x \geqslant 0, y \geqslant 0$.

Solution

SECTION A - 65 MARKS

1. In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(a) **(c)** $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

Explanation:

Given, $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$\therefore B^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$\therefore A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

(b) **(a)** $\frac{1}{2}\log\tan\left(\frac{x}{2} + \frac{\pi}{3}\right) + C$

Explanation:

$I = \int \frac{1}{\cos x + \sqrt{3}\sin x} dx$

$I = \frac{1}{2}\int \frac{1}{\frac{\cos x}{2} + \frac{\sqrt{3}}{2}\sin x} dx$

$I = \frac{1}{2}\int \frac{1}{\cos\left(x - \frac{\pi}{6}\right)} dx$

$I = \frac{1}{2}\int \sec\left(x - \frac{\pi}{6}\right) dx$

$I = \frac{1}{2}\ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{3}\right)\right| + c$

(c) **(a)** $\frac{\pi}{3}$

Explanation:

$\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$ =$\cos^{-1}\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3}$, because, $\cos\theta$ is positive in fourth quadrant.

(d) **(a)** $log|y| = 2(x - log|1 + x|) + c$

Explanation:

$(x+1)\frac{dy}{dx} = 2xy$

$\int \frac{1}{y} dy = 2\int \frac{x}{1+x} dx$

$\Rightarrow \int \frac{1}{y} dy = 2\int \left(1 - \frac{1}{1+x}\right) dx$

$log|y| = 2(x - log|1 + x|) + c$

(e) **(d)** $\frac{3}{4}$

Explanation:

P (problem will be solved)

= 1 - P (problem will not solved by A, B and C)

= 1 - $\left\{\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\right\}$

= 1 - $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$ = 1 -$\frac{1}{4} = \frac{3}{4}$

(f) **(c)** not symmetric

Explanation:

Here, R is not reflexive as x is not 7 cm taller than x.
R is not symmetric as if x is exactly 7 cm taller than y, then y cannot be 7 cm taller than x and R is not transitive as if x is exactly 7 cm taller than y and y is exactly 7 cm taller than z, then x is exactly 14 cm taller than z.

(g) **(c)** $1 - y^2$
Explanation:
Solution. $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)$
$=\frac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}=\frac{(e^x+e^{-x})^2}{(e^x+e^{-x})^2}-\frac{(e^x-e^{-x})^2}{(e^x+e^{-x})^2}=1-y^2$.
Which is the required solution.

(h) **(d)** 25 y
Explanation:
We have,
$y = ae^{5x} + be^{-5x}$
On differentiating w.r.t x, we get
$\frac{d^2y}{dx^2} = 5ae^{5x} - 5be^{-5x}$
$\frac{d^2y}{dx^2} = 25(ae^{5x}+be^{-5x})$
$\frac{d^2y}{dx^2} = 25y$
Hence, this is the answer.

(i) **(b)** $(A + B)^{-1} = B^{-1} + A^{-1}$
Explanation:
Given A and B are invertible matrices.
Now, $(A\cdot B)B^{-1}A^{-1} = A(BB^{-1})A^{-1} = AIA^{-1} = (AI)A^{-1} = AA^{-1} = I$
$\Rightarrow (AB)^{-1}=B^{-1}A^{-1}$
$\Rightarrow A A^{-1} = I$
$\Rightarrow |A A^{-1}| = |I|$
$\Rightarrow |A||A^{-1}| = 1$
$\Rightarrow |A^{-1}|=\frac{1}{|A|}$
$\Rightarrow \det(A)^{-1}=[\det(A)]^{-1}$
Also we know that $A^{-1}=\frac{adj\cdot A}{|A|}$
$\Rightarrow$ adj. $A=|A|\cdot A^{-1}$
$(A+B)^{-1}=\frac{1}{|A+B|}\text{adj.}(A+B)$
But $B^{-1}+A^{-1}=\frac{1}{|B|}$ adj. $B+\frac{1}{|A|}\text{adj.}A$
$\Rightarrow (A+B)^{-1}\neq B^{-1}+A^{-1}$

(j) **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Explanation: Here, A = A'
$$\therefore \begin{bmatrix}2 & x-3 & x-2\\ 3 & -2 & -1\\ 4 & -1 & -5\end{bmatrix}=\begin{bmatrix}2 & 3 & 4\\ x-3 & -2 & -1\\ x-2 & -1 & -5\end{bmatrix}$$
On comparing, we get
x - 3 = 3
or x = 6

(k) We observe that f(x) = 0 for any $x \in R-\{-4\}$ Therefore, $\frac{1}{f}: R-\{-4\} \rightarrow R$ is given by

$\left(\frac{1}{f}\right)(x)=\frac{1}{f(x)}=\frac{1}{1/(x+4)}=$ (x + 4)

(l) We have to find X,

Given 2A – B + X = 0

$2\left(\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}\right)-\begin{bmatrix}-2 & 1\\ 0 & 3\end{bmatrix}$ + X = 0

$X=\begin{bmatrix}-2 & 1\\ 0 & 3\end{bmatrix}-2\left(\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}\right)$

$=\begin{bmatrix}-2 & 1\\ 0 & 3\end{bmatrix}-\begin{bmatrix}6 & 2\\ 0 & 4\end{bmatrix}$

$=\begin{bmatrix}-8 & -1\\ 0 & -1\end{bmatrix}$

(m) f : Z $\rightarrow$ Z : f(x) = x^3 is one - one into

f(x) = x^3

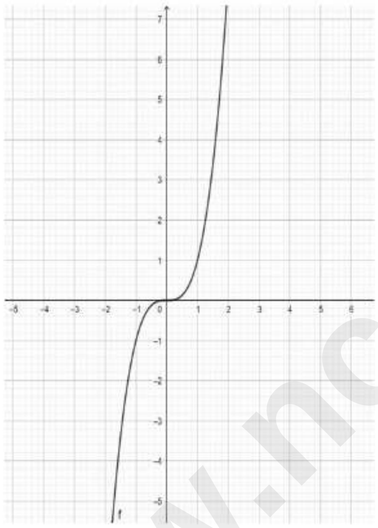
Since the function f(x) is monotonically increasing from the domain Z $\rightarrow$ Z

$\therefore$ f(x) is one –one

Range of f(x) = (- ∞,∞) $\neq$ Z(codomain)

$\therefore$ f(x) is into

$\therefore$ f : Z $\rightarrow$ Z : f(x) = x^3 is one - one into.



(n) We have,

P(B/A) = $\frac{\mathrm{P(A\cap B)}}{\mathrm{P(A)}}$

$=\frac{4}{13} \div \frac{7}{13}$

$=\frac{4}{7}$.

(o) Two event A and B are independent if $P(A \cap B)=P(A).P(B)$

Sample space of the experiment is, S = {1, 2, 3, 4, 5, 6}

Now E = {3, 6}, F = { 2, 4, 6} and E ∩ F = {6}

Then P(E) = $\frac{2}{6}=\frac{1}{3}$, P(F) = $\frac{3}{6}=\frac{1}{2}$ and $\mathrm{P(E\cap F)}=\frac{1}{6}$

Clearly P(E ∩ F) = P(E). P(F) $=\frac{1}{6}$

Hence E and F are independent events.

2. We have, $f'(x)=\sqrt{2x^2-1}$

and y = f(x^2)

Differentiating 'y' w.r.t. 'x'

$\Rightarrow \frac{dy}{dx}=\frac{d}{dx} f\left(x^2\right)$

$\Rightarrow \frac{dy}{dx}=f'\left(x^2\right) \frac{d}{dx}\left(x^2\right)$

$\Rightarrow \frac{dy}{dx} = f'\left(x^2\right) \times 2x$
$\Rightarrow \frac{dy}{dx} = f'\left(x^2\right) \times 2x$
Putting x = 1, we get,
$\frac{dy}{dx} = 2(1) f'\left(1^2\right)$
$\Rightarrow \frac{dy}{dx} = 2 \times f'(1)$
$\Rightarrow \frac{dy}{dx} = 2 \times 1 \left[\because f'(1) = \sqrt{2(1)^2 - 1} = \sqrt{2-1} = 1\right]$
$\Rightarrow \frac{dy}{dx} = 2$

OR

Given: $f(x) = x^8 + 6x^2$
$P(x) = \frac{d}{dx}\left(8x^7 + 12x\right)$
$\Rightarrow \mathrm{f}'(\mathrm{x}) = 8\mathrm{x}^7 + 12\mathrm{x}$
To find the critical point of f(x), we must have
f′(x) = 0
$\Rightarrow 4x\left(2x^6 + 3\right) = 0$
$\Rightarrow \mathrm{x}\left(2\mathrm{x}^6 + 3\right) = 0$
$x = 0, \sqrt[6]{-\frac{3}{2}}$
Since $\sqrt[6]{-\frac{3}{2}}$, is a complex number, therefore we only check for x = 0, on both of its sides.
Clearly, f′(x) > 0 if x > 0
and f′(x) < 0 if x < 0
Thus, f(x) increases on (0, ∞)
and f(x) is decreasing on interval x ∈ (–∞, 0)

3. Let $I = \int_0^{\frac{x}{2}} (2\log\cos x - \log\sin 2x)dx$, then we have
$= \int_0^{\frac{\pi}{2}} \left(\log\cos^2 x - \log\sin 2x\right) dx$
$= \int_0^{\frac{\pi}{2}} \log\frac{\cos^2 x}{\sin x} dx$
$= \int_0^{\frac{\pi}{2}} \log\frac{\cos^2 x}{2\sin x \cdot \cos x} dx$
$= \int_0^{\frac{\pi}{2}} \log\frac{\cos x}{2\sin x} dx$
$= \int_0^{\frac{\pi}{2}} (\log\cos x - \log\sin x - \log 2) dx$
$= \int_0^{\frac{\pi}{2}} \log\cos x\, dx - \int_0^{\frac{x}{2}} \log\sin x\, dx - \int_0^{\frac{x}{2}} \log 2$
We know that $\int_0^{\frac{\pi}{2}}$ log cos x dx =$\int_0^{\frac{\pi}{2}}$ log sin x dx .....(i)
Hence from equation (i), we have
$I = -\int_0^{\frac{\pi}{2}} \log 2 = -\frac{\pi}{2}\log 2$

4. Given: f(x) = $\log_a$ x , 0 < a < 1
Theorem: Let f be a differentiable real function defined on an open interval (a,b), then
   i. If f′(x) > 0 for all $\mathrm{x} \in (\mathrm{a}, \mathrm{b})$, then f(x) is increasing on (a, b)
   ii. If f′(x) < 0 for all $\mathrm{x} \in (\mathrm{a}, \mathrm{b})$, then f(x) is decreasing on (a, b)

For the value of x obtained in (i) f(x) is increasing and for remaining points in its domain it is decreasing.
Here we have,
f(x) = $\log_a$ x, 0 < a < 1
$\Rightarrow \mathrm{f}'(\mathrm{x}) = \frac{\mathrm{d}}{\mathrm{dx}}(\log_\mathrm{a} \mathrm{x})$
$\Rightarrow f(x) = \frac{1}{x\log a}$
As given 0 < a < 1
log(a) < 0 and for x > 0

$= \frac{1}{x \log_a} < 0$
Therefore f′(x) is
$= \frac{1}{x \log_a} < 0$
f′(x) < 0
Hence, it is the condition for f(x) to be decreasing
Thus f(x) is decreasing for all x > 0.

5. Let I = $\int \frac{1}{(x-1)(x+1)(x+2)}$
Using partial fractions,
$\frac{1}{(x-1)(x+1)(x+2)}$ = $\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2}$
$\Rightarrow$ 1 = A(x + 1) (x + 2) + B(x - 1) (x + 2) + C (x^2 - 1)
Put x = 1
$\Rightarrow$ 1 = 6A $\Rightarrow$ A = $\frac{1}{6}$
Put x = -1
$\Rightarrow$ 1 = -2B $\Rightarrow$ B = -$\frac{1}{2}$
Put x = -2
$\Rightarrow$ 1 = 3C $\Rightarrow$ C = $\frac{1}{3}$
So,
I = $\frac{1}{6}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{3}\int \frac{dx}{x+2}$
I = $\frac{1}{6}\log|x-1| - \frac{1}{2}\log|x \ \ +1| + \frac{1}{3}\log|x+2| + c$

OR

Let I = $\int_0^{\pi/4} x^2 \sin x dx$, where
$\int x^2 \sin x dx = x^2 \int \sin x dx - \int 2x \left(\int \sin x dx\right) dx = x^2 \cos x + \int 2x \cos x dx$
$= x^2 \cos x + 2\left[x \int \cos x dx - \int \left(\int \cos x dx\right) dx\right]$
$= -x^2 \cos x + 2\left[x \sin x - \int \sin x dx\right]$
$\therefore \quad \int_0^{\frac{\pi}{4}} x^2 \sin x dx = \left[-x^2 \cos x + 2x \sin x + 2\cos x\right]_0^{\frac{x}{4}}$
$= \frac{1}{\sqrt{2}}\left[\frac{-\pi^2}{16} + \frac{\pi}{2} + 2\right] - 2$
$= \frac{1}{\sqrt{2}}\left[\frac{-\pi^2}{16} + \frac{\pi}{2} + 2\right] - 2$
$= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$
$\therefore \int_0^{\frac{\pi}{4}} x^2 \sin x dx = \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$

6. Let R = $\{(\Delta_1, \Delta_2) : \Delta_1 \sim \Delta_2\}$ be a relation defined on A.
Now,
For reflexivity:
R is Reflexive if $(\Delta, \Delta) \subseteq R \ \forall \Delta \subseteq A$
We observe that for each $\Delta \in$ A we have,
$\Delta \sim \Delta$ since, every triangle is similar to itself.
$= (\Delta, \Delta) \in R \forall \Delta \in A$
$\Rightarrow$ R is reflexive.
For symmetry:
R is Symmetric if $(\Delta_1, \Delta_2) \in \mathrm{R} \Rightarrow (\Delta_2, \Delta_1) \in \mathrm{R}, \ \forall \ \Delta_1, \Delta_2 \in \mathbb{A}$
Let $\Delta_1 \sim \Delta_2$
= $\Delta_1 \sim \Delta_2$
= $\Delta_2 \sim \Delta_1$
$\Rightarrow (\Delta_2, \Delta_1) \in \mathrm{R}$
$\Rightarrow$ R is symmetric
For Transitivity:

R is Transitive if $(\Delta_1, \Delta_2) \in R$ and $(Q_2, \Delta_3) \in R \Rightarrow (\Delta_1, \Delta_3) \in R \neq A_1, \Delta_2, A_3 \in A$

$(\Delta_1, \Delta_2) \in \mathrm{R}$ and $((\Delta_2, \Delta_3) \in \mathrm{R} \forall \Delta_1, \Delta_2, \Delta_3 \in \mathrm{A}$

$\Rightarrow \Delta_1 \sim \Delta_2$ and $\Delta_2 - \Delta_3$

$\Rightarrow \Delta_1 \sim \Delta_3$

$\Rightarrow \Delta_1 \sim \Delta_3$

= R is transitive.

Since R is reflexive, symmetric and transitive, it is an equivalence relation on A

7. $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

RHS = $\frac{\pi}{4} - \frac{x}{2}$

LHS $= \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$= \tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$

$[\because \cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}]$

$[\because\ 1 = \sin^2\frac{x}{2} + cos^2\frac{x}{2}]$

$[\because sinx = 2\sin\frac{x}{2}\cos\frac{x}{2}]$

$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right]$

$= \tan^{-1}\left(\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}\right)$

Dividing the numerator and Denominator by $cos\frac{\pi}{2}$,

$\therefore$ LHS = $\tan^{-1}\left[\frac{\frac{\cos\frac{x}{2}}{\cos\frac{x}{2}} - \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{\frac{\cos\frac{x}{2}}{\cos\frac{x}{2}} + \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}\right]$

$= \tan^{-1}\left(\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}\right)$

$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$

$\Rightarrow \frac{\pi}{4} - \frac{x}{2}$

$\left[\because \tan^{-1}(\tan\theta) = \theta; \forall\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

= RHS

Hence proved

8. Let the given integral be, I$= \int \frac{dx}{x(x-2)(x-4)}$

Now using partial fractions we have, $\frac{1}{x(x-2)(x-4)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-4}$ ... (i)

which implies,

A(x - 2)(x - 4) + Bx(x - 4) + Cx(x - 2) = 1

Now put x - 2 = 0

Therefore, x = 2

A(0) + B $\times$ 2(2 - 4) + C(0) = 1

B $\times$ 2(-2) = 1

$B = -\frac{1}{4}$

Now put x - 4 = 0

Therefore, x = 4

A(0) + B × (0) + C $\times$ 4(4 - 2) = 1

C $\times$ 4(2) = 1

$C = \frac{1}{8}$

Now put x = 0

A(0 - 2)(0 - 4) + B(0) + C(0)=1

$A=\frac{1}{8}$

Now From equation (1) we get

$\frac{1}{x(x-2)(x-4)}=\frac{1}{8}\times\frac{1}{x}-\frac{1}{4}\times\frac{1}{x-2}+\frac{1}{8}\times\frac{1}{x-4}$

$\int\frac{dx}{x(x-2)(x-4)}=\frac{1}{8}\int\frac{1}{x}dx-\frac{1}{4}\int\frac{1}{x-2}dx+\frac{1}{8}\int\frac{1}{x-4}dx$

$=\frac{1}{8}\log|x|-\frac{1}{4}\log|x-2|+\frac{1}{8}\log|x-4|+c$

9. Given, $y=\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Put, x = tan $\theta\Rightarrow\theta$ = $\tan^{-1}$x

$\therefore y=\tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$

= $\tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right]$

= $\tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$

= $\tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}\right]$

= $\tan^{-1}\left[\tan\frac{\theta}{2}\right]$

$\Rightarrow y=\frac{\theta}{2}=\frac{1}{2}\tan^{-1}x$

$\therefore\frac{dy}{dx}=\frac{1}{2(1+x^2)}$

OR

We have, y = sinx sin 2x sin 3x sin 4x ...(i)

Taking log on both sides

log y = log(sin x sin 2x sin 3x sin 4x)

$\Rightarrow$ log y = log sin x + log sin 2x + log sin 3x + log sin 4x

Differentiating with respect to x using chain rule,

$\frac{1}{y}\frac{dy}{dx}=\frac{d}{dx}(\log\sin x)+\frac{d}{dx}(\log\sin 2x)+\frac{d}{dx}(\log\sin 3x)+\frac{d}{dx}(\log\sin 4x)$

$\Rightarrow\frac{1}{y}\frac{dy}{dx}=\frac{1}{\sin x}\frac{d}{dx}(\sin x)+\frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x)+\frac{1}{\sin 3x}\frac{d}{dx}(\sin 3x)+\frac{1}{\sin 4x}\frac{d}{dx}(\sin 4x)$

$\Rightarrow\frac{1}{y}\frac{dy}{dx}=\frac{1}{\sin x}(\cos x)+\frac{1}{\sin 2x}(\cos 2x)\frac{d}{dx}(2x)+\frac{1}{\sin 3x}(\cos 3x)\frac{d}{dx}(3x)+\frac{1}{\sin 4x}(\cos 4x)\frac{d}{dx}(4x)$

$\Rightarrow\frac{1}{y}\frac{dy}{dx}$ = [cot x + cot 2x(2) + cot 3x (3) + cot 4x (4)]

$\Rightarrow\frac{dy}{dx}$ = y[cot x + 2 cot 2x + 3 cot 3x + 4 cot 4x]

$\Rightarrow\frac{dy}{dx}$ = (sin x sin 2x sin 3x sin 4x) [cot x + 2 cot 2x + 3 cot 3x + 4 cot 4x] [Using equation (i)]

The differentiation of the given function y is as above.

10. **Read the text carefully and answer the questions:**

To hire a marketing manager, it's important to find a way to properly assess candidates who can bring radical changes and has leadership experience.

Ajay, Ramesh and Ravi attend the interview for the post of a marketing manager. Ajay, Ramesh and Ravi chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8, and 0.5. If the change does take place.

(a) Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{4}{7}\times 0.3}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{1.2}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{1.2}{7}}{\frac{3}{7}}$

$= \frac{1.2}{3} = \frac{12}{30} = \frac{2}{5}$

(b) Let E_1: Ajay(A) is selected, E_2: Ramesh(B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_2/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{1}{7}\times 0.8}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{0.8}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{0.8}{7}}{\frac{3}{7}}$

$= \frac{0.8}{3} = \frac{8}{30} = \frac{4}{15}$

(c) Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_3/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{2}{7}\times 0.5}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{1}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{1}{3}$

(d) Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

Ramesh or Ravi

$\Rightarrow P(E_2/A) + P(E_3/A) = \frac{4}{15} + \frac{1}{3} = \frac{9}{15} = \frac{3}{5}$

OR

Read the text carefully and answer the questions:

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(a) Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

$E_1 = PQ$, $E_2 = \bar{P}\bar{Q}$, $E_3 = \bar{P}Q$, $E_4 = P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

$P(E_1) = 0.3 \times 0.2 = 0.06$, $P(E_2) = 0.7 \times 0.8 = 0.56$, $P(E_3) = 0.7 \times 0.2 = 0.14$, $P(E_4) = 0.3 \times 0.8 = 0.24$

$$P\left(\frac{E}{E_1}\right) = 0, P\left(\frac{E}{E_2}\right) = 0, P\left(\frac{E}{E_3}\right) = 1, P\left(\frac{E}{E_4}\right) = 1$$

$P(E) = P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right) + P(E_3)\cdot P\left(\frac{E}{E_3}\right) + P(E_4)\cdot P\left(\frac{E}{E_4}\right)$

= 0.14 + 0.24 + = 0.38

(b) By Bayes' Theorem, $\mathrm{P}\left(\frac{E_3}{E}\right) = \frac{P(E_3)\cdot P\left(\frac{E}{E_3}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right)+P(E_2)\cdot P\left(\frac{E}{E_2}\right)+P(E_3)\cdot P\left(\frac{E}{E_3}\right)+P(E_4)\cdot P\left(\frac{E}{E_4}\right)}$

$= \frac{0.14}{0.38} = \frac{7}{19}$

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses E_1 and E_2 are actually eliminated as

$$P\left(\frac{E}{E_1}\right) = P\left(\frac{E}{E_2}\right) = 0$$

(c) By Bayes' Theorem, $\mathrm{P}\left(\frac{E_4}{E}\right) = \frac{P(E_4)\cdot P\left(\frac{E}{E_4}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right)+P(E_2)\cdot P\left(\frac{E}{E_2}\right)+P(E_3)\cdot P\left(\frac{E}{E_3}\right)+P(E_4)\cdot P\left(\frac{E}{E_4}\right)}$

$= \frac{0.24}{0.38} = \frac{12}{19}$

(d) Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

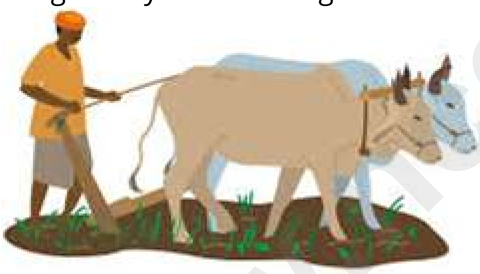
$E_1 = PQ$, $E_2 = \bar{P}\bar{Q}$, $E_3 = \bar{P}Q$, $E_4 = P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

$P(E_1) = 0.3 \times 0.2 = 0.06$, $P(E_2) = 0.7 \times 0.8 = 0.56$, $P(E_3) = 0.7 \times 0.2 = 0.14$, $P(E_4) = 0.3 \times 0.8 = 0.24$

11. **Read the text carefully and answer the questions:**

Two farmers Ankit and Girish cultivate only three varieties of pulses namely Urad, Massor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.



September sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$$

October sales (in ₹):

$$A = \begin{pmatrix} Urad & Masoor & Mung \\ 5000 & 10000 & 6000 \\ 20000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$$

(a) $A + B = \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} + \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$

$= \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix}$

The combined sales of Masoor in September and October, for farmer Girish ₹40000.

(b)

A + B = $\begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix}$ + $\begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$

$= \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix}$

The combined sales of Urad in September and October, for farmer Ankit is ₹15000.

(c)

A - B = $\begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix}$ - $\begin{bmatrix} 5000 & 10,000 & 6000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}$

= $\begin{bmatrix} 10,000-5000 & 20,000-10,000 & 30,000-6000 \\ 50,000-20,000 & 30,000-10,000 & 10,000-10,000 \end{bmatrix}$

A - B = $\begin{bmatrix} 5000 & 10,000 & 24,000 \\ 30,000 & 20,000 & 0 \end{bmatrix}$ Ankit Girish

12. Given, $e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)+\left(1+e^{\frac{y}{x}}\right)\frac{dx}{dy}$ = 0

Substitute x = vy $\Rightarrow$ $\frac{dx}{dy}$ = v + y$\frac{dv}{dy}$

e^v - ve^v + v + ve^v +(1 + e^v)y$\frac{dv}{dy}$ = 0

$\Rightarrow$ (v + ev) + (1 + e^v)y$\frac{dv}{dy}$ = 0

$\Rightarrow$ $\left(\frac{1+e^v}{v+e^v}\right)dv+\frac{dy}{y}$ = 0

Integrating

$\int\left(\frac{1+e^v}{v+e^v}\right)$dv = - $\int\frac{dy}{y}$

$\Rightarrow$ log|v + e^v|= -log|y| + log c

$\Rightarrow$ log|v + e^v| + log|y| = log c

$\Rightarrow$ log|{(v + e^v)y}| = log c

$\Rightarrow$ log$\left|\left\{\left(\frac{x}{y}+e^{\frac{x}{y}}\right)y\right\}\right|$ = log c

$\Rightarrow x+e^{\frac{x}{y}}y$ = c

When x = 0 and y = 1 $\Rightarrow$ c = 1

$x+e^{\frac{x}{y}}y=1$

OR

Given, $\frac{dy}{dx}-3y\cot x$ = sin 2 x

This is a linear equation of the form $\frac{dy}{dx}+Py=Q$

where, p = -3cot x, Q = sin 2x

$\therefore$ I.F. $=e^{\int\rho dx}$

$\therefore$ I.F. $=e^{\int-3\cot x dx}$ $=e^{-3\int\cot x dx}$

$=e^{-3\log|\sin x|}$ $=e^{\log|\sin x|^{-3}}$

$=(\sin x)^{-3}=\frac{1}{\sin^3 x}$

$\therefore$ Solution is given as,

$y\times$ (I.F.) $=\int Q\times$ (I.F.)dx

$\Rightarrow$ $y\cdot\frac{1}{\sin^3 x}=\int\sin 2x\cdot\frac{1}{\sin^3 x}dx$

$\Rightarrow y\cdot\frac{1}{\sin^3 x}=\int\frac{2\sin x\cdot\cos x}{\sin^3 x}dx$ [$\because$ sin 2x = 2 sinx cosx]

$\Rightarrow \frac{y}{\sin^3 x}=2\int\operatorname{cosec}x\cdot\cot x\cdot dx$

$\Rightarrow \frac{y}{\sin^3 x}$ = -2 cosec x + c

Putting y = 2, x = $\frac{\pi}{2}$

$\frac{2}{\sin^3\frac{\pi}{2}}=-2\operatorname{cosec}(\frac{\pi}{2})+c$

2 = -2 + c ⇒ c = 4

$\therefore \frac{y}{\sin^3 x} = -\frac{2}{\sin x} + 4$

⇒ y = -2 $\sin^2$x + 4 $\sin^3$x

13. **Given:** Total surface area of cylinder

$= 2\pi r^2 + 2\pi rh$

$A = 2\pi r^2 + 2\pi rh$

$\therefore h = \frac{A - 2\pi r^2}{2\pi r}$ ...(i)

[Where r is the radius and h is the height of the cylinder]

Volume of cylinder (V) = $\pi r^2 h$

$V = \pi r^2 \frac{(A - 2\pi r^2)}{2\pi r}$ [from (i)]

$V = \frac{r^r}{2}\left(A - 2\pi r^2\right)$

$V = \frac{Ar}{2} - \pi r^3$

Now, $\frac{dV}{dr} = \frac{d}{dr}\frac{Ar}{2} - \frac{d}{dr}\left(\pi r^3\right)$

$\frac{dV}{dr} = \frac{A}{2} - 3\pi r^2$

For maximum/minimum;

$\frac{dV}{dr} = 0$

$\therefore \frac{A}{2} - 3\pi r^2 = 0$

$\frac{A}{2} = 3\pi r^2$

A = $6\pi r^2$

So, $2\pi r^2 + 2\pi rh = 6\pi r^2$

$2\pi rh = 4\pi r^2$

h = 2r

$r = \frac{h}{2}$ ...(ii)

Again, $\frac{dV}{dr} = \frac{A}{2} - 3\pi r^2$

And, $\frac{d^2V}{w^2} = 0 - 6\pi r$

for any value of r i.e., r > 0

$\frac{d^2V}{dr^2}$ < 0

Hence, volume is maximum when r = $\frac{h}{2}$

Hence Proved

OR

Let ABCD be a rectangle in a given circle of radius 'a' with centre at O.

Let AB = 2x and AD = 2y be the sides of the rectangle.

Applying Pythagoras theorem in ΔOAM, we get

$AM^2 + OM^2 = OA^2$

⇒ $x^2 + y^2 = a^2$

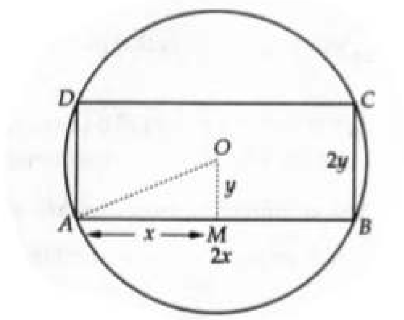
⇒ y = $\sqrt{a^2 - x^2}$

Let P be the perimeter of the rectangle ABCD. Then,we have,

P = 4x + 4y

⇒ P = 4x + 4$\sqrt{a^2 - x^2}$ [Using (i)]

⇒ $\frac{dP}{dx}$ = 4 $-\frac{4x}{\sqrt{a^2 - x^2}}$ [differentiating both sides w.r.t x]



The critical points of P are given by $\frac{dP}{dx}$ = 0

$\because \frac{dP}{dx}$ = 0.

$\Rightarrow 4 - \frac{4x}{\sqrt{a^2-x^2}} = 0$

$\Rightarrow 4 = \frac{4x}{\sqrt{a^2-x^2}} \Rightarrow \sqrt{a^2-x^2} = x$

$\Rightarrow a^2 - x^2 = x^2$

$\Rightarrow 2x^2 = a^2$

$\Rightarrow x = \frac{a}{\sqrt{2}}$

Now, $\frac{dP}{dx} = 4 - \frac{4x}{\sqrt{a^2-x^2}}$

$$\Rightarrow \frac{d^2P}{dx^2} = \frac{4\left\{\sqrt{a^2-x^2} - \frac{x(-x)}{\sqrt{a^2-x^2}}\right\}}{\{\sqrt{a^2-x^2}\}^2} = \frac{-4a^2}{(a^2-x^2)^{3/2}}$$

$$\Rightarrow \left(\frac{d^2P}{dx^2}\right)_{x=a/\sqrt{2}} = \frac{-4a^2}{\left(a^2-\frac{a^2}{2}\right)^{3/2}} = \frac{-8\sqrt{2}}{a} < 0$$

Thus, P is maximum when $x = \frac{a}{\sqrt{2}}$.

Putting $x = \frac{a}{\sqrt{2}}$ in (i), we obtain $y = \frac{a}{\sqrt{2}}$.

$\because x = y = a\sqrt{2} \Rightarrow 2x = 2y \Rightarrow AB = BC$

$\Rightarrow$ ABCD is a square.

Hence, P is maximum when the rectangle is square of side $2x = \frac{2a}{\sqrt{2}} = \sqrt{2}a$.

14. **Read the text carefully and answer the questions:**

Family photography is all about capturing groups of people that have family ties. These range from the small group, such as parents and their children. New-born photography also falls under this umbrella. Mr Ramesh, His wife Mrs Saroj, their daughter Sonu and son Ashish line up at random for a family photograph, as shown in figure.

(a) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

Let A denotes the event that daughter is at one end n(A) = 12 and B denotes the event that father, and mother are in the middle n(B) = 4

Also, n(A ∩ B) = 4

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{4}{24}}{\frac{4}{24}} = 1$$

(b) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

Let A denotes the event that mother is at right end. n(A) = 6 and B denotes the event that son and daughter are together. n(B) = 12

Also, n (A ∩ B) = 4

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{4}{24}}{\frac{12}{24}} = \frac{1}{3}$$

(c) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

Let A denotes the event that father, and mother are in the middle. n(A) = 4 and B denote the event that son is at right end. n(B) = 6

Also, n (A ∩ B) = 2

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{2}{24}}{\frac{6}{24}} = \frac{1}{3}$$

(d) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24

Let A denotes the event that father and son are standing together. n(A) = 12 and B denote the event that mother and daughter are standing together. n(B) = 12

Also, n (A ∩ B) = 8

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{8}{24}}{\frac{12}{24}} = \frac{2}{3}$$

SECTION B - 15 MARKS

15. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(a) **(b)** $\frac{3}{2}(i+j)$

Explanation:

Let $\vec{a} = a_1\hat{i} + \alpha_2\hat{j} + a_3\hat{k}$, $\vec{\beta} = \beta_1\hat{i} + \beta_2\hat{j} + \beta_3\hat{k}$

$\vec{b} = 3\hat{i} + 4\hat{k}$

$\vec{\alpha} + \vec{\beta} = 3\overrightarrow{i} + 4\vec{k}$

$(\alpha_1 + \beta_1) i + (\alpha_2 + \beta_2) j + (\alpha_3 + \beta_3)\hat{k} = \bar{3} + 4\hat{k}$

$\Rightarrow \alpha_1 + \beta_1 = 3$

$\alpha_2 + \beta_2 = 0$

$a_3 + \beta_3 = 4$

Given that $\vec{a}$ is parllel to $\overrightarrow{\mathrm{a}}$

$\vec{\alpha} \times \vec{a} = 0$

$\begin{vmatrix} i & \hat{j} & \hat{k} \\ \alpha_1 & \alpha_2 & \alpha_3 \\ 1 & 1 & 0 \end{vmatrix} = 0$ {Given $\vec{a} = \vec{i} + \vec{j}$}

$-\alpha_3\hat{i} + \alpha_3\hat{j} + (\alpha_1 - \alpha_2)\hat{k} = 0$

$\alpha_3 = 0, \alpha_1 - \alpha_2 = 0$

$\alpha_3 = 0, \alpha_1 = \alpha_2$

Given $\vec{\beta}$ is perpendicular to $\overrightarrow{\mathrm{a}}$

$\vec{\beta} \cdot \vec{a} = 0$

$\left(\beta_1\tilde{i} + \beta_2\tilde{j} + \beta_3\tilde{k}\right) \cdot (\tilde{i} + \tilde{j}) = 0$

$\beta_1 + \beta_2 = 0$

$\beta_1 = -\beta_2$

Solving $\alpha_3 = 0, \alpha_1 = \alpha_2, \alpha_1 + \beta_1 = 3$
$\alpha_2 + \beta_2 = 0, \alpha_3 + \beta_3 = 4, \beta_1 = -\beta_2$
$\Rightarrow \alpha_1 = \alpha_2 = \frac{3}{2}, \alpha_3 = 0$
$\vec{\alpha} = \alpha_1 \hat{i} + \alpha_2 \hat{j} + \alpha_3 \hat{k}$
$\vec{\alpha} = \frac{3}{2}(i + \vec{j})$

(b) (1, 0, 0), (0, 1, 0), (0, 0, 1) respectively.

(c)
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$
$= \hat{i}\,(-2 - 15) - \hat{j}\,(-4 - 9) + \hat{k}\,(10 - 3)$
$= -17\hat{i} + 13\hat{j} + 7\hat{k}$

(d) **(b)** $\frac{1}{6}$

Explanation:

Multiplying the first equation of the plane by 2 ;
4x + 4y - 2z + 4 = 0
4x + 4y - 2z = -4 ...(i)
The second equation of the plane is
4x + 4y - 2z + 5 = 0
4x + 4y - 2z = -5 ...(ii)
We know that the distance between two planes ax + by + cz = d_1 and ax + by + cz = d_2 is
$\frac{|d_2 - d_1|}{\sqrt{a^2+b^2+c^2}}$
So, the required distance
$= \frac{|-5+4|}{\sqrt{4^2+4^2+(-2)^2}}$
$= \frac{|-1|}{\sqrt{16+16+4}}$
$= \frac{1}{\sqrt{36}}$
$= \frac{1}{6}$ units

(e) The required distance = the length of the perpendicular from P(2, 1, -1) to the plane x - 2y + 4z - 9 = 0
$= \frac{|2-2\times1+4\times(-4)-9|}{|\sqrt{1^2+(-2)^2+4^2}|}$ = $\frac{13}{\sqrt{21}}$ units.

16. $|\vec{a} \times \vec{b}| = |a||b|$ sin θ
$\therefore$ sin θ = $\frac{8}{2\times5} = \frac{4}{5}$
$\therefore$ cos θ = $\sqrt{1 - \sin^2\theta}$
$\sqrt{1 - \frac{16}{25}} = \frac{3}{5}$
$\therefore$ a.b = $|\vec{a}||\vec{b}|$ cos θ
= 2 x 5 x $\frac{3}{5}$ = 6

OR

$\vec{a} \perp \vec{b}$
$\Rightarrow \vec{a} \cdot \vec{b}$ = 0 & $|\vec{a}|$ = 5
Now, $|\vec{a} + \vec{b}|$ = 13
$\therefore |\vec{a} + \vec{b}|^2$ = 169
$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a} \cdot \vec{b})$ = 169
$\Rightarrow 25 + |\vec{b}|^2$ + 2·0 = 169
$\therefore |\vec{b}|$ = 12

17. We are given that

$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$ and $\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$

The vector equations of the given lines can be re-written as

$\vec{r} = -\hat{i} + \hat{j} - \hat{k} + \lambda(\hat{i} + \hat{j} - \hat{k})$ and $\vec{r} = \hat{i} - \hat{j} + 2\hat{k} + \mu(-\hat{i} + 2\hat{j} + \hat{k})$

Comparing the given equations with the equations

$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$

We get,

$\overrightarrow{a_1} = -\hat{i} + \hat{j} - \hat{k}$

$\overrightarrow{a_2} = \hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{b_1} = \hat{i} + \hat{j} - \hat{k}$

$\overrightarrow{b_2} = -\hat{i} + 2\hat{j} + \hat{k}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 2\hat{i} - 2\hat{j} + 3\hat{k}$

and $|\overrightarrow{b_1} \times \overrightarrow{b_2}|$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$

= $3\hat{i} + 3\hat{k}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}|$ = $\sqrt{3^2 + 3^2}$

= $\sqrt{9+9}$

= $3\sqrt{2}$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})$ = $(2\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} + 3\hat{k})$

= 6 + 9 = 15

Hence the shortest distance between the lines,

$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is given by

d = $\left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$

= $\left|\frac{15}{3\sqrt{2}}\right|$

= $\frac{5}{\sqrt{2}}$

OR

Given the equation of the plane is,

2x - 3y - 6z = 14

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} - 3\hat{j} - 6\hat{k})$ = 14

Dividing the equation by $\sqrt{(2)^2 + (-3)^2 + (-6)^2}$

$\vec{r} \cdot \frac{(2\hat{i} - 3\hat{j} - 6\hat{k})}{\sqrt{4+9+36}} = \frac{14}{\sqrt{4+9+36}}$

$\vec{r} \cdot \left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = \frac{14}{7}$

$\vec{r} \cdot \left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right)$ = 2 ...(i)

We know that the vector equation of a plane with distance d from the origin and normal to unit vector $\hat{n}$ is given by

$\vec{r} \cdot \hat{n}$ = d ...(ii)

Comparing (i) and (ii),

d = 2 and

$\hat{n} = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$

So, the distance of the plane from origin = 2 unit

Direction cosine of normal to plane = $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$.

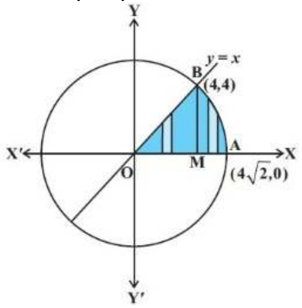
18. The given equations are

$y = x$ ... (1)

and $x^2 + y^2 = 32$ ... (2)

Solving (1) and (2), we find that the line and the circle meet at B(4, 4) in the first quadrant.

Draw perpendicular BM to the x-axis.



Therefore, the required area = area of the region OBMO + area of the region BMAB.

Now, the area of the region OBMO

$= \int_0^4 y dx = \int_0^4 x dx$ ...(3)

$= \frac{1}{2}\left[x^2\right]_0^4 = 8$

Again, the area of the region BMAB

$= \int_4^{4\sqrt{2}} y dx = \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\, dx$

$= \left[\frac{1}{2}x\sqrt{32 - x^2} + \frac{1}{2} \times 32 \times \sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$

$= \left(\frac{1}{2}4\sqrt{2} \times 0 + \frac{1}{2} \times 32 \times \sin^{-1}1\right) - \left(\frac{4}{2}\sqrt{32 - 16} + \frac{1}{2} \times 32 \times \sin^{-1}\frac{1}{\sqrt{2}}\right)$

$= 8\pi - (8 + 4\pi) = 4\pi - 8$ ...(4)

Adding (3) and (4), we get, the required area = 4π.

SECTION C - 15 MARKS

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(a) **(d)** Both MP and AP rise

Explanation:

Both MP & AP rise

(b) **(a)** R

Explanation:

Corner points	Value of Z = 2x + 5y
P(0, 5)	Z = 2(0) + 5(5) = 25
Q(1, 5)	Z = 2(1) + 5(5) = 27
R(4, 2)	Z = 2(4) + 5(2) = 18 $\rightarrow$ Minimum
S(12, 0)	Z = 2(12) + 5(0) = 24

Thus, minimum value of Z occurs ar R(4, 2)

(c) Given $\bar{x}$ = 18, $\bar{y}$ = 100, σ_x = 14, σ_y = 20, r = 0.8

$b_{xy} = r \cdot \frac{\sigma_y}{\sigma_x} = 0.8 \times \frac{20}{14} = \frac{8}{7}$

Regression equation y on x

y - $\bar{y}$ = $b_{yx} \cdot (x - \bar{x})$

y - 100 = $\frac{8}{7}$(x - 18)

7y - 700 = 8x - 144

8x - 7y + 556 = 0

(d) Given x = $\frac{24-2p}{3}$ and price per unit = p.

So, R = p·x = p·$\left(8 - \frac{2}{3}p\right)$ = $8p - \frac{2}{3}p^2$

(e) Given P = $\frac{100}{q+2}$ - 4

So, R = p·q = $\frac{100q}{q+2}$ - 4q

MR = $\frac{d(\mathrm{R})}{dq}$ = $\frac{(q+2)\cdot 100 - 100q\cdot 1}{(q+2)^2}$ - 4

$\Rightarrow$ MR = $\frac{200}{(q+2)^2}$ - 4

20. Given C(x) = $\frac{1}{3}x^3$ + x^2 - 8x + 5

i. MC = $\frac{d}{dx}$C(x) = $\frac{d}{dx}(\frac{1}{3}x^3$ + x^2 - 8x + 5)

$\Rightarrow$ MC = x^2 + 2x - 8

ii. AC = $\frac{C(x)}{x}$ = $\frac{\frac{1}{3}x^3+x^2-8x+5}{x}$

$\Rightarrow$ AC = $\frac{1}{3}x^2$ + x - 8x + $\frac{5}{x}$

iii. Slope of average cost function = $\frac{d}{dx}$(AC) = $\frac{2}{3}x$ + 1 - $\frac{5}{x^2}$

OR

Let x be the number of units produced and sold.

$\Rightarrow$ Revenue function = R(x) = 6x

Given that variable cost is estimated to 35% of R(x)

$\Rightarrow$ V(x) = 35% of R(x)

= $\frac{35}{100} \times 6x$

$\Rightarrow V(x) = \frac{21x}{10}$

Hence, cost function = C(x) = Fixed cost + V(x)

= 20000 + $\frac{21x}{10}$ [Given, fixed cost = 20000]

Now, profit function P(x) = R(x) - C(x)

= 6x - 20000 - $\frac{21x}{10}$

$\Rightarrow P(x) = \frac{39x}{10} - 20000$

21.

x	y	xy	x^2
1	7	7	1
2	6	12	4
3	5	15	9
4	4	16	16
5	3	15	25
Σx = 15	Σy = 25	Σxy = 65	Σx^2 = 55

$\bar{x}$ = $\frac{\sum x}{n}$ = $\frac{15}{5}$ = 3

and $\bar{y}$ = $\frac{\sum y}{n}$ = $\frac{25}{5}$ = 5

$b_{yx} = \frac{\sum xy - \frac{\sum x \cdot \Sigma y}{n}}{\sum x^2 - \frac{(\Sigma x)^2}{n}}$

= $\frac{65 - \frac{15.25}{5}}{55 - \frac{(15)^2}{5}}$

$= \frac{65-75}{55-45} = \frac{-10}{10}$ = -1

Regression line y on x is given by

(y - $\bar{y}$) = b_{yx} (x - $\bar{x}$)

$\therefore$ y - 5 = -1(x - 3)

$\Rightarrow$ y - 5 = -x + 3

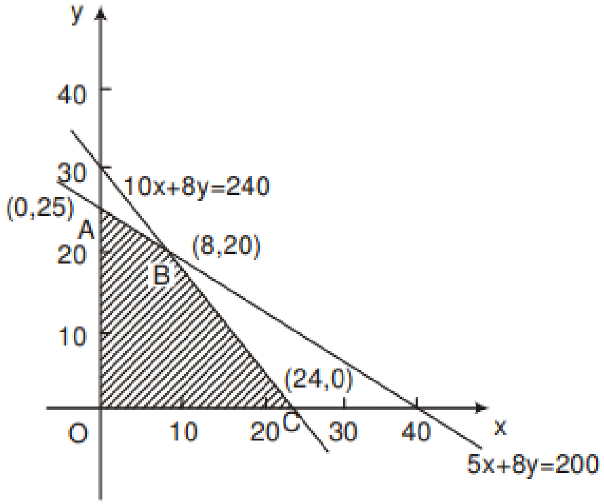
$\Rightarrow$ x + y = 8 ...(i)

When x = 6,

From (i), we get

$y = 8 - 6$
$\Rightarrow y = 2$

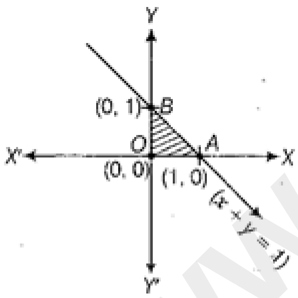
22. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise P = 50x + 60y
such that $5x + 8y \leq 200$
$10x + 8y \leq 240$
$x, y \geq 0$
P(at A) = ₹ 1500
P(at B) = ₹ (400 + 1200) = ₹ 1600
P(at C) = ₹ (1200)
$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20.

OR

Maximise Z = 3x + 4y. Subject to the constraints
$x + y \leqslant 1, x \geqslant 0, y \geqslant 0$
The Shaded region shown in the figure as OAB is bounded and the coordinates of corner points O, A and B are (0, 0), (1, 0) and (0, 1), respectively.



Corner Points	Corresponding value of Z
(0, 0)	0
(1, 0)	3
(0, 1)	4 (Maximum)

Hence, the maximum value of Z is 4 at (0, 1).