ISC 2026 EXAMINATION
Sample Question Paper - 4
Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

This Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions* ***EITHER*** *from* ***Section B OR Section C.***

Section A*: Internal choice has been provided in* ***two questions of two marks each, two questions of four marks each and two questions of six marks each.***

Section B: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

Section C: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

1. **In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.** **[15]**

(a) A square matrix A can be expressed as $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$, where **[1]**

a) $\frac{1}{2}(A + A')$ is a skew-symmetric matrix of A
b) $\frac{1}{2}(A - A')$) is a skew-symmetric matrix of A
c) $\frac{1}{2}(A' - A)$ is a symmetric matrix of A
d) $\frac{1}{2}(A - A')$ is a symmetric matrix of A

(b) $\int_0^{2\pi} |\sin x| dx = ?$ **[1]**

a) 3
b) 2
c) 1
d) 4

(c) If $\cot^{-1}\frac{1}{5} + \tan^{-1}x = \frac{\pi}{2}$, than value of x is: **[1]**

a) 0
b) $\frac{-1}{5}$
c) 1
d) $\frac{1}{5}$

(d) The degree and order respectively of the differential equation $\frac{dy}{dx} = \frac{1}{x+y+1}$ are **[1]**

a) 2, 1
b) 1, 1
c) 2, 2
d) 1, 2

(e) If A and B are two independent events such that P(A) = 0.3, P(A ∪ B) = 0.5, then P(A / B) - P(B / A) = **[1]**

a) $\frac{2}{7}$
b) $\frac{1}{7}$
c) $\frac{3}{35}$
d) $\frac{1}{70}$

(f) Let R be a relation on set N given by R = {(a, b) : a = b - 2, b > 6). Then **[1]**

a) (6, 8) ∈ R
b) (2, 4) ∈ R
c) (3, 8) ∈ R
d) (8, 7) ∈ R

(g) The derivative of an even function is **[1]**

a) an odd function
b) negative
c) an even function
d) Positive

(h) If $e^{x + y}$ = xy then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{(x-xy)}{(xy-y)}$
b) $\frac{1}{2}\frac{(x-xy)}{(xy-y)}$
c) $\frac{y(1-x)}{x(y-1)}$
d) $\frac{x(1-y)}{y(x-1)}$

(i) Which of the following is not correct? **[1]**

a) If A is a skew-symmetric matrix of odd order, then |A| = 0
b) $|kA| = k^3|A|$, where A = $[a_{ij}]_{3\times3}$
c) $|A| = |A^T|$, where A = $[a_{ij}]_{3\times3}$
d) $\begin{vmatrix} a+b & c+d \\ e+f & g+h \end{vmatrix} = \begin{vmatrix} a & c \\ e & g \end{vmatrix} + \begin{vmatrix} b & d \\ f & h \end{vmatrix}$

(j) Let A and B be two symmetric matrices of order 3. **[1]**

Assertion (A): A(BA) and (AB) A are symmetric matrices.

Reason (R): AB is symmetric matrix, if matrix multiplication of A with B is commutative.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

(k) Let S = {1, 2, 3} Determine whether the function f: S → S defined as below have inverse. Find f^{-1}, if it exists. **[1]**

f = { (1, 2) (2, 1) (3, 1) }

(l) If 3A - B = $\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A. **[1]**

(m) Let f : R → R be the function defined by f(x) = 4x - 3 ∀ x ∈ R. Then write f^{-1} **[1]**

(n) A coin is tossed three times. Let the events A and B be defined as follows: A = first toss is head, B = second toss is head. Check the independence of A and B. **[1]**

(o) If E_1 and E_2 are independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$, find $\mathrm{P}\left(\overline{\mathrm{E}}_1 \cap \mathrm{E}_2\right)$. **[1]**

2. Differentiate $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, if $x > \frac{1}{\sqrt{3}}$ **[2]**

OR

A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

3. Evaluate: $\int \frac{dx}{(\sin x - \sin 2x)}$. **[2]**

4. Find the intervals in which $f(x) = \frac{x}{\log x}$ is increasing or decreasing. **[2]**

5. Evaluate the Integral: $\int \frac{1}{x\sqrt{x^4-1}} dx$ **[2]**

OR

Evaluate: $\int \frac{\tan x \sec^2 x}{(1-\tan^2 x)} dx$.

6. Let R_+ be the set of all positive real numbers. Show that the function **[2]**

$f : R_+ \to [-5, \infty) \quad : f(x) = 9x^2 + 6x - 5$ invertible Find f^{-1}

7. Find the value of $\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$ **[4]**

8. Evaluate the integral: $\int \frac{x^2+x+1}{(x+1)^2(x+2)} dx$ **[4]**

9. Let f(x + y) = f(x) + f(y) for all x, y $\in$ R. If f(x) is continuous at x = 0, show that f(x) is continuous at all x. **[4]**

OR

If $x = a.sin(2t).(1 + cos2t)$ and $y = b.cos2t.(1 - cos2t)$, show that $\left(\frac{dy}{dx}\right)_{at\ t=\frac{\pi}{4}} = \frac{b}{a}$

10. **Read the text carefully and answer the questions:** **[4]**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(a) Find the probability that Ajay gets Grade A in all subjects.

(b) Find the probability that he gets Grade A in no subjects.

(c) Find the probability that he gets Grade A in two subjects.

(d) Find the probability that he gets Grade A in at least one subject.

OR

Read the text carefully and answer the questions: **[4]**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(a) Find the value of x.

(b) Find the value of y.

(c) Find $P\left(\frac{C}{B}\right)$.

(d) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

11. **Read the text carefully and answer the questions:** **[6]**

Three shopkeepers A, B and C go to a store to buy stationery. A purchase 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs ₹40, a pen costs ₹12 and a pencil costs ₹3.

(a) How are the number of items purchased by shopkeepers A, B, and C represented in matrix form?

(b) If X represents a matrix, and Y represents the matrix formed by the cost of each item, what does the product X Y equal?

(c) If $A^2 = A$, then what is the value of $(A + I)^3 - 7A$?

12. Solve $(x^2 - y^2)dx + 2xy\,dy = 0$ **[6]**

OR

Show that the differential equation of $\left(x^2 + xy\right) dy = \left(x^2 + y^2\right) dx$ is homogeneous and solve it.

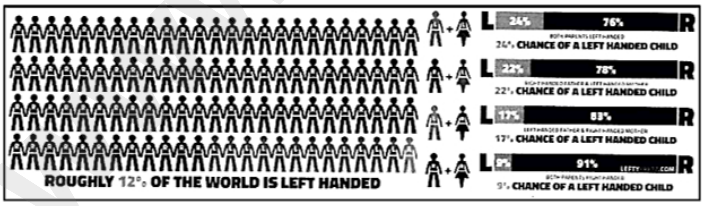
13. Show that the curves intersect orthogonally at the indicated points $y^2 = 8x$ and $2x^2 + y^2 = 10$ at $(1, 2\sqrt{2})$. **[6]**

OR

Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

14. **Read the text carefully and answer the questions:** **[6]**

Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows:

A. When both father and mother are left handed:
Chances of left handed child is 24%.

B. When father is right handed and mother is left handed:
Chances of left handed child is 22%.

C. When father is left handed and mother is right handed:
Chances of left handed child is 17%.

D. When both father and mother are right handed:
Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

(a) Find $P\left(\frac{L}{C}\right)$

(b) Find $P\left(\frac{\bar{L}}{A}\right)$

(c) Find $P\left(\frac{A}{L}\right)$

(d) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

SECTION B - 15 MARKS

15. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) If $\vec{a}\cdot\vec{b} = \frac{1}{2}|\vec{a}||\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$is **[1]**

a) 90^{o} b) 30^{o}

c) 0^{o} d) 60^{o}

(b) Find the angle between the pair of lines **[1]**

$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$

$\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

(c) Find a vector of magnitude 9, which is perpendicular to both the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. **[1]**

(d) If a line makes an angle of 30^{o} with the positive direction of x-axis, 120^{o} with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is: **[1]**

a) 0^{o} b) 120^{o}

c) 90^{o} d) 60^{o}

(e) Find the value of p, so that lines $\frac{x-1}{-2} = \frac{y-4}{3p} = \frac{z-3}{4}$ and $\frac{x-2}{4p} = \frac{y-5}{2} = \frac{1-z}{7}$ are perpendicular to each other. **[1]**

16. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, show that $(\vec{a} + \vec{b})$ is perpendicular $(\vec{a} - \vec{b})$. **[2]**

OR

Find the angle between the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$

17. Find the angle between the planes whose vector equations are $\vec{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 5$ and $\vec{r}.\left(3\hat{i} - 3\hat{j} + 5\hat{k}\right) = 3$. **[4]**

OR

Find the image of the point (3, -2, 1) in the plane 3x - y + 4z = 2.

18. Make a sketch of the region $\left\{(x, y) : 0 \le y \le x^2 + 3; 0 \le y \le 2x + 3; 0 \le x \le 3\right\}$ and find its area using integration. **[4]**

SECTION C - 15 MARKS

19. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) If the cost function of a certain commodity is C(x) = 2000 + 50x - $\frac{1}{5}x^2$, then the average **[1]** cost of producing 5 units is

a) ₹ 2245 b) ₹ 451

c) ₹ 449 d) ₹ 450

(b) A linear programming problem deals with the optimization of a/an: **[1]**

a) linear function b) logarithmic function

c) quadratic function d) exponential function

(c) Two regression lines are represented by 2x + 3y - 10 = 0 and 4x + y - 5 = 0. Find the line of regression of y on x. **[1]**

(d) A company has fixed costs of ₹ 26,000. The cost of producing one item is ₹ 30. If this item sells for ₹ 43, find the break-even point. **[1]**

(e) A firm knows that the demand function for one of its products is linear. It also knows that it can sell 1400 units when the price is ₹ 4 per unit and it can sell 1800 units at a price ₹ 2 per unit. Find the marginal revenue function of this product. **[1]**

20. A firm knows that the demand function for one of its products is linear. It also knows that it can sell 1000 units when the price is ₹ 4 per unit, and it can sell 1500 units when the price is ₹ 2a unit. Determine: **[2]**

i. the demand function
ii. the total revenue function
iii. the average revenue function
iv. the marginal revenue function

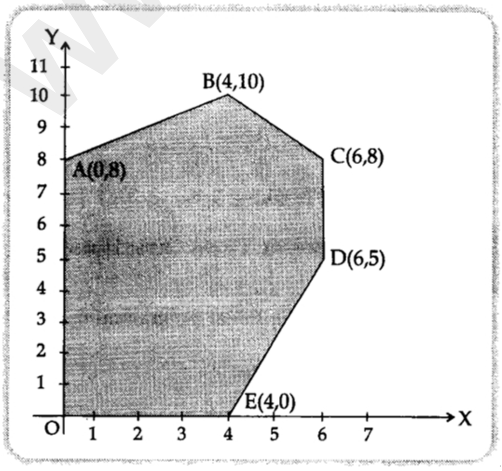
OR

Suppose the cost to produce some commodity is a linear function of output. Find cost as a function of output, if costs are ₹ 4000 for 250 units and ₹ 5000 for 350 units.

21. By using the data $\bar{x}$ = 25, $\bar{y}$ = 30, b_{yx} = 1.6 and b_{xy} = 0.4, find: **[4]**

i. The regression equation y on x.
ii. What is the most likely value of y when x = 60?
iii. What is the coefficient of correlation between x and y?

22. The corner points of the feasible region determined by the system of linear constraints are as shown below: **[4]**



Answer each of the following:

i. Let Z = 3x - 4y be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs.

ii. Let Z = px + qy, where p, q > 0 be the objective function. Find the condition on p and q so that the maximum value of Z occurs at B(4, 10) and C(6, 8). Also mention the number of optimal solutions in this case.

OR

A gardener has a supply of fertilizers of the type I which consists of 10% nitrogen and 6% phosphoric acid, and of the type II which consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, he finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If the type-I fertilizer costs 60 paise per kg and the type-II fertilizer costs 40 paise per kg, determine how many kilograms of each type of fertilizer should be used so that the nutrient requirements are met at a minimum cost. What is the minimum cost?

Solution

SECTION A - 65 MARKS

1. In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(a) **(b)** $\frac{1}{2}(A-A')$) is a skew-symmetric matrix of A

Explanation:

Let B $=\frac{1}{2}(A-A')$

$B'=\frac{1}{2}(A-A')'=\frac{1}{2}[A'-(A')']$

$=\frac{1}{2}(A'-A)=-\frac{1}{2}(A-A')=-B$

$\therefore \frac{1}{2}(A-A')$ is skew-symmetric matrix of A.

(b) **(d)** 4

Explanation:

Let the given integral be = $\int_0^{2\pi}|\sin x|dx$

Now, let us find the equivalent expression to $|\sin x|$ at $0\leq\times\leq 2\pi$

In $0\leq\times\leq\pi$

$|\sin x|$ = sin x

In $\pi\leq\times\leq 2\pi$

$|\sin x|$ = -sin x

$\therefore$ I = $\int_0^{\pi}\sin x dx+\int_{\pi}^{2\pi}-\sin x dx$

$=-\cos\pi-(-\cos 0)+\cos 2\pi-\cos\pi$

= -(-1) + 1 + 1 - (-1)

= 2 + 2

= 4

(c) **(d)** $\frac{1}{5}$

Explanation:

cot$^{-1}\frac{1}{5}$ + tan^{-1} x = $\frac{\pi}{2}$

Also, cot^{-1} x + tan^{-1} x = $\frac{\pi}{2}$

From (i) & (ii), we get x = $\frac{1}{5}$

(d) **(b)** 1, 1

Explanation:

1, 1

(e) **(d)** $\frac{1}{70}$

Explanation:

P(A) = 0.3 , P(A∪B) = 0.5 (Given)

Since, A and B are two independent events,

P(A∩B) = P(A) P(B)

P(A∩B) = 0.3 $\times$ P(B) ...(i)

Also, according to the addition theorem of probability,

P(A $\cup$ B) = P(A) + P(B) - P(A $\cap$ B)

0.5 = 0.3 + P(B) - 0.3 P(B) From (Given) & (i)

0.7 P(B) = 0.2

P(B) = $\frac{0.2}{0.7}$ = $\frac{2}{7}$ ...(ii)

Putting value of P(B) in equation (i) we get,

P(A $\cap$ B) = 0.3 $\times\frac{2}{7}=\frac{3}{10}\times\frac{2}{7}$

P(A $\cap$ B) = $\frac{6}{70}$ ...(iii)

Now,

$P\left(\frac{A}{B}\right)-P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(B)}-\frac{P(A\cap B)}{P(A)}$

= $\frac{\frac{6}{70}}{\frac{2}{7}}-\frac{\frac{6}{70}}{\frac{3}{10}}$ From (iii) & (ii) and (Given)

= $\frac{6}{70}\times\frac{7}{2}-\frac{6}{70}\times\frac{10}{3}$

= $\frac{3}{10}-\frac{2}{7}$

= $\frac{1}{70}$

(f) **(a)** (6, 8) $\in$ R

Explanation:

For option (6, 8)

b = 8 > 6

also, b = 8 - 2 = 6 $\Rightarrow$ a = b - 2

(g) **(a)** an odd function

Explanation:

The derivative of an even function is an odd function.

(h) **(c)** $\frac{y(1-x)}{x(y-1)}$

Explanation:

Given that xy = e^{x+y}

Taking log both sides, we get

$\log_e xy = x + y$ (Since $\log_a b^c = c\log_a b$)

Since $\log_a bc = \log_a b + \log_a c$, we get

$\log_e x + \log_e y = x + y$

Differentiating with respect to x, we obtain

$\frac{1}{x}+\frac{1}{y}\frac{dy}{dx}=1+\frac{dy}{dx}$

Or

$\frac{dy}{dx}\left(\frac{y-1}{y}\right)=\frac{1-x}{x}$

Therefore, $\frac{dy}{dx}=\frac{y(1-x)}{x(y-1)}$

(i) **(d)** $\begin{vmatrix}a+b & c+d\\ e+f & g+h\end{vmatrix}=\begin{vmatrix}a & c\\ e & g\end{vmatrix}+\begin{vmatrix}b & d\\ f & h\end{vmatrix}$

Explanation:

For adding the determinants, we need to find the value of the determinants and add them.We cannot apply the method applicable for matrix addition.

(j) **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation:

Assertion: Since, A and B are symmetric matrices.

$\therefore$ A^T = A and B^T = B.

Now, to check A (BA) is symmetric.

Consider $[A(BA)]^T = (BA)^T \cdot A^T = (A^T B^T)A^T$

= (AB)A = A(BA)

So, $[A(BA)]^T$ = A (BA)

$\Rightarrow$ A(BA) is symmetric.

Similarly, (AB) A is symmetric.

So, Assertion is true.

Reason: Now, (AB)' = B'A'

= BA

This will be symmetric, if A and B is commutative i.e. AB = BA.

Hence, both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(k) f(2) = 1, f(3) =1 , as two elements of domain have same image in co-domain.
f is not one – one so that f is not invertible
Hence no inverse.

(l) Given 3A - B = $\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

or 3A - $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$

or 3A = $\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

or 3A = $\begin{bmatrix} 9 & 3 \\ 3 & 6 \end{bmatrix}$

or A = $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

(m) Given that f(x) = 4x - 3 = y (say), then 4x = y + 3

$\Rightarrow x = \frac{y+3}{4}$

Hence $f^{-1}(y) = \frac{y+3}{4} \Rightarrow f^{-1}(x) = \frac{x+3}{4}$

(n) The sample space is given by,
S = {(H H H),(H H T),(H T H),(H T T),(T H H),(T H T),(T T H),(T T T)}

$P(A) = \frac{4}{8} = \frac{1}{2}$

$P(B) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = \frac{2}{8} = \frac{1}{4} = P(A)P(B)$

Thus, A and B are independent events .

(o) We are given that, E_1 and E_2 are two independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$

$P\left(\overline{E_1} \cap E_2\right) = P(\overline{E_1}) \times P\left(E_2\right)$

= 0.7 $\times$ 0.4 = 0.28

Thus, $P\left(\overline{E_1} \cap E_2\right)$ = 0.28

2. Let $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

Put, x = tan θ, we get

$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right) = \tan^{-1}(\tan 3\theta)$

If $x > \frac{1}{\sqrt{3}}$, then

$x = \tan\theta \Rightarrow \tan\theta > \frac{1}{\sqrt{3}} \Rightarrow \frac{\pi}{6} < \theta < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < 3\theta < \frac{3\pi}{2}$

$\therefore \quad y = \tan^{-1}(\tan 3\theta)$

$\Rightarrow \quad y = \tan^{-1}(-\tan(\pi - 3\theta)\}$

$\Rightarrow \quad y = \tan^{-1}\{\tan(3\theta - \pi)\}$

$\Rightarrow \quad y = 3\theta - \pi \quad \left[\because \frac{\pi}{2} < 3\theta < \frac{3\pi}{2} \Rightarrow -\frac{\pi}{2} < 3\theta - \pi < \frac{\pi}{2}\right]$

$\Rightarrow \quad y = 3\tan^{-1}x - \pi \quad \left[\because x = \tan\theta \Rightarrow \theta = \tan^{-1}x\right]$

$\Rightarrow \quad \frac{dy}{dx} = \frac{3}{1+x^2} - 0 = \frac{3}{1+x^2}$

OR

Let x cm be the radius and y be the enclosed area of the circular wave at any time t.
Rate of increase of radius of circular wave = 5 cm/sec

$\Rightarrow \frac{dx}{dt}$ is positive and = 5 cm/sec

$\Rightarrow \frac{dx}{dt} = 5cm/\sec$ ...(i)

$y=\pi x^2$

$\therefore$ Rate of change of area $=\frac{dy}{dt}=\pi\frac{d}{dt}x^2$

$=\pi.2x\frac{dx}{dt}=2\pi x\left(5\right)$ (from (i)

$=10\pi x cm^2/\sec$

Putting $x=8cm$ (given),

$\frac{dy}{dt}=10\pi\left(8\right)=80\pi cm^2/\sec$

Since $\frac{dy}{dt}$ is positive, therefore area of circular wave is increasing at the rate of $80\pi cm^2/\sec$.

3. Let I = $\int\frac{dx}{(\sin x-\sin 2x)}$, then

$I=\int\frac{dx}{(\sin x-2\sin x\cos x)}$

$=\int\frac{dx}{\sin x(1-2\cos x)}=\int\frac{\sin x}{\sin^2 x(1-2\cos x)}dx$

$=-\int\frac{dt}{(1-t^2)(1-2t)}$, where cos x = t

$=\int\frac{dt}{(t-1)(t+1)(1-2t)}$ .... (i)

Let $\frac{1}{(t-1)(t+1)(1-2t)}=\frac{A}{(t-1)}+\frac{B}{(t+1)}+\frac{C}{(1-2t)}$

Then, 1 = A(t + 1)(1 - 2t) + B(t - 1)(1 - 21) + C(t - 1)(t + 1) .... (ii)

Putting t = 1 in (ii), we get A $=\frac{-1}{2}$

Putting t = -1 in (ii), we get B $=\frac{-1}{6}$

Putting t $=\frac{1}{2}$ in (ii), we get C $=\frac{-4}{3}$

$\therefore I=-\frac{1}{2}\int\frac{dt}{(t-1)}-\frac{1}{6}\cdot\int\frac{dt}{(t+1)}-\frac{4}{3}\cdot\int\frac{dt}{(1-2t)}$

$=-\frac{1}{2}\log|t-1|-\frac{1}{6}\log|t+1|+\frac{2}{3}\cdot\int\frac{-2dt}{(1-2t)}$

$=-\frac{1}{2}\log|t-1|-\frac{1}{6}\log|t+1|+\frac{2}{3}\log|1-2t|+C$

$=-\frac{1}{2}\log|\cos x-1|-\frac{1}{6}\log|\cos x+1|+\frac{2}{3}\log|1-2\cos x|+C$

4. Note that the domain of f(x) is the set of all positive real numbers other than unity ie; (0,1) $\cup(1,\infty)$

Now $f(x)=\frac{x}{\log x}$

$\Rightarrow\quad f'(x)=\frac{\log x-1}{(\log x)^2}$

For f(x) to be increasing function, we must have

$f'(x)>0$

$\Rightarrow\quad\frac{\log x-1}{(\log x)^2}>0$

$\Rightarrow\quad\log x-1>0$

$\log x>1$

$x>e^1$

So, f(x) is increasing on (e,∞)

For f(x) to be decreasing we must have

$\Rightarrow\quad\frac{\log x-1}{(\log x)^2}<0$

$\Rightarrow\quad\log x-1<0$

$\Rightarrow\quad\log x<1$

$\Rightarrow\quad x<e^1$

So f(x) is decreasing on $(0,e)$

5. Let I = $\int\frac{1}{x\sqrt{x^4-1}}dx$

Since $\int\frac{1}{x\sqrt{x^2-1}}dx=\sec^{-1}x+c$

We have, $I=\int\frac{1}{x\sqrt{x^4-1}}dx$ .... (i)

Multiplying numerator and denominator with x

$I=\int \frac{x}{x^2\sqrt{(x^2)^2-1}}dx$

Let x^2=t

$\Rightarrow 2x=\frac{dt}{dx}$

$\Rightarrow$ xdx = $\frac{dt}{2}$

Putting this value in equation (i), we get

$I=\frac{1}{2}\int \frac{dt}{t\sqrt{t^2-1}}\left[x^2=t\right]$

$\Rightarrow \mathrm{I}=\frac{1}{2}\sec^{-1}\mathrm{t}+\mathrm{c}$

$\Rightarrow \mathrm{I}=\frac{1}{2}\sec^{-1}\left(\mathrm{x}^2\right)+\mathrm{c}$

OR

Take tan x = a

Hence, $\sec^2$x dx = da

$\therefore \int \frac{\tan x \sec^2 x}{1-\tan^2 x}dx$

$=\int \frac{\text{ada}}{1-a^2}$

Now, taking 1-a^2 = k , -2a da = dk i.e. a da = -dk/2

$\therefore \int \frac{\text{ada}}{1-a^2}$

$=\int \frac{-\mathrm{dk}}{2\mathrm{k}}$

$=-\frac{1}{2}\ln|\mathrm{k}|+\mathrm{c}$

Replacing the value of k,

$-\frac{1}{2}\ln|\mathrm{k}|+\mathrm{c}$

$=-\frac{1}{2}\ln\left|1-a^2\right|+c$

Replacing the value of a,

$-\frac{1}{2}\ln\left|1-a^2\right|+c$

$=-\frac{1}{2}\ln\left|1-\tan^2 x\right|+c$

Where c is the integrating constant

6. Given, f(x) = $9x^2$ + 6x – 5

For, a,b $\in$ R_+

$\Rightarrow$ f(a) = f(b)

$\Rightarrow$ $9a^2$ + 6a -5 =$9b^2$ + 6b -5

$\Rightarrow$ a = b

This, f is a one one function

Now, Let y = f(x)= ($9x^2$ + 6x – 5)

So x = $\frac{-1+\sqrt{y+6}}{3}$ [Range of f(x) = Domain of y]

So Domain of y = Range of f(x) = [-5, ∞]

Hence, Range of f(x) = co-domain of f(x) =[-5, ∞]

So, f(x) is onto function

As it is bijective function.

So it is invertible

Inverse of f(x) is f^{-1}(x) =$\frac{-1+\sqrt{x+6}}{3}$

7. Let $\tan^{-1}\frac{2}{3}=x$ and $\tan^{-1}\sqrt{3}=y$

so that $\tan x=\frac{2}{3}$ and $\tan y=\sqrt{3}$

Therefore,

$\sin\left(2\tan^{-1}\frac{2}{3}\right)+\cos\left(\tan^{-1}\sqrt{3}\right)$

= sin(2x) + cos y

$= \frac{2\tan x}{1+\tan^2 x} + \frac{1}{\sqrt{1+\tan^2 y}}$

$= \frac{2.\frac{2}{3}}{1+\frac{4}{9}} + \frac{1}{\sqrt{1+(\sqrt{3})^2}}$

$= \frac{12}{13} + \frac{1}{2} = \frac{37}{26}$

8. We have,

$I = \int \frac{x^2+x+1}{(x+1)^2(x+2)}\text{dx}$

To solve this we use partial fraction.

Let $\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$ ...(i)

$\Rightarrow$ $x^2 + x + 1 = A(x + 1)(x + 2) + B(x + 2) + C(x + 1)^2$ ...(ii)

Putting x = -1 in (ii), we get

B = 1

Putting x = -2 in (ii), we get

C = 3

Putting x = 0 in (ii), we get

1 = 2A + 2B + C

$\Rightarrow$ 1 = 2A + 2 + 3

$\Rightarrow$ -4 = 2A

$\Rightarrow$ A = -2

Now, (i) becomes

$\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{x+2}$

Therefore, integral becomes

$I = \int \left[\frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{x+2}\right] dx$

= -2 log|x + 1| - $\frac{1}{(x+1)}$ + 3 log |x + 2| + C

9. Since f(x) is continuous at x = 0.

Therefore,

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\Rightarrow \quad \lim_{h\to 0} f(0-h) = \lim_{h\to 0} f(0+h) = f(0)$

$\Rightarrow \quad \lim_{h\to 0} f(0+(-h)) = \lim_{h\to 0} f(0+h) = f(0)$

$\Rightarrow \quad \lim_{h\to 0}[f(0)+f(-h)] = \lim_{h\to 0}[f(0)+f(h)] = f(0)$ ......[Using: f (x + y) = f(x) + f(y)]

$\Rightarrow \quad f(0) + \lim_{h\to 0} f(-h) = f(0) + \lim_{h\to 0} f(h) = f(0)$

$\Rightarrow \quad \lim_{h\to 0} f(-h) = \lim_{h\to 0} f(h) = 0$ ....(i)

Let 'a' be any real number.

Then,

$\lim_{x\to a^-} f(x) = \lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(a+(-h))$

$\Rightarrow \quad \lim_{x\to a^-} f(x) = \lim_{h\to 0}[f(a)+f(-h)]$ .....[∵ f (x + y) = f(x) + f(y)]

$\Rightarrow \quad \lim_{x\to a^-} f(x) = f(a) + \lim_{h\to 0} f(-h)$

$\Rightarrow \quad \lim_{x\to a^-} f(x) = f(a) + 0$ [Using (i)]

$\Rightarrow \quad \lim_{x\to a^-} f(x) = f(a)$

and, $\lim_{x\to a^+} f(x) = \lim_{h\to 0} f(a+h)$

$\Rightarrow \quad \lim_{x\to a^+} f(x) = \lim_{h\to 0}[f(a)+f(h)]$ ... [∵ f (x + y) = f(x) + f(y)]

$\Rightarrow \quad \lim_{x \to a^+} f(x) = f(a) + \lim_{h \to 0} f(h)$

$\Rightarrow \quad \lim_{x \to a^+} f(x) = f(a) + 0 = f(a)$ ...[Using (i)]

Thus, we have

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

$\therefore$ f(x) is continuous at x = a

Since 'a' is an arbitrary real number.

So, f(x) is continuous at all x $\in$ R

OR

Here,

$x = a.sin(2t).(1 + cos2t) \;\; and \;\; y = b.cos(2t).(1 - cos2t)$

$\therefore \frac{dx}{dt} = a\left[\sin 2t \cdot \frac{d}{dt}(1 + \cos 2t) + (1 + \cos 2t) \cdot \frac{d}{dt}\sin 2t\right]$

$= a\left[\sin 2t.(-\sin 2t) \cdot \frac{d}{dt}2t + (1 + \cos 2t) \cdot \cos 2t \cdot \frac{d}{dt}2t\right]$

$= -2a.sin^2(2t) + 2a.cos(2t)(1 + cos2t)$

$\Rightarrow \frac{dx}{dt} = -2\alpha\left[\sin^2 2t - \cos 2t(1 + \cos 2t)\right]$ .... (i)

and $\frac{dy}{dt} = b\left[\cos 2t \cdot \frac{d}{dt}(1 - \cos 2t) + (1 - \cos 2t) \cdot \frac{d}{dt}\cos 2t\right]$

$= b\left[\cos 2t.(\sin 2t)\frac{d}{dt}2t + (1 - \cos 2t)(-\sin 2t) \cdot \frac{d}{dt}2t\right]$

$= b[2\sin 2t \cdot \cos 2t + 2(1 - \cos 2t)(-\sin 2t)]$

$= b[2\sin 2t \cdot \cos 2t + 2(1 - \cos 2t)(-\sin 2t)]$ ..... (ii)

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2b[-\sin 2t \cdot \cos 2t + (1 - \cos 2t)\sin 2t]}{-2a\left[\sin^2 2t - \cos 2t(1 + \cos 2t)\right]}$

$= \frac{b}{a} \cdot \frac{(0+1)}{(1-0)} \quad \ldots\left[\because \sin\frac{\pi}{2} = 1 \text{ and } \cos\frac{\pi}{2} = 0\right]$

$= \frac{b}{a}$

Hence proved.

10. **Read the text carefully and answer the questions:**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(a) P(Grade A in Maths) = P(M) = 0.2
P(Grade A in Physics) = P(P) = 0.3
P(Grade A in Chemistry) = P(C) = 0.5
P(Grade A in all subjects) = P(M $\cap$ P $\cap$ C) = P(M)·P(P)·P(C)
P(Grade A in all subjects) = 0.2 $\times$ 0.3 $\times$ 0.5 = 0.03

(b) P(Grade A in Maths) = P(M) = 0.2
P(Grade A in Physics) = P(P) = 0.3
P(Grade A in Chemistry) = P(C) = 0.5
P(Grade A in no subjects) = $P(\bar{M} \cap \bar{P} \cap \bar{C})$ = $P(\bar{M}) \cdot P(\bar{P}) \cdot P(\bar{C})$
P(Grade A in no subjects) = 0.8 $\times$ 0.7 $\times$ 0.5 = 0.280

(c) P(Grade A in Maths) = P(M) = 0.2
P(Grade A in Physics) = P(P) = 0.3
P(Grade A in Chemistry) = P(C) = 0.5
P(Grade A in 2 subjects) = $P(M \cap P \cap \bar{C}) + P(P \cap C \cap \bar{M})$ + $P(M \cap C \cap \bar{P})$

P(Grade A in 2 subjects) = 0.2 × 0.3 × 0.5 + 0.3 × 0.5 × 0.8 + 0.2 × 0.5 × 0.7 = 0.03 + 0.12 + 0.07 = 0.22

P(Grade A in 2 subjects) = 0.22

(d) P(Grade A in Maths) = P(M) = 0.2

P(Grade A in Physics) = P(P) = 0.3

P(Grade A in Chemistry) = P(C) = 0.5

P(Grade A in atleast one subject) = 1 − P(grade A in no subject) = 1 − $P(\bar{M} \cap \bar{P} \cap \bar{C})$

P(Grade A in atleast one subjects) = 1 − 0.280 = 0.72

OR

Read the text carefully and answer the questions:

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(a) x + 0.21 = 0.44 ⇒ x = 0.23

(b) 0.41 + y + 0.44 + 0.11 = 1 ⇒ y = 0.04

(c) $P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)}$

P(B) = 0.09 + 0.04 + 0.23 = 0.36

$P\left(\frac{C}{B}\right) = \frac{0.23}{0.36} = \frac{23}{36}$

(d) P(A or B but not C)

= 0.32 + 0.09 + 0.04

= 0.45

11. **Read the text carefully and answer the questions:**

Three shopkeepers A, B and C go to a store to buy stationery. A purchase 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs ₹40, a pen costs ₹12 and a pencil costs ₹3.

(a) Number of items purchased by shopkeepers A, B and C can be written in matrix form as

$$X = \begin{array}{c} \begin{matrix} Notebooks & pens & pencils \end{matrix} \\ \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \end{array} \begin{matrix} A \\ B \\ C \end{matrix}$$

(b)

$$\text{Since, } Y = \begin{bmatrix} 40 \\ 12 \\ 3 \end{bmatrix} \begin{matrix} Note\ book \\ Pen \\ Pencil \end{matrix}$$

$$\therefore XY = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{bmatrix} 40 \\ 12 \\ 3 \end{bmatrix}$$

$= \begin{bmatrix} 5760+720+216 \\ 4800+864+252 \\ 5280+1872+288 \end{bmatrix} = \begin{bmatrix} 6696 \\ 5916 \\ 7440 \end{bmatrix}$

(c) $(A + I)^2 = A^2 + 2A + I = 3A + I$

$\Rightarrow (A + I)^3 = (3A + I)(A + I)$

$= 3A^2 + 4A + I = 7A + I$

$\therefore (A + I)^3 - 7A = I$

12. $(x^2 - y^2)dx + 2xy\,dy = 0$

$\Rightarrow (x^2 - y^2)dx = -2xy\,dy$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

$\Rightarrow \frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)}$ (1)

Put y = vx, then, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Put $\frac{dy}{dx}$ in eq (i),we get,

$v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{-1 - v^2}{2v}$

$\Rightarrow \int \frac{2v dv}{v^2 + 1} = \int \frac{-dx}{x}$

$\Rightarrow \log(v^2 + 1) = -\log x + c$

$\Rightarrow \log((v^2 + 1).x) = c$

$\Rightarrow (v^2 + 1).x = e^c$

$\Rightarrow \left(\frac{y^2}{x^2} + 1\right).x = e^c \ \left[v = \frac{y}{x}\right]$

$\Rightarrow \frac{x^2 + y^2}{x} = A \ [\because e^c = A]$

$\Rightarrow x^2 + y^2 = Ax$

OR

We have $\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$

Let $f(x, y) = \frac{x^2 + y^2}{x^2 + xy}$

Here, putting x = kx and y = ky

$f(kx, ky) = \frac{(kx)^2 + (ky)^2}{(kx)^2 + kx.ky}$ = k^0.f(x,y)

Therefore, the given differential equation is homogeneous.

$(x^2 + xy)dy = (x^2 + y^2)dx$

$\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$

To solve it we make the substitution.

y = vx

Differentiating above eq. with respect to x, we get

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{x^2 + (vx)^2}{x^2 + x \cdot vx}$

$v + x\frac{dv}{dx} = \frac{x^2(1 + v^2)}{x^2(1 + v)}$

$v + x\frac{dv}{dx} = \frac{1 + v^2}{1 + v}$

$\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}} = \frac{1 + \mathrm{v}^2}{1 + \mathrm{v}} - \mathrm{v} = \frac{1 + \mathrm{v}^2 - \mathrm{v} - \mathrm{v}^2}{1 + \mathrm{v}}$

$x\frac{dv}{dx}=\frac{1-v}{1+v}$
$\frac{1+v}{1-v}dv=\frac{1}{x}dx$
Integrating on both side,
$\int\frac{1+v}{1-v}dv=\int\frac{1}{x}dx$
$\int\left(-1+\frac{2}{1-v}\right)dv=\int\frac{1}{x}dx$
- v - 2log|1 - v| = log|x| + log c
$-\frac{y}{x}-2\log\left|1-\frac{y}{x}\right|=\log|x|+\log C$
$-\frac{y}{x}=2\log\left|1-\frac{y}{x}\right|+\log|x|+\log C$
$-\frac{y}{x}=\log\frac{(x-y)^2}{x^2}+\log|x|+\log C$
$-\frac{y}{x}=\log\frac{(x-y)^2}{x^2}\cdot Cx$
$-\frac{y}{x}=\log\frac{(x-y)^2}{x}c$
$-\frac{y}{x}=\log\frac{(x-y)^2}{x}c$
$\frac{\mathrm{C(x-y)^2}}{\mathrm{x}}=\mathrm{e^{-y/x}}$
$\mathrm{C(x-y)^2=xe^{-y/x}}$
$(x-y)^2=kxe^{-y/x}$
Which is the required solution of the given differential equation.

13. Given:
Curves y^2 = 8x .... (i)
& $2x^2+y^2$ = 10 ... (ii)
The point of intersection of two curves are (0, 0) & (1, $2\sqrt{2}$)
Now, Differentiating curves (i) & (ii) w.r.t. x, we get
$\Rightarrow y^2=8x$
$\Rightarrow 2y\cdot\frac{dy}{dx}=8$
$\Rightarrow\frac{dy}{dx}=\frac{8}{2y}$
$\Rightarrow\frac{dy}{dx}=\frac{4}{y}$ ... (iii)
$\Rightarrow 2x^2+y^2=10$
Differentiating above w.r.t. x,
$\Rightarrow 4x+2y\cdot\frac{dy}{dx}=0$
$\Rightarrow 2x+y\cdot\frac{dy}{dx}=0$
$\Rightarrow y\cdot\frac{dy}{dx}=-2x$
$\Rightarrow\frac{dy}{dx}=\frac{-2x}{y}$ ... (iv)
Substituting (1, $2\sqrt{2}$) for m_1 & m_2, we get,
$m_1=\frac{4}{y}$
$\Rightarrow\frac{4}{2\sqrt{2}}$
$m_1=\sqrt{2}$ ... (v)
$\mathrm{m_2=\frac{-2x}{y}}$
$\Rightarrow\frac{-2\times1}{2\sqrt{2}}$
$m_2=-\frac{-1}{\sqrt{2}}$ ... (vi)
When $\mathrm{m_1}=\sqrt{2}$ & $m_2=\frac{-1}{\sqrt{2}}$
Two curves intersect orthogonally if m_1m_2 = -1, where m_1 and m_2 the slopes of the two curves.

$\Rightarrow \sqrt{2} \times \frac{-1}{\sqrt{2}} = -1$

$\therefore$ Two curves $y^2 = 8x$ & $2x^2 + y^2 = 10$ intersect orthogonally.

OR

Let ABC be right-circular cone having radius 'r' and height 'h'. If V and S are its volume and surface area (curved) respectively, then

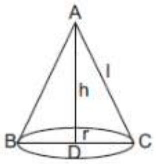
$S = \pi r l$

$S = \pi r \sqrt{h^2 + r^2}$ ..... (i)

Putting the value of h in (i), we get

$S = \pi r \sqrt{\frac{9V^2}{\pi^2 r^4} + r^2}$

$\Rightarrow \quad S^2 = \pi^2 r^2 \left(\frac{9V^2 + \pi^2 r^6}{\pi^2 r^4}\right)$



$\left[\begin{array}{l} \because V = \frac{1}{3}\pi r^2 h \\ \quad h = \frac{3V}{\pi r^2} \end{array}\right.$

[Maxima or Minima is same for S or S^2]

$\Rightarrow \quad S^2 = \frac{9V^2}{r^2} + \pi^2 r^4$

$\Rightarrow \quad \left(S^2\right)' = \frac{-18V^2}{r^3} + 4\pi^2 r^3$ ..... (ii) [Differentiating w.r.t. 'r']

Now, $(S^2)' = 0$

$\Rightarrow \quad -18\frac{V^2}{r^3} + 4\pi^2 r^3 = 0$

$\Rightarrow \quad 4\pi^2 r^6 = 18V^2$

$\Rightarrow \quad 4\pi^2 r^6 = 18 \times \frac{1}{9}\pi^2 r^4 h^2$ [Putting value of V]

$\Rightarrow \quad 2r^2 = h^2 \quad \Rightarrow \quad r = \frac{h}{\sqrt{2}}$

Differentiating (ii) w.r.t. 'r', again

$\left(S^2\right)'' = \frac{54V^2}{r^4} + 12\pi^2 r^2$

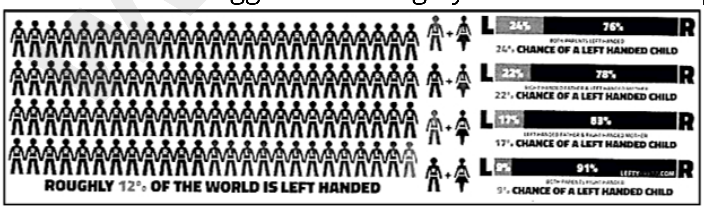
$\Rightarrow \quad \left(S^2\right)''\Big]_{r=\frac{h}{\sqrt{2}}} > 0$ (for any value of r)

Hence, S^2 i.e., is minimum for $r = \frac{h}{\sqrt{2}}$ or $h = \sqrt{2}r$.

i.e., For least curved surface, altitude is equal to $\sqrt{2}$ times the radius of the base.

14. **Read the text carefully and answer the questions:**

Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows:

A. When both father and mother are left handed:
Chances of left handed child is 24%.

B. When father is right handed and mother is left handed:
Chances of left handed child is 22%.

C. When father is left handed and mother is right handed:
Chances of left handed child is 17%.

D. When both father and mother are right handed:
Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

(a) $P\left(\frac{L}{C}\right) = \frac{17}{100}$

(b) $P\left(\frac{\bar{L}}{A}\right) = 1 - P\left(\frac{L}{A}\right) = 1 - \frac{24}{100} = \frac{76}{100}$ or $\frac{19}{25}$

(c) $P\left(\frac{A}{L}\right) = \frac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100}+\frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}+\frac{1}{4}\times\frac{9}{100}} = \frac{24}{72} = \frac{1}{3}$

(d) Probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed.

$= P\left(\frac{L}{B\cup C}\right) = \frac{22}{100} + \frac{17}{100} = \frac{39}{100}$

SECTION B - 15 MARKS

15. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(a) **(d)** 60^{o}

Explanation:

$\vec{a}\cdot\vec{b} = \frac{1}{2}|\vec{a}||\vec{b}|$

$|\vec{a}|\ |\vec{b}|\ \cos\theta = \frac{1}{2}|\vec{a}||\vec{b}|$

$\cos\theta = \frac{1}{2}$

$\theta = 60^{o}$

(b) The direction ratios of the first line are (3, 5, 4) and the direction ratios of the second line are (1, 1, 2).

If θ is the angle between them, then

$\cos\theta = \left|\frac{3.1+5.1+4.2}{\sqrt{3^2+5^2+4^2}\sqrt{1^2+1^2+2^2}}\right| = \frac{16}{\sqrt{50}\sqrt{6}} = \frac{16}{5\sqrt{2}\sqrt{6}} = \frac{8\sqrt{3}}{15}$

Hence, the required angle is $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

(c) Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$. Then vector perpendicular to both vectors a & b is ;

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = (2-3)\hat{i} - (-8+6)\hat{j} + (4-2)\hat{k} = -\hat{i} + 2\hat{j} + 2\hat{k}$$

$\Rightarrow |\vec{a}\times\vec{b}| = \sqrt{(-1)^2 + 2^2 + 2^2} = 3$

$\therefore$ Required vector $= 9\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{9}{3}(-\hat{i} + 2\hat{j} + 2\hat{k}) = -3\hat{i} + 6\hat{j} + 6\hat{k}$

(d) **(c)** 90^{o}

Explanation:

To find the angle with the z-axis, we use the relation:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

where $\alpha = 30^\circ$ and $\beta = 120^\circ$. Calculating, we get:

$\cos^2 30^\circ = \frac{3}{4}, \cos^2 120^\circ = \frac{1}{4}$

Thus,

$\frac{3}{4} + \frac{1}{4} + \cos^2\gamma = 1 \Longrightarrow \cos^2\gamma = 0 \Longrightarrow \gamma = 90^\circ.$

So, the angle with the z-axis is 90^{o}.

(e) d.r.'s of lines are < -2, 3p, 4 > and < 4p, 2, -7 >

As lines are perpendicular

-8p + 6p - 28 = 0

$\Rightarrow$ p = -14

16. We have, $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$,

$\vec{a} + \vec{b}$ = 1 + 2 $\hat{\jmath} - 3\hat{k} + 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\Rightarrow \vec{a} + \vec{b}$ = 4 $\hat{\imath} + \hat{\jmath} - \hat{k}$

$\vec{a} - \vec{b}$ = $\hat{\imath}$ + 2 $\hat{\jmath} - 3\hat{k} - (3\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$\Rightarrow \vec{a} - \vec{b}$ = -2 $\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$

Now $(\vec{\mathrm{a}} + \vec{\mathrm{b}}) \cdot (\vec{\mathrm{a}} - \vec{\mathrm{b}})$ = $(4\hat{\mathrm{i}} + \hat{\mathrm{j}} - \hat{\mathrm{k}}) \cdot (-2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} - 5\hat{\mathrm{k}})$

= (4 × -2) + (1 × 3) + (-1 × -5) = -8 + 3 + 5 = 0

Here,we see that the dot product of two vectors is zero so the vector $(\vec{\mathrm{a}} + \vec{\mathrm{b}})$ is perpendicular to $(\vec{\mathrm{a}} - \vec{\mathrm{b}})$

Hence,proved.

OR

Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$ ...(i)

$\vec{a} \cdot \vec{b} = (\hat{i} - \hat{j})(\hat{j} + \hat{k})$

$= (\hat{i} - \hat{j} + 0 \times \vec{k})(0 \times \hat{i} + \hat{j} + \vec{k})$

$= (1)(0) + (-1)(1) + (0)(1)$

= 0 - 1 + 0

$\vec{a} \cdot \vec{b} = -1$

$|\vec{a}| = |\hat{i} - \hat{j}|$

$= |\hat{i} - \hat{j} + 0 \times \hat{k}|$

$= \sqrt{(1)^2 + (-1)^2 + (0)^2}$

$= \sqrt{1 + 1 + 0}$

$|\vec{a}| = \sqrt{2}$

$|\vec{b}| = |\hat{j} + \vec{k}|$

$= |0 \times \hat{i} + \hat{j} + \hat{k}|$

$= \sqrt{(0)^2 + (1)^2 + (1)^2}$

$= \sqrt{0 + 1 + 1}$

$|\vec{b}| = \sqrt{2}$

put $\vec{a} \times \vec{b}, |\vec{a}|$ and $|\vec{b}|$ in equation (i)

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$= \frac{-1}{\sqrt{2}\times\sqrt{2}}$

$\cos\theta = \frac{-1}{2}$

$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

$\theta = \pi - \frac{\pi}{3}$

$\theta = \frac{2\pi}{3}$

Therefore, the angle between $\vec{a}$ and $\vec{b} = \frac{2\pi}{3}$

17. Equation of one plane is $\vec{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 5$ ...(i)

Comparing this equation with $\vec{r}.\vec{n}_1 = d_1$, we have

Normal vector to plane (i) is $\vec{n}_1 = 2\hat{i} + 2\hat{j} - 3\hat{k}$

Again, equation of second plane is $\vec{r}.\left(3\hat{i} - 3\hat{j} + 5\hat{k}\right) = 3$ ...(ii)

Comparing this equation with $\vec{r}.\vec{n}_2 = d_2$, we have

Normal vector to plane (i) is $\vec{n}_2 = 3\hat{i} - 3\hat{j} + 5\hat{k}$

Let θ be the acute angle between planes (i) and (ii).

$\therefore$ angle between normals $\vec{n}_1$ and $\vec{n}_2$ to planes (i) and (ii) is also θ

$\therefore \cos\theta = \frac{|\vec{n}_1.\vec{n}_2|}{|\vec{n}_1|.|\vec{n}_2|} = \frac{|2(3)+2(-3)+(-3)5|}{\sqrt{4+4+9}\sqrt{9+9+25}}$

$= \frac{|6-6-15|}{\sqrt{17}\sqrt{13}}$

$= \frac{|-15|}{\sqrt{731}} = \frac{15}{\sqrt{731}}$

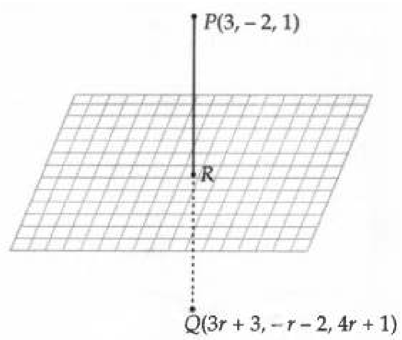
$\Rightarrow \theta = \cos^{-1}\frac{15}{\sqrt{731}}$

OR

Let Q be the image of the point P (3, - 2 ,1) in the plane 3x - y + 4z = 2.

Then, PQ is normal to the plane. Therefore, direction ratios of PQ are proportional to 3, - 1, 4. Since PQ passes through P (3, - 2,1) and has direction ratios proportional to 3, -1,4.

Therefore, the equation of PQ is $\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-1}{4} = r$



Let the coordinates of Q be (3 r+3,-r-2,4 r+1). Let R be the mid-point of PQ .

Then, R lies on the plane 3x - y + 4z = 2. The coordinates of R are

$\left(\frac{3r+3+3}{2}, \frac{-r-2-2}{2}, \frac{4r+1+1}{2}\right)$ or, $\left(\frac{3r+6}{2}, \frac{-r-4}{2}, 2r+1\right)$

Since R lies on the plane 3x - y + 4z = 2

$3\left(\frac{3r+6}{2}\right) - \left(\frac{-r-4}{2}\right) + 4(2r+1) = 2 \Rightarrow 13r = -13 \Rightarrow r = -1$

Putting r = -1 in (3r + 3, -r -2, 4r + 1), we obtain the coordinates of Q as (0, -1, -3)

Hence, the image of (3, -2, 1) in the plane 3x - y + 4z = 2 is (0, -1, -3).

18. To find area of region

$\{(x,y) : 0 \le y \le x^2 + 3; 0 \le \ y \le 2x + 3; 0 \le x \le 3\}$

$\Rightarrow$ y = x^2 + 3 ...(i)

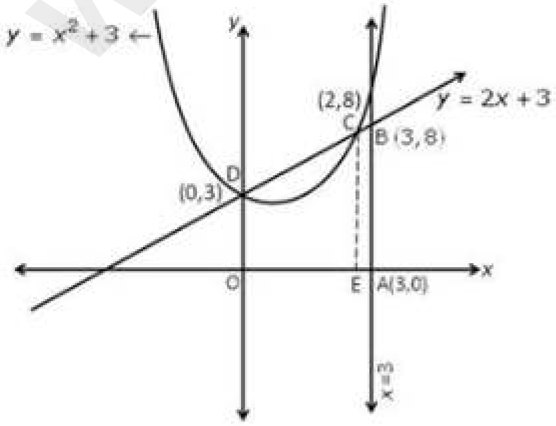
y = 2x +3 ...(ii)

and x = 0, x = 3

Equation (i) represents a parabola with vertex (3, 0) and axis as y-axis.

Equation (ii) represents a line a passing through (0, 3) and $\left(-\frac{3}{2}, 0\right)$.

A rough sketch of curve is as under:-



Thus Required area of the region= Area of the bounded Region ABCDOA

A = Region ABCEA + Region ECDOE

$= \int_2^3 y_1 dx + \int_0^2 y_z dx$
$= \int_2^3 (2x+3) dx + \int_0^2 \left(x^2+3\right) dx$
$= \left(x^2+3x\right)_2^3 + \left(\frac{x^3}{3}+x\right)_0^2$
$= [(9+9)-(4+6)] + \left[\left(\frac{8}{3}+2\right)-(0)\right]$
$= [18-10] + \left[\frac{14}{3}\right]$
$= 8 + \frac{14}{3}$
$A = \frac{38}{3}$ sq. units.

SECTION C - 15 MARKS

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(a) **(c)** ₹ 449

Explanation:

AC = $\frac{Cx}{x}$ $\Rightarrow$ AC = $\frac{2000}{x}$ + 50 - $\frac{1}{5}$x

Now, $[AC]_{x=5}$ = $\frac{2000}{5}$ + 50 - $\frac{5}{5}$ = ₹ 449

(b) **(a)** linear function

Explanation:

linear function

(c) Let the line of regression of y on x be

2x + 3y - 10 = 0

$\Rightarrow$ 3y = -2x + 10

$\Rightarrow$ y = -$\frac{2}{3}$x - $\frac{10}{3}$

$\therefore$ b_{yx} = -$\frac{2}{3}$

Let the line of regression of x on y be

4x + y - 5 = 0

$\Rightarrow$ 4x = -y + 5

$\Rightarrow$ x = -$\frac{1}{4}$y + $\frac{5}{4}$

$\therefore$ bxy = -$\frac{1}{4}$

Here, $b_{yx} \times b_{xy}$ = $\left(-\frac{2}{3}\right)\left(-\frac{1}{4}\right) = \frac{1}{6} < 1$

Which is true. Hence, our assumption is correct and line of regression of y on x is 2x + 3y - 10 = 0

(d) Let x be the number of items produced and sold. Let C(x) be the total cost incurred in producing x items. Then,

C(x) = Fixed cost + Variable cost = 26,000 + 30x

Let R(x) be the total revenue received in selling x items. Then, R(x) = 43x

At the break-even point, we have C(x) = R(x)

$\Rightarrow$ 26,000 + 30x = 43x

$\Rightarrow$ 13x = 26,000

$\Rightarrow$ x = 2000

Hence, the break-even point is 2000 items.

(e) The demand function is linear, so assume $p = a + bx$. Using given points $(1400, 4)$ and $(1800, 2)$, solve for a and b.

Slope $b = \frac{2-4}{1800-1400} = -\frac{2}{400} = \frac{1}{200}$

Substituting $p = 4$ at $x = 1400 : 4 = a - \frac{1400}{200}$

$a = 4 + 7 = 11$ So, $p = 11 - \frac{x}{200}$

Revenue function: $R = p \cdot x = x\left(11 - \frac{x}{200}\right) = 11x - \frac{x^2}{200}$ Marginal revenue function:

$MR = \frac{dR}{dx} = 11 - \frac{x}{100}$

Thus, the marginal revenue function is $MR = 11 - \frac{z}{100}$.

20. i. Since the demand function is given to be linear. So, let it be

x = ap + b ... (i)

where p is the price per unit and x is the quantity demanded at this price.

When p = 4, x = 1000 and when p = 2, x = 1500

$\therefore$ 1000 = 4a + b and 1500 = 2a + b

Solving these two equations, we get: a = -250 and b = 2000

Putting the values of a and b in (i), we get

x = - 250p + 2000 ... (ii)

$\Rightarrow$ p = 8 - $\frac{x}{250}$

Hence, the demand function is x = - 250p + 2000

ii. Let R be the total revenue function. Then, R = px $\Rightarrow$ R = 8x - $\frac{x^2}{250}$ [using (iii)]

iii. The average revenue function is AR = $\frac{R}{x}$ = p $\Rightarrow$ AR = 8 - $\frac{x}{250}$ [using (iii)]

iv. The marginal revenue function is MR = $\frac{dR}{dx}$ = 8 - $\frac{x}{125}$ [using(iii)]

OR

Let C denote the cost and x the output. It is given that the cost C is a linear function of output x.

$\therefore$ C = ax + b, where a, b are constants.

When x = 250, we have C = 4000

$\therefore$ 4000 = 250 a + b ...(i)

When x = 350, we have C = 5000

$\therefore$ 5000 = 350 a + b ... (ii)

Solving (i) and (ii), we get a = 10 and b = 1500.

Substituting these values in C = ax + b, we get C = 10x + 1500

21. Given, $\bar{x}$ = 25, $\bar{y}$ = 30, b_{yx} = 1.6 and b_{xy} = 0.4

i. Regression equation y on x:

y - $\bar{y}$ = b_{yx}(x - $\bar{x}$)

$\Rightarrow$ y - 30 = 1.6(x - 25)

$\Rightarrow$ y - 30 = 1.6x - 40

$\Rightarrow$ y = 1.6x - 10 ...(i)

ii. From eq. (i)

When x = 60, y = (1.6) $\times$ (60) - 10

= 96 - 10 = 86

iii. Coefficient of correlation:

r = $\pm\sqrt{b_{yx}b_{xy}}$

= $\pm\sqrt{(1.6)(0.4)}$

= $\pm\sqrt{0.64}$

($\because$ both regression coefficients are +ve)

22. i.

Corner points	Z = 3x - 4y
O(0, 0)	0
A(0, 8)	-32
B(4, 10)	-28
C(6, 8)	-14
D(6, 5)	-2
E(4, 0)	12

Max Z = 12 at E(4, 0)

Min Z = -32 at A(0, 8)

ii. Since maximum value of Z occurs at B(4, 10) and C(6, 8)

$\therefore$ 4p + 10p = 6p + 8p

$\Rightarrow$ 2p = 2p

$\Rightarrow$ p = q

Number of optimal solution are infinite.

OR

Let x kg of fertilizer type I and y kg of fertilizer type II be used, therefore the problem can be formulated as an L.P.P as under:

Minimize the cost (in Rs) Z = 0.6x + 0.4y subject to the constraints

i.e. $\frac{10}{100}x + \frac{5}{100}y \geq 14$ i.e. $2x + y \geq 280$

6% of $x + 10\%$ of $y \geq 14$

i.e. $\frac{6}{100}x + \frac{10}{100}y \geq 14$

i.e. $3x + 5y \geq 700$

Draw the lines 2x + y = 280 and 3x + 5 y = 700

and shaded the region satisfied by the above inequalities.

The feasible region convex and unbounded is shown in the figure.Therefore,we have,

The corner points are $A\left(\frac{700}{3}, 0\right), B(100, 80)$ and $C(0, 280)$

The values in z = 0.6 +0.4 at the points A, B and C are 140 92 and 112 respectively.

Among the values, z the least value is 92 as the feasible region is unbounded we draw the half has minimum value Rs 92 at the point B (100.80). Therefore, the minimum cost = Rs 92, when 100 kg of fertilizer type 1 and 80 kg of fertilizer type II are mixed.