ISC 2026 EXAMINATION
Sample Question Paper - 3
Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

This Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions*

EITHER** from **Section B OR Section C.

Section A**: Internal choice has been provided in **two questions of two marks each, two questions of four marks each and two questions of six marks each.

Section B:** Internal choice has been provided in **one question of two marks and one question of four marks.

Section C:** Internal choice has been provided in **one question of two marks and one question of four marks.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

1. **In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.** **[15]**

(a) If |A| = |kA|, where A is a square matrix of order 2, then sum of all possible values of k is **[1]**

a) 1 b) -1

c) 0 d) 2

(b) $\int \sqrt{x^2-8x+7}\,dx$ is equal to **[1]**

a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log|x-4+\sqrt{x^2-8x+7}|+C$ b) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log|x-4+\sqrt{x^2-8x+7}|$

c) $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log|x+4+\sqrt{x^2-8x+7}|+C$ d) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log|x-4+\sqrt{x^2-8x+7}|$

(c) Range of cosec^{-1}x is **[1]**

a) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ - {0} b) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

c) $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ d) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ - {1}

(d) What is the solution of the differential equation $\frac{dx}{dy}+\frac{x}{y}-y^2$ = 0? **[1]**

a) xy= x^4 + C b) xy = y^4 + C

c) 4xy = y^4 + C d) 3xy = y^3 + C

(e) The probability that a man will live for 10 more years is $\frac{1}{4}$ and that his wife will live 10 more years is $\frac{1}{3}$. The probability that neither will be alive in 10 years is **[1]**

a) $\frac{1}{2}$ b) $\frac{7}{12}$

c) $\frac{5}{12}$ d) $\frac{11}{12}$

(f) Let A = {1, 2, 3} and consider the relation R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)}. Then R is **[1]**

a) neither symmetric, nor transitive b) reflexive but not transitive

c) reflexive but not symmetric d) symmetric and transitive

(g) If the function $f(x)=\begin{cases}3x-1 & x<2\\ k, & x=2\\ 2x+1, & x>2\end{cases}$ is continuous at x = 2, then the value of **k** is: **[1]**

a) 2

b) 1

c) 5

d) 3

(h) If $x^2 + y^3 = 42$, then $\frac{dy}{dx}$ is: **[1]**

a) $\frac{dy}{dx} = \frac{-3y^2}{2x}$

b) $\frac{dy}{dx} = \frac{2x}{3y^2}$

c) $\frac{dy}{dx} = \frac{3y^2}{2x}$

d) $\frac{dy}{dx} = \frac{-2x}{3y^2}$

(i) If A is an invertible matrix of order 3, then which of the following information is NOT true? **[1]**

a) $(A^{-1})^{-1} = A$

b) $(AB)^{-1} = B^{-1} A^{-1}$, where $B = [b_{ij}]_{3\times3}$ and $|B| \neq 0$

c) If BA = CA, then $B \neq C$, where B and C are square matrices of order 3

d) $|\text{adj } A| = |A|^2$

(j) **Assertion (A):** If A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$, then B is the inverse of A. **[1]**

Reason (R): If A is a square matrix of order m and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the inverse of A.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

(k) Check whether the relation R defined on the set {1, 2, 3, 4} as R = {(a, b) : b = a + 1} is transitive. Justify your answer. **[1]**

(l) Find the value of A^2, where A is a 2×2 matrix whose elements are given by $a_{ij} = \begin{cases} 1 & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$ **[1]**

(m) Let f : $N \to Y : f(x) = 4x^2 + 12x + 15$ and y = range (f) show that f is invertible and find f^{-1} **[1]**

(n) If P(A) = $\frac{7}{13}$, P(B) = $\frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, evaluate P(A|B). **[1]**

(o) The probability that a student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university none will graduate. **[1]**

2. Show that the function defined by $f(x) = \cos(x^2)$ is a continuous function. **[2]**

OR

For what values of a the function f given by $f(x) = x^2 + ax + 1$ is increasing on [1, 2]?

3. Evaluate the integral: $\int \frac{(4x-5)}{(2x^2-5x+1)} dx$ **[2]**

4. Prove that the function $f(x) = x^3 - 6x^2 + 12x + 5$ is increasing on $\mathbb{R}$. **[2]**

5. Evaluate: $\int_0^{\pi} \sin 2x \cos 3x \, dx$ **[2]**

OR

Integrate the function: $\frac{x+2}{\sqrt{x^2-1}}$

6. Let f :$Q \to Q : f(x) = 3x - 4$, show that f is invertible and find f^{-1} **[2]**

7. Prove that $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$. **[4]**

8. Evaluate the integral: $\int \sin^{-1}(3x - 4x^3) dx$ **[4]**

9. Differentiating the function w.r.t. x: $\cot^{-1}\left(\frac{1+x}{1-x}\right)$. **[4]**

OR

If $y = \tan^{-1}\left\{\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\}$, $-1 < x < 1$, $x \neq 0$, find $\frac{dy}{dx}$.

10. **Read the text carefully and answer the questions:** **[4]**

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(a) Find the probability that it is due to the appointment of Ajay (A).
(b) Find the probability that it is due to the appointment of Ramesh (B).
(c) Find the probability that it is due to the appointment of Ravi (C).
(d) Find the probability that it is due to the appointment of Ramesh or Ravi.

OR

Read the text carefully and answer the questions: **[4]**

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.
The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(a) What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work.
(b) An application selected at random is found to be under the below 18 category. Find the probability that it is under the folk genre. Show your work.
(c) If P(A) = 0.4, P(B) = 0.8 and P(B|A) = 0.6, then P(A ∪ B) is equal to
(d) If A and B are two independent events with P(A) = $\frac{3}{5}$ and P(B) = $\frac{4}{9}$, then find P(A' ∩ B').

11. **Read the text carefully and answer the questions:** **[6]**

A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below:

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
I	10,000	2,000	18,000
II	6,000	20,000	8,000

If the unit Sale price of Pencil, Eraser and Sharpener are ₹2.50, ₹1.50 and ₹1.00 respectively,Based on the information given above, answer the following questions:

(a) What is the total revenue collected from Market-I?
(b) What is the total revenue collected from Market-II?
(c) What is the gross profit from both markets considering the unit costs of the three commodities as ₹2.00, ₹1.00, and 50 paise respectively?

12. Find the general sol. of the differential equation $\frac{dy}{dx} - y = \cos x$ **[6]**

OR

Solve $\cos\left(\frac{dy}{dx}\right) = a, y = 1,$ when x = 0

13. A metal box with a square base and vertical sides is to contain 1024 cm^3. The material for the top and bottom costs ₹ 5/cm^2 and the material for the sides costs ₹ 2.50/cm^2. Find the least cost of the box. **[6]**

OR

A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, then find the dimensions of the rectangle that will produce the largest area of the window.

14. **Read the text carefully and answer the questions:** **[6]**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

(a) Find the probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics?

(b) Find the probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics?

(c) Find the probability that the selected student has passed in Mathematics, if it is known that he has failed in Economics?

(d) Find the probability that the selected student has passed in Economics, if it is known that he has failed in Mathematics?

SECTION B - 15 MARKS

15. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively, having $\vec{a} \cdot \vec{b} = \sqrt{6}$ **[1]**

a) $\frac{\pi}{5}$ b) $\frac{\pi}{3}$

c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

(b) Find the vector equation of the plane with intercepts 3, -4 and 2 on X, Y and Z-axes respectively. **[1]**

(c) For any two vectors $\bar{a}$ and $\bar{b}$ of magnitudes 3 and 4 respectively, write the value of $[\bar{a} \quad \bar{b} \quad \bar{a} \times \bar{b}] + (\vec{a})^2(\bar{b})^2$. **[1]**

(d) The distance d from a point P(x_1, y_1, z_1) to the plane Ax + By + Cz + D = 0 is **[1]**

a) d = $\left|\frac{Ax_1+By_1+Cz_1+2D}{\sqrt{A^2+B^2+C^2}}\right|$ b) d = $\left|\frac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$

c) d = $\left|\frac{Ax_1+By_1+2Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$ d) d = $\left|\frac{Ax_1+2By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$

(e) Find the angle between the line $\vec{r} = (\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$. **[1]**

16. For any vector $\vec{a}$, find the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ **[2]**

OR

Find the angle between the vectors $\vec{a}$ *and* $\vec{b}$, where: $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$

17. Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, reduce it to vector form. **[4]**

OR

Find the distance between the line $\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j})$ and the line passing through (0, -1, 2) and (1, -2, 3).

18. Using integration, find the area of the region enclosed by the parabola y = $3x^2$ and the line 3x - y + 6 = 0. **[4]**

SECTION C - 15 MARKS

19. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) If C(x) and R(x) are respectively Cost function and Revenue function, then profit function P(x) is given by **[1]**

a) P(x) = R(x)

b) P(x) = C(x) + R(x)

c) P(x) = R(x) - C(x)

d) P(x) = R(x)·C(x)

(b) The maximum value of Z = x + 3y such that 2x + y $\leq$ 20, x + 2y $\leq$ 20, x $\geq$ 0, y $\geq$ 0 is **[1]**

a) $\frac{80}{3}$

b) 10

c) 30

d) 60

(c) For 5 observations of pairs (x, y) of variables X and Y, the following results are obtained: **[1]**
Σx = 15, Σy = 25, Σx^2 = 55, Σy^2 = 135, Σxy = 83
Calculate the value of b_{xy} and b_{yx}.

(d) The total variable cost of manufacturing x units in a firm is ₹ $\left(3x + \frac{x^5}{25}\right)$. Show that the average variable cost increases with output x. **[1]**

(e) The total cost function of producing and marketing x units of a commodity is given by C = 16 - 12x + 2x^2. Find the level of output at which it is minimum. **[1]**

20. The total revenue in rupees received from the sale of x units of a product is given by R(x) = 300x - $\frac{x^2}{5}$. Find **[2]**

i. the average revenue
ii. the marginal revenue
iii. the total revenue when MR = 0

OR

The average cost function AC for a commodity is given by AC = x + 5 + $\frac{36}{x}$ in terms of output x. Find the total cost C and the marginal cost MC as the function of x. Also, find the outputs for which AC increases.

21. For the given lines of regression, 3x - 2y = 5 and x - 4y = 7, find **[4]**

i. regression coefficients b_{yx} and b_{xy}
ii. coefficient of correlation r(x, y).

22. A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximise the total profit of the society? **[4]**

OR

If a young man drives his vehicle at 25 km /hr, he has to spend ₹ 2 per km on petrol. If he drives it at a faster speed of 40 km /hr, the petrol cost increases to ₹ 5 per km. He has ₹ 100 to spend on petrol and travel within one hour. Express this as an LPP and solve the same.

Solution

SECTION A - 65 MARKS

1. In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(a) **(c)** 0

Explanation:

0

(b) **(d)** $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log|x-4+\sqrt{x^2-8x+7}|+\mathrm{C}$

Explanation:

$I=\int\sqrt{x^2-8x+7}\,dx$

$=\int\sqrt{(x^2-8x+16)-9}\,dx$

$=\int\sqrt{(x-4)^2-(3)^2}\,dx$

We know that

$\Rightarrow\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log|x+\sqrt{x^2-a^2}|+C$

Therefore,

$\Rightarrow I=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log|(x-4)+\sqrt{x^2-8x+7}|+C$

(c) **(a)** $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ - {0}

Explanation:

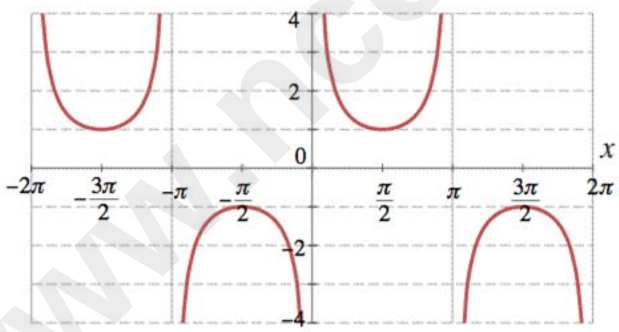
To Find: The range of $\operatorname{coses}^{-1}(x)$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $\operatorname{coses}^{-1}(x)$ can be obtained from the graph of

$Y = \operatorname{coses}^{-1}(x)$ by interchanging x and y axes.i.e, if a, b is a point on Y = cosec x then b, a is the point on the function $y = \operatorname{coses}^{-1}(x)$

Below is the Graph of the range of $\operatorname{coses}^{-1}(x)$



From the graph, it is clear that the range of $\operatorname{coses}^{-1}(x)$ is restricted to interval $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ - {0}

(d) **(c)** $4xy = y^4 + C$

Explanation:

Consider the given differential equation, $\frac{dx}{dy}+\frac{x}{y}=y^2$

On comparing with linear differential equation of the form $\frac{dx}{dy}$ + Px = Q. Here, P = $\frac{1}{y}$ and Q = y^2

$\therefore$ IF = $e^{\int P dy}=e^{\int\frac{1}{y}dy}=e^{\int\ln(y)}=y$

Now, the solution of given differential equation is

$xy=\int\left(y\cdot y^2\right)dy+C_1\Rightarrow xy=\int y^3\,dy+C_1$

$\Rightarrow xy=\frac{y^4}{4}+C_1\Rightarrow 4xy=y^4+C,$

where $C = 4C_1$

(e) **(a)** $\frac{1}{2}$

Explanation:

Probability that a man will not live 10 more years $= 1 - \frac{1}{4} = \frac{3}{4}$

Probability that a man will not live for 10 more years $= 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore$ Required probability $= \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$

(f) **(c)** reflexive but not symmetric

Explanation:

As (1, 1), (2, 2), (3, 3) $\in$ R, therefore R is reflexive. Since (1,2) $\in$ R, but (2,1) $\notin$ R. Therefore,R is not symmetric.

(g) **(c)** 5

Explanation:

Given the function f (x) is continous at x = 2.

Then $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$ = f(2)

Now, $\lim_{x \to 2^-} (3x - 1)$ = 3(2) - 1 = 5

and $\lim_{x \to 2^+} (2x + 1)$ = 2(2) + 1 = 5

Hence, k = 5

(h) **(d)** $\frac{dy}{dx} = \frac{-2x}{3y^2}$

Explanation:

$x^2 y^3 = 42$

$\Rightarrow \frac{d}{dx}x^2 + \frac{d}{dx}y^3 = \frac{d}{dx}42$

$\Rightarrow \frac{d}{dx}x^2 + \frac{d}{dx}y^3 = \frac{d}{dx}42$

$\Rightarrow 2x + 3y^2\frac{dy}{dx}$ = 0

$\Rightarrow \frac{dy}{dx} = -\frac{2x}{3y^2}$

(i) **(c)** If BA = CA, then B $\neq$ C, where B and C are square matrices of order 3

Explanation:

BA = CA

$\Rightarrow BAA^{-1} = CAA^{-1}$

$\Rightarrow$ BI = CI

$\Rightarrow$ B = C

(j) **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Let A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ be two matrices.

Then, AB = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4-3 & -6+6 \\ 2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Also, BA = $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4-3 & 6-6 \\ -2+2 & -3+4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Thus, B is the inverse of A.

(k) (1, 2), (2, 3) $\in$ R but (1, 3) $\notin$ R $\therefore$ R is not transitive.

(l) A $= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow$ A$^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(m)f is one one then,we have

$f(x_1) = f(x_2) \Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$

$\Rightarrow 4\left(x_1^2 - x_2^2\right) + 12(x_1 - x_2) = 0$

$\Rightarrow \left(x_1^2 - x_2^2\right) + 3(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 3) = 0$

$\Rightarrow x_1 - x_2 = 0$

$x_1 + x_2 + 3 \neq 0$

$\Rightarrow x_1 = x_2$

also, range (f) = Y thus, f is onto

Therefore,f is one one onto and therefore f invertible.

Let y $\in$ Y then, f being onto there exists x such that y = f (x)

Now, y = f (x) = y = 4x +12 +15

= $y = (2x + 3)^2 + 6$

$\Rightarrow (2x + 3) = \sqrt{y - 6}$

$\Rightarrow f^{-1}(y) = \frac{1}{2}(\sqrt{y-6} - 3)$

Therefore, we define:

$f^{-1} : Y \to N : f^{-1}(y) = \frac{1}{2}(\sqrt{y-6} - 3)$

(n) We have P(A|B) = $\frac{P(A\cap B)}{P(B)} = \frac{\frac{4}{13}}{\frac{9}{13}} = \frac{4}{9}$

(o) Let X be a random variable denoting number of students that graduate from among 3 students.

Let p = probability that a student entering a university will graduate.

Here, n = 3, p = 0.4 and q = 0.6

Therefore, the distribution is given by

$P(X = r) = ^3C_r(0.4)^r(0.6)^{3-r}$, r = 0, 1, 2, 3

P(X = 0) = q^3 = 0.216

2. Let $f(x) = x^2$ and $g(x) = \cos x$, then

$(gof)(x) = g[f(x)] = g\left(x^2\right) = \cos x^2$

Now f and g being continuous it follows that their composite (gof) is continuous.

Hence $\cos x^2$ is continuous function.

OR

f(x) = x^2 + ax + 1

$\therefore$ f'(x) = 2x + a

Now, function f will be increasing in (1, 2), if f'(x) > 0 in (1, 2).

$\Rightarrow$ 2x + a > 0

$\Rightarrow$ 2x > -a

$\Rightarrow$ x > $\frac{-a}{2}$

Therefore, we have to find the least value of a such that

$\Rightarrow$ x > $\frac{-a}{2}$, when x $\in$ (1, 2).

Thus, the least value of a for f to be increasing on (1, 2) is given by,

$\frac{-a}{2}$ = 1 $\Rightarrow$ a = -2

Hence, the required value of a is -2.

3. Let I = $\int \frac{4x-5}{(2x^2-5x+1)}dx$

Since $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \frac{4x-5}{(2x^2-5x+1)}dx$ ... (i)

Let 2x^2 - 5x + 1 = t

$\Rightarrow \frac{d(2x^2-5x+1)}{dx} = \frac{dt}{dx}$
$\Rightarrow 4x - 5 = \frac{dt}{dx}$
⇒ (4x - 5)dx = dt
Putting this value in equation (i), we get
$I = \int \frac{dt}{t}$
I = log |t| + c
I = log $|2x^2 - 5x + 1|$ + c

4. f(x) = $x^3 - 6x^2 + 12x + 5$
   f'(x) = $3x^2 - 12x + 12$
   = $3(x^2 - 4x + 4)$
   = $3(x - 2)^2 \geq 0$ for all $x \in \mathbb{R}$
   $\therefore$ f'(x) $\geq 0 \ \forall \ x \in \mathbb{R}$
   $\therefore$ f(x) increasing function on $\mathbb{R}$
5. Let I = $\int_0^\pi$ sin2x cos3x xdx, then
   I = $\frac{1}{2}\int_0^\pi$ (sin5x - sinx) dx
   $= \frac{1}{2}\left[\frac{-\cos 5x}{5} + \cos x\right]$
   $= \frac{1}{2}\left[-\frac{\cos(5\pi)}{5} + \cos(\pi)\right] - \frac{1}{2}\left[-\frac{\cos(0)}{5} + \cos(0)\right]$
   $= \frac{1}{2}\left[\frac{-(-1)}{5} - 1\right] - \frac{1}{2}\left[-\frac{1}{5} + 1\right]$
   $= \frac{1}{2}\left[\frac{-4}{5} - \frac{4}{5}\right]$
   $= \frac{1}{2}2\left(-\frac{4}{5}\right)$
   $= -\frac{4}{5}$

OR

   Clearly, we can write, x + 2 = $\frac{1}{2}(2x) + 2$
   $\Rightarrow \int \frac{x+2}{\sqrt{x^2-1}}dx = \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$
   $= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx + 2\int \frac{1}{\sqrt{x^2-1}}dx$
   Now in, $I_1 = \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx$
   Let $x^2 - 1 = t$
   ⇒ (2x)dx = dt
   $\Rightarrow I_1 = \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}[2\sqrt{t}]$ = $\frac{1}{2}[2\sqrt{x^2-1}]$
   And $2\int \frac{1}{\sqrt{x^2-1}}dx = 2\log|x + \sqrt{x^2-1}|$
   $\Rightarrow \int \frac{x+2}{\sqrt{x^2-1}}dx = \sqrt{x^2-1} + 2\log|x + \sqrt{x^2-1}| + C$
6. Onto function: If range = co-domain then f(x) is onto functions.
   So, We need to prove that the given function is one-one and onto function.
   Let $x_1, x_2 \in Q$ and f(x) = 3x-4.
   So $f(x_1) = f(x_2) \rightarrow 3x_1 - 4 = 3x_2 - 4 \rightarrow x_1 = x_2$
   So $f(x_1) = f(x_2) \rightarrow x_1 = x_2$, f(x) is one-one function.
   Given co-domain of f(x) is Q.
   Let y = f(x) = 3x- 4, So x = $\frac{y+4}{3}$ [Range of f(x) = Domain of y]
   So Domain of y is Q = Range of f(x)
   Hence, Range of f(x) = co-domain of f(x) = Q
   So, f(x) is onto function.
   As it is a bijective function. So it is invertible function.
   Inverse of f(x) is $f^{-1}(x) = \frac{x+4}{3}$

7. Let $\sin^{-1}\frac{3}{5}=\theta$, then

$\sin\theta=\frac{3}{5}$,

where $\theta\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

Thus,

$\tan\theta=\frac{3}{4}$, which gives $\theta=\tan^{-1}\frac{3}{4}$

Therefore,

$2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}$

$=2\theta-\tan^{-1}\frac{17}{31}=2\tan^{-1}\frac{3}{4}-\tan^{-1}\frac{17}{31}$

$=\tan^{-1}\left(\frac{2.\frac{3}{4}}{1-\frac{9}{16}}\right)-\tan^{-1}\frac{17}{31}=\tan^{-1}\frac{24}{7}-\tan^{-1}\frac{17}{31}$

$=\tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\cdot\frac{17}{31}}\right)$

$=\tan^{-1}\left(\frac{625}{625}\right)$

$=\tan^{-1}1$

$=\frac{\pi}{4}$

8. Let the given integral be,

$I=\int\sin^{-1}\left(3x-4x^3\right)dx$

Putting $\mathbf{x}=\sin\theta\Rightarrow\theta=\sin^{-1}x$

$\Rightarrow dx=\cos\theta d\theta$

$\therefore I=\int\sin^{-1}\left(3\sin\theta-4\sin^3\theta\right)\cos\theta d\theta$

$=\int\sin^{-1}(\sin 3\theta)\cos\theta\mathrm{d}\theta$

$=3\int\theta\cos\theta\mathrm{d}\theta$

$=3\left[\theta(\sin\theta)-\int 1\sin\theta\mathrm{d}\theta\right]$

$=3[\theta\sin\theta+\cos\theta]+C$

$=3[\theta\sin\theta+\sqrt{1-\sin^2\theta}]+C$

$=3\left[\sin^{-1}x\times x+\sqrt{1-x^2}\right]+C$

$=3\left[x\sin^{-1}x+\sqrt{1-x^2}\right]+C$

9. To find: value of cot$^{-1}\left(\frac{1+x}{1-x}\right)$

The formula used cos θ = sin $\left(\frac{\pi}{2}-\theta\right)$

$\frac{d\left(\sin^{-1}x\right)}{dx}=\frac{1}{\sqrt{1-x^2}}$

We have, cot$^{-1}\left(\frac{1+x}{1-x}\right)$

Putting x = tan θ

θ = tan-1 x ……(i)

Putting x = tan θ in the equation

$\Rightarrow$ cot$^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

= cot$^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right)$

= cot$^{-1}\left(\tan\frac{\pi}{4}+\theta\right)$

= cot^{-1} $\left(\cot\left(\frac{\pi}{2}-\left(\frac{\pi}{4}+\theta\right)\right)\right)$

= cot^{-1} $\left(\cot\left(\frac{\pi}{2}-\left(\frac{\pi}{4}+\theta\right)\right)\right)$

= cot$^{-1}\left(\cot\left(\frac{\pi}{4}-\theta\right)\right)$

= $\frac{\pi}{4}-\theta$

Now, we can see that cot$^{-1}\left(\frac{1+x}{1-x}\right)=\frac{\pi}{4}-\theta$

Now Differentiating

$\Rightarrow \frac{d\left(\frac{\pi}{4}-\theta\right)}{dx}$

= $\frac{d\left(\frac{\pi}{4}\right)}{dx} - \frac{d(\theta)}{dx}$

= $0 - \frac{d(\theta)}{dx}$

= $-\frac{d}{dx}\left(\tan^{-1} x\right)$

= $-\frac{1}{1+x^2}$

OR

Put x^2 = cos 2θ, we get

$y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right) = \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}\right)$

$\Rightarrow \quad y = \tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right) = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)$

$\Rightarrow \quad y = \frac{\pi}{4} + \theta \quad \left[\because 0 < x^2 < 1 \Rightarrow 0 < \cos 2\theta < 1 \Rightarrow 0 < 2\theta < \frac{\pi}{2} \Rightarrow 0 < \theta < \frac{\pi}{4} \Rightarrow \frac{\pi}{4} < \frac{\pi}{4} + \theta < \frac{\pi}{2}\right]$

$\Rightarrow \quad y = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2 \quad \left[\because \cos 2\theta = x^2 \Rightarrow \theta = \frac{1}{2}\cos^{-1} x^2\right]$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{4}\right) + \frac{1}{2}\frac{d}{dx}\left(\cos^{-1} x^2\right)$

$\Rightarrow \quad \frac{dy}{dx} = 0 + \frac{1}{2}\frac{(-1)}{\sqrt{1-x^4}}\frac{d}{dx}\left(x^2\right) = -\frac{1}{2} \times \frac{2x}{\sqrt{1-x^4}} = \frac{-x}{\sqrt{1-x^4}}$

10. **Read the text carefully and answer the questions:**

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above. The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(a) Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3,\ P(A/E_2) = 0.8,\ P(A/E_3) = 0.5$

$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

= $\frac{\frac{4}{7}\times 0.3}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5}$ = $\frac{\frac{1.2}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{1.2}{7}}{\frac{3}{7}}$

= $\frac{1.2}{3} = \frac{12}{30} = \frac{2}{5}$

(b) Let E_1: Ajay(A) is selected, E_2: Ramesh(B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3,\ P(A/E_2) = 0.8,\ P(A/E_3) = 0.5$

$P(E_2/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

= $\frac{\frac{1}{7}\times 0.8}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5}$ = $\frac{\frac{0.8}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{0.8}{7}}{\frac{3}{7}}$

= $\frac{0.8}{3} = \frac{8}{30} = \frac{4}{15}$

(c) Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3,\ P(A/E_2) = 0.8,\ P(A/E_3) = 0.5$

$P(E_3/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

= $\frac{\frac{2}{7}\times 0.5}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5}$ = $\frac{\frac{1}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{1}{3}$

(d) Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

Ramesh or Ravi

$\Rightarrow P(E_2/A) + P(E_3/A) = \frac{4}{15} + \frac{1}{3} = \frac{9}{15} = \frac{3}{5}$

OR

Read the text carefully and answer the questions:

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above. The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(a) According to given information, we construct the following table.

Given, total applications = 2000

	Folk Genre	**Classical Genre**
	960 (given)	2000 - 960 = 1040
Below 18	192 (given)	1040 - 104 = 936
18 or Above 18	960 - 192 = 768	104 (given)

Let E_1 = Event that application for folk genre

E_2 = Event that application for classical genre

A = Event that application for below 18

B = Event that application for 18 or above 18

$\therefore P(E_2) = \frac{1040}{2000}$

and $P(B \cap E_2) = \frac{104}{2000}$

Required Probability = $\frac{P(B \cap E_2)}{P(E_2)}$

$= \frac{\frac{104}{2000}}{\frac{1040}{200}} = \frac{1}{10}$

(b) Required probability = $P\left(\frac{\text{folk}}{\text{below 18}}\right)$

$= P\left(\frac{E_1}{A}\right)$

$= \frac{P(E_1 \cap A)}{P(A)}$

Now, $P(E_1 \cap A) = \frac{192}{2000}$

and $P(A) = \frac{192+936}{2000} = \frac{1128}{2000}$

$\therefore$ Required probability = $\frac{\frac{192}{2000}}{\frac{1128}{2000}} = \frac{192}{1128} = \frac{8}{47}$

(c) Here,

$P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$

$\because P(B|A) = \frac{P(B \cap A)}{P(A)}$

$\Rightarrow P(B \cap A) = P(B|A).P(A)$

$= 0.6 \times 0.4 = 0.24$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.8 - 0.24$

$= 1.2 - 0.24 = 0.96$

(d) Since, A and B are independent events, A' and B' are also independent. Therefore,

$P(A' \cap B') = P(A') \cdot P(B')$

= (1 - P(A)(1 - P(B))

= $\left(1-\frac{3}{5}\right)\left(1-\frac{4}{9}\right)$

= $\frac{2}{5}\cdot\frac{5}{9}$

= $\frac{2}{9}$

11. **Read the text carefully and answer the questions:**

A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below:

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
I	10,000	2,000	18,000
II	6,000	20,000	8,000

If the unit Sale price of Pencil, Eraser and Sharpener are ₹2.50, ₹1.50 and ₹1.00 respectively,Based on the information given above, answer the following questions:

(a) Let A be the 2 $\times$ 3 matrix representing the annual sales of products in two markets.

$\therefore$ A = $\begin{bmatrix} x & y & z \\ 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}$

Let B be the column matrix representing the sale price of each unit of products x, y, z.

$\therefore$ B = $\begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$

Now, revenue = sale price $\times$ number of items sold

= $\begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}\begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$

= $\begin{bmatrix} 25000+3000+18000 \\ 15000+30000+8000 \end{bmatrix}$ = $\begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$

Therefore, the revenue collected from Market I = ₹ 46000

(b) Let A be the 2 $\times$ 3 matrix representing the annual sales of products in two markets.

$\therefore$ A = $\begin{bmatrix} x & y & z \\ 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}$

Let B be the column matrix representing the sale price of each unit of products x, y, z.

$\therefore$ B = $\begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$

Now, revenue = sale price $\times$ number of items sold

$= \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 25000+3000+18000 \\ 15000+30000+8000 \end{bmatrix} = \begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$

The revenue collected from Market II = ₹ 53000.

(c) Let C be the column matrix representing cost price of each unit of products x, y, z.

Then, C = $\begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$

$\therefore$ Total cost in each market is given by

AC = $\begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$

$= \begin{bmatrix} 20000+2000+9000 \\ 12000+20000+4000 \end{bmatrix} = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$

Now, Profit matrix = Revenue matrix - Cost matrix

$= \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} - \begin{bmatrix} 31000 \\ 36000 \end{bmatrix} = \begin{bmatrix} 15000 \\ 17000 \end{bmatrix}$

Therefore, the gross profit from both the markets = ₹ 15000 + ₹ 17000 = ₹ 32000

12. $\frac{dy}{dx} - y = \cos x$

Given differential equation is of the form

$\frac{dy}{dx} + Py = Q$

$P = -1, Q = \cos x$

$I.F = e^{\int p dx}$

$= e^{\int -1 dx}$

$= e^{-x}$

$\therefore$ Solution is,$y \times e^{-x} = \int \cos x \times e^{-x} dx + c$

let $I = \int \cos x \times e^{-x} dx$

$= \cos x . \frac{e^{-x}}{-1} - \int -\sin x . (-e^{-x}) dx$

$= \cos x \times e^{-x} - \int \sin x \times e^{-x} dx$

$= \cos x \times e^{-x} - [\sin x(-e^{-x}) - \int \cos x(-e^{-x}) dx]$

$= \cos x \times e^{-x} + \sin x \times e^{-x} - \int \cos x \times e^{-x} dx$

$\Rightarrow I = \cos x e^{-x} + \sin x e^{-x} - I$

$\Rightarrow 2I = e^{-x}(\sin x + \cos x)$

$\Rightarrow I = \frac{e^{-x}}{2}(\sin x + \cos x)$

Therefore, (i) gives,

$y \times e^{-x} = \frac{1}{2} e^{-x}(\sin x + \cos x) + c$ [from (i)]

$\Rightarrow y = \frac{1}{2}(\sin x + \cos x) + ce^{x}$

OR

$\cos\left(\frac{dy}{dx}\right) = a$

$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$

$\Rightarrow \int dy = \int \cos^{-1} a dx$

$\Rightarrow y = \cos^{-1} a . x + c$ ....(1)

$\Rightarrow 1 = 0 + c \left[\because \begin{matrix} y = 1 \\ x = 0 \end{matrix}\right.$

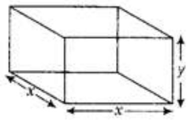
$\Rightarrow c = 1$

Therefore, from(1),we have,

$y = \cos^{-1} a . x + 1$

13. Since, volume of the box = 1024 cm^3

Let length of the side of square base be x cm and height of the box be y cm.



$\therefore$ Volume of the box (V) $= x^2 . y = 1024$

Since, $x^2 y = 1024 \Rightarrow y = \frac{1024}{x^2}$

Let C denotes the cost of the box.

$\therefore C = 2x^2 \times 5 + 4xy \times 2.50$

$= 10x^2 + 10xy = 10x(x + y)$

$= 10x\left(x + \frac{1024}{x^2}\right)$

$= \frac{10x}{x^2}\left(x^3 + 1024\right)$

$\Rightarrow C = 10x^2 + \frac{1024}{x}$ ...(i)

On differentiating both sides w.r.t. x, we get

$\frac{dC}{dx} = 20x - 10240(x)^{-2}$

$= 20x - \frac{10240}{x^2}$ ...(ii)

Now, $\frac{dC}{dx} = 0$

$\Rightarrow 20x = \frac{10240}{x^2}$

$\Rightarrow 20x^3 = 10240$

$\Rightarrow x^3 = 512 = 8^3 \Rightarrow x = 8$

Again, differentiating Eq. (ii) w.r.t. x, we get

$\frac{d^2C}{dx^2} = 20 - 10240(-2) . \frac{1}{x^3}$

$= 20 + \frac{20480}{x^3}$

$\therefore \left(\frac{d^2C}{dx^2}\right)_{x=8} = 20 + \frac{20480}{512} = 60 > 0$

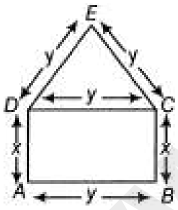
For x = 8, cost is minimum and the corresponding least cost of the box

$C(8) = 10.8^2 + \frac{10240}{8}$

$\therefore$ Least cost = ₹ 1920

OR

Let ABCD be the rectangle which is surmounted by an equilateral ΔEDC.



Now, given that

Perimeter of window = 12m

$\Rightarrow$ 2x + 2y + y = 12

$x = 6 - \frac{3}{2}y$ ...(i)

Let A denotes the combined area of the window.

Then, A = area of rectangle + area of equilateral triangle

$\Rightarrow \quad A = xy + \frac{\sqrt{3}}{4}y^2$

$\Rightarrow \quad A = y\left(6 - \frac{3}{2}y\right) + \frac{\sqrt{3}}{4}y^2 \quad \left[\because x = 6 - \frac{3}{2}y \text{ from Eq. (i)}\right]$

$\Rightarrow \quad A = 6y - \frac{3}{2}y^2 + \frac{\sqrt{3}}{4}y^2$

On differentiating both sides w.r.t. y, we get,

$\frac{dA}{dy} = 6 - 3y + \frac{\sqrt{3}}{2}y$

For maxima or minima, put $\frac{dA}{dy} = 0$

$\Rightarrow \quad 6-3y+\frac{\sqrt{3}}{2}y=0$

$\Rightarrow \quad y\left(\frac{\sqrt{3}}{2}-3\right)=-6$

$\Rightarrow \quad y=\frac{12}{6-\sqrt{3}}$

Now, $\frac{d^2A}{dy^2}=\frac{d}{dy}\left(\frac{dA}{dy}\right)=\frac{d}{dy}\left(6-3y+\frac{\sqrt{3}}{2}y\right)$

$=-3+\frac{\sqrt{3}}{2}$

$=\frac{-6+\sqrt{3}}{2}<0$

$\therefore$ A is maximum.

Now, on putting $y=\frac{12}{6-\sqrt{3}}$ in Eq. (i), we get

$x=6-\frac{3}{2}\left(\frac{12}{6-\sqrt{3}}\right)\Rightarrow x=\frac{36-6\sqrt{3}-18}{6-\sqrt{3}}$

$\therefore \quad x=\frac{18-6\sqrt{3}}{6-\sqrt{3}}$

Hence, the area of the window is largest when the dimensions of the window are

$x=\frac{18-6\sqrt{3}}{6-\sqrt{3}}$ and $y=\frac{12}{6-\sqrt{3}}$

14. **Read the text carefully and answer the questions:**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

(a) Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$, P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$

The probability that the selected student has failed in Economics if it is known that he has failed in Mathematics.

Required probability = $P(\frac{E}{M})$

= $\frac{P(E\cap M)}{P(M)}=\frac{\frac{1}{4}}{\frac{7}{20}}$ = $\frac{1}{4}\times\frac{20}{7}=\frac{5}{7}$

(b) Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$, P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$

The probability that the selected student has failed in Mathematics if it is known that he has failed in Economics.

Required probability = P(M/E)

$=\frac{P(M\cap E)}{P(E)}=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$

(c) Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$, P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$

The probability that the selected student has passed in Mathematics if it is known that he has failed in Economics

Required probability = P(M'/E)

$\Rightarrow P(M'/E)=\frac{P(M'\cap E)}{P(E)}$

= $\frac{P(E)-P(E\cap M)}{P(E)}$

$= \frac{\frac{1}{2}-\frac{1}{4}}{\frac{1}{2}}$

$\Rightarrow P(M'/E) = \frac{1}{2}$

(d) Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Economics if it is known that he has failed in Mathematics

Required probability = P(E'/M)

$\Rightarrow P(E'/M) = \frac{P(E' \cap M)}{P(M)}$

$= \frac{P(M) - P(E \cap M)}{P(M)}$

$= \frac{\frac{7}{20}-\frac{1}{4}}{\frac{7}{20}}$

$\Rightarrow P(E'/M) = \frac{2}{7}$

SECTION B - 15 MARKS

15. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(a) **(d)** $\frac{\pi}{4}$

Explanation:

$\left|\vec{a}\right| = \sqrt{3}, \left|\vec{b}\right| = 2, \vec{a} . \vec{b} = \sqrt{6},$

$\Rightarrow \vec{a} . \vec{b} = \left|\vec{a}\right| . \left|\vec{b}\right| \cos\theta \Rightarrow \sqrt{6}$

$= 2\sqrt{3}\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

(b) The equation is $\frac{x}{3} + \frac{y}{-4} + \frac{z}{2} =$ 1 or 4x - 3y + 6z = 12

The vector equation is

$\vec{r} \cdot (4\hat{i} - 3\hat{j} + 6\hat{k}) = 12$

(c) $\left[\vec{a} \quad \vec{b} \quad \vec{a} \times \vec{b}\right] + (\vec{a})^2(\vec{b})^2$

$= 0 + (12)^2$ [given |a|=3,|b|=4]

= 144

(d) **(b)** d = $\left|\frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}}\right|$

Explanation:

The distance d from a point P(x_1, y_1, z_1) to the plane Ax + By + Cz + D = 0 is given by : d = $\left|\frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}}\right|$.

(e) Give line is $\vec{r} = \vec{a} + \lambda\vec{b}$ and the equation of given plane is $\vec{r} \cdot \vec{n} = p$

$\sin\theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}||\vec{n}|} = \frac{|(\hat{i} - \hat{j} + \hat{k}) \cdot (2\hat{i} - \hat{j} + \hat{k})|}{\{\sqrt{1^2 + (-1)^2 + 1^2}\}\{\sqrt{2^2 + (-1)^2 + 1^2}\}}$

$= \frac{|(2\times1) + (-1)\times1\times1|}{(\sqrt{3}\times\sqrt{6})} = \frac{4}{\sqrt{18}} = \left(\frac{4}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{2\sqrt{2}}{3} \Rightarrow \theta = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

16. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore \vec{a}^2$ = $x^2 + y^2 + z^2$

$\therefore \vec{a} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix}$

= $\hat{i}[0] - \hat{j}[-z] + \hat{k}[-y]$

$= z\hat{j} - y\hat{k}$

$\therefore (\vec{a} \times \hat{j})^2 = (z\hat{j} - y\hat{k})(z\hat{j} - y\hat{k})$

$= y^2 + z^2$

Similarly, $(\vec{a} \times \hat{j})^2 = x^2 + z^2$ and $(\vec{a} \times \hat{k})^2 = x^2 + z^2$

$(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2 = y^2 + z^2 + x^2 + z^2 + x^2 + y^2$

$= 2(x^2 + y^2 + z^2)$

$= 2\vec{a}^2$

OR

Let θ be the angle between given vectors $\vec{a}$ and $\vec{b}$, then

$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|\vec{b}|}$ ...(i)

$\vec{a}.\vec{b} = (2\hat{i} - 3\hat{j} + \hat{k}).(\hat{i} + \hat{j} - 2\hat{k})$

$= (2)(1) + (-3)(1) + (1)(-2)$

$= 2 - 3 - 2$

$\vec{a} \cdot \vec{b} = -3$

$|\vec{a}| = |2\hat{i} - 3\hat{j} + \hat{k}|$

$= \sqrt{(2)^2 + (-3)^2 + (-1)^2}$

$= \sqrt{4 + 9 + 1}$

$= \sqrt{14}$

$|\vec{b}| = |i + \hat{j} - 2\hat{k}|$

$|\vec{b}| = \sqrt{(1)^2 + (1)^2 + (-2)^2}$

$= \sqrt{1 + 1 + 4}$

$|\vec{b}| = \sqrt{6}$

put $\vec{a} \cdot \vec{b}, |\vec{a}|$ and $|\vec{b}|$ in equation (i)

$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$= \frac{-3}{\sqrt{14} \times \sqrt{6}}$

$\cos\theta = \frac{-3}{\sqrt{84}}$

$\theta = \cos^{-1}\left(\frac{-3}{\sqrt{84}}\right)$

angle between vector $\vec{a}$ and $\vec{b} = \cos^{-1}\left(\frac{-3}{\sqrt{84}}\right)$

17. The equation of the line is

$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

But to make it cartesian equation the coefficient of x, y, z must be one so the above equation becomes as

$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$

Now the direction ratios of this line is -2, 6, -3

Direction cosines of line

$\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$

Are given as

$\frac{b_1}{\sqrt{b_1^2+b_2^2+b_3^2}}, \frac{b_2}{\sqrt{b_1^2+b_2^2+b_3^2}}, \frac{b_3}{\sqrt{b_1^2+b_2^2+b_3^2}}$

Here

$b_1^2 + b_2^2 + b_3^2 = 2^2 + 6^2 + (-3)^2 = 49$

$\sqrt{b_1^2 + b_2^2 + b_3^2} = \sqrt{49} = 7$

Hence direction cosines of the above line are

$\left(\frac{2}{7}, \frac{6}{7}, \frac{-3}{7}\right)$

Cartesian equation of the line is

$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$

Let this be equal to λ

$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$

Hence, x = 4 - 2λ, y = 6λ, z = 1 - 3λ ....(i)

$\vec{r} = \mathrm{x}\hat{i} + \mathrm{y}\hat{\mathrm{j}} + \mathrm{z}\hat{\mathrm{k}}$ .....(ii)

Hence comparing (i) and (ii), we get,

$\vec{\mathrm{r}} = 4\hat{\mathrm{i}} + \hat{\mathrm{k}} + \lambda(-2\hat{\mathrm{i}} + 6\hat{\mathrm{j}} - 3\hat{\mathrm{k}})$

OR

The vector equation of the line passing through points (0, -1, 2) and (1, -2, 3) is

$\vec{r} = (-\hat{j} + 2\hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$

Thus, the equations of two lines are

$l_1 : \vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j})$

$l_2 : \vec{r} = (-\hat{j} + 2\hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$

Clearly, line l_1 passes through the point $\vec{a} = -\hat{i} + 3\hat{k}$.

The plane containing line l_2 and parallel to line l_1 is normal to the vector $\vec{n}$given by

$$\vec{n} = (\hat{i} - 2\hat{j}) \times (\hat{i} - \hat{j} + \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 0 \\ 1 & -1 & 1 \end{vmatrix} = -2\hat{i} - \hat{j} + \hat{k}$$

Also, it passes through the point $\vec{a} = -\hat{j} + 2\hat{k}$

Thus, the equation of the plane containing l_2 and parallel to l_1 is

$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$ or, $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$ or, $\vec{r} \cdot (-2\hat{i} - \hat{j} + \hat{k}) = 3$ ....(iii)

Let d be the shortest distance between the lines l_1 and l_2. Then

d = Length of the perpendicular from $\vec{a} = -\hat{i} + 3\hat{k}$ on the plane (iii)

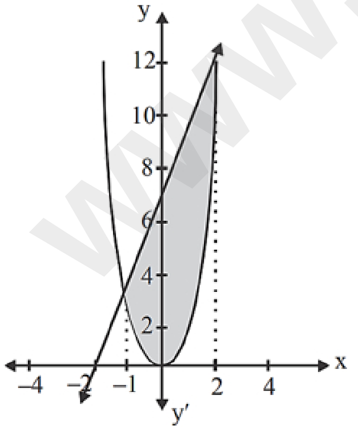
$\Rightarrow d = \frac{|(-\hat{i}+3\hat{k})\cdot(-2\hat{i}-\hat{j}+\hat{k})-3|}{\sqrt{4+1+1}} = \frac{|2+3-3|}{\sqrt{6}} = \frac{2}{\sqrt{6}}$.

18. Points of intersection x = -1, 2

$= \int_{-1}^{2} 3(x + 2)dx - 3\int_{-1}^{2} x^2\, dx$

$= \frac{3}{2}\left[(x+2)^2\right]_{-1}^{2} - \left[x^3\right]_{-1}^{2}$

$= \frac{3}{2} \times 15 - 9 = \frac{27}{2}$



SECTION C - 15 MARKS

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

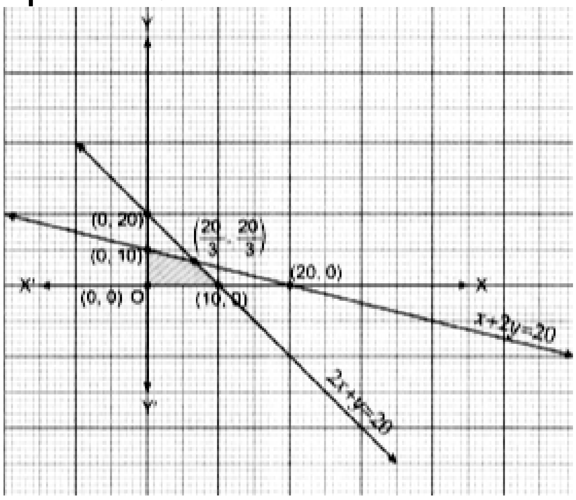
(a) **(c)** P(x) = R(x) - C(x)

Explanation:

P(x) = R(x) - C(x)

(b) **(c)** 30

Explanation:



Feasible region is shaded region which is shown in the figure with corner points (0, 0), (10, 0), $\left(\frac{20}{3}, \frac{20}{3}\right)$ L and (0,10).

Z(0, 0) = 0 Z(10,0) = 10

$Z\left(\frac{20}{3}, \frac{20}{3}\right) = \frac{20}{3} + 20 = \frac{80}{3}$

Z (0,10) = 30 ⟵ Maximum

Z_{max} = 30 obtained at (0, 10).

(c) Given, Σx = 15, Σy = 25, Σx^2 = 55, Σy^2 = 135, Σxy = 83 and n = 5

$b_{xy} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum y^2 - \frac{(\Sigma y)^2}{n}}$

$= \frac{83 - \frac{15 \times 25}{5}}{135 - \frac{(25)^2}{5}}$

$= \frac{83-75}{135-125}$

$= \frac{8}{10}$

= 0.8

and $b_{yx} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$

$= \frac{83 - \frac{15 \times 25}{5}}{55 - \frac{(15)^2}{5}}$

$= \frac{83-75}{55-45}$

$= \frac{8}{10}$ = 0.8

(d) Given TVC = 3x + $\frac{x^5}{25}$

$\therefore$ AVC = $\frac{\text{TVC}}{x} = \frac{3x + \frac{x^5}{25}}{x} = 3 + \frac{x^4}{25}$

$\Rightarrow \frac{d}{dx}$(AVC) = $\frac{4x^3}{25}$

$\therefore \frac{d}{dx}$(AVC) > 0 when x > 0

So, the average variable cost increases with output x.

(e) We have,

= 16 - 12x + $2x^2$

$\Rightarrow \frac{dC}{dx}$ = 0 - 12 + 4x and, $\frac{d^2C}{dx^2}$ = 4

For C to be minimum or maximum, we must have

$\frac{dC}{dx} = 0 \Rightarrow -12 + 4x = 0$

$\Rightarrow$ x = 3

Clearly, $\frac{d^2C}{dx^2}$ = 4 > 0 for all x

Hence, C is minimum when the level of output is 3 units.

20. Given R(x) = 300x $-\frac{x^2}{5}$

i. AR = $\frac{\mathrm{R}(x)}{x}$ = $\frac{300x-\frac{x^2}{5}}{x}$ = $300-\frac{x}{5}$

ii. MR = $\frac{d}{dx}$(R(x)) = 300 - $\frac{2x}{5}$

iii. When MR = 0

i.e. 300 - $\frac{2x}{5}$ = 0 $\Rightarrow$ x = 750

$\therefore$ R(750) = ₹ $\left(300\times750-\frac{(750)^2}{5}\right)$ = ₹(225000 - 112500) = ₹ 112500

OR

Let C be the total cost function. Then,

AC = $\frac{\mathrm{C}}{x}$

$\Rightarrow$ C = $AC\cdot x$

$\Rightarrow$ C = $\left(x+5+\frac{36}{x}\right)x$

$\Rightarrow$ C = x^2 + 5x + 36

Let MC be the marginal cost function. Then,

MC = $\frac{dC}{dx}$

$\Rightarrow$ MC = $\frac{d}{dx}$(x^2 + 5x + 36)

$\Rightarrow$ MC = 2x + 5

Now, AC = x + 5 + $\frac{36}{x}$

$\Rightarrow$ $\frac{d}{dx}(AC)$ = 1 - $\frac{36}{x^2}$

For AC to be increasing, we have

$\frac{d}{dx}(AC)>0$

$\Rightarrow 1-\frac{36}{x^2}$ > 0

$\Rightarrow \frac{x^2-36}{x^2}>0$

$\Rightarrow$ x^2 - 36 > 0

$\Rightarrow$ (x - 6) (x + 6) > 0

$\Rightarrow$ x < - 6 or x > 6 $\Rightarrow$ x > 6 [$\because$ x > 0]

Thus, the average cost increases, if the output x > 6.

21. i. Let the line of regression of x on y be

3x - 2y = 5

and the line of regression of y on x be

x - 4y = 7

Written the first equation in the form

x = $\frac{2}{3}$y + $\frac{5}{3}$

we get, b_{xy} = $\frac{2}{3}$

Written the second equation in the form

y = $\frac{1}{4}$x - $\frac{7}{4}$

we get, b_{yx} = $\frac{1}{4}$

ii. Now, r^2 = b_{yx} x b_{xy}

= $\frac{1}{4}\times\frac{2}{3}=\frac{1}{6}$

$\therefore r=\frac{1}{\sqrt{6}}$

Since r, b_{yx} and b_{xy} all have same sign.

22. Let x hectare of land be allocated to crop X and y hectare to crop Y. Obviously, $x\geq0, y\geq0$

Profit per hectare on crop X = ₹ 10500

Profit per hectare on crop Y = ₹ 9000

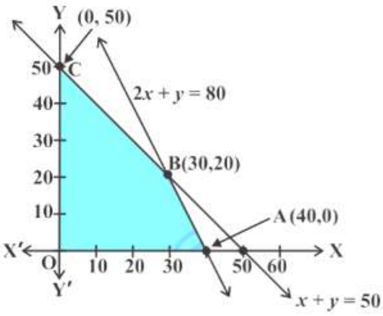
Therefore, total profit = ₹ (10500x + 9000y)

The mathematical formulation of the problem is as follows: Maximise Z = 10500x + 9000y

subject to the constraints:

$x + y \leq 50$ ((constraint related to land) ......(i)
$20x + 10y \leq 800$ (constraint related to use of herbicide)
i.e. $2x + y \leq 80$ ......(ii)
$x \geq 0, y \geq 0$ (non negative constraint) ......(iii)
Let us draw the graph of the system of inequalities (i) to (iii). The feasible region OABC is shown (shaded) in the Figure. Observe that the feasible region is bounded.
The coordinates of the corner points O, A, B and C are (0, 0), (40, 0), (30, 20) and (0, 50) respectively. Let us evaluate the objective function Z = 10500x + 9000y at these vertices to find which one gives the maximum profit.

Corner point	Z = 10500x + 9000y
O(0,0)	0
A (40, 0)	420000
B(30, 20)	**495000 Maximum**
C(0, 50)	450000



Hence, the society will get the maximum profit of ₹4,95,000 by allocating 30 hectares for crop X and 20 hectares for crop Y.

OR

Suppose the youngman travels x km at the speed 25 km/hour and y km at the speed of 40 km/hour.

Speed	Distance (d)	Time	Cost
25 km/hour	x	$\frac{x}{25}$	2x
40 km/hour	y	$\frac{y}{40}$	5y

We have to maximize $d = x + y$ ...(i)
subjects to constraints $x \geq 0, y \geq 0$ ...(ii)
$\frac{x}{25} + \frac{y}{40} \leq 1$ ...(iii)
$2x + 5y \leq 100$ ....(iv)
First of all,we shall convert all the inequalities into equations as follows:
Consider the line $\frac{x}{25} + \frac{y}{40} = 1$
$\Rightarrow 8x + 5y = 200$ ...(v)
When $x = 0, y = 40$
and $y = 0, x = 25$
$A(0,40); B(25,0)$
Consider the line
$2x + 5y = 100$ ...(vi)
When $x = 0, y = 20$
and $y = 0, x = 50$
$C(0,20); D(50,0)$; the lines (v) and (vi) intersect at $\left(\frac{50}{3}, \frac{40}{3}\right)$.
Now, feasible region shaded have corner points
$B(25,0), \mathrm{E}\left(\frac{50}{3}, \frac{40}{3}\right) \, and \, C(0,20)$.
Now, corner points of feasible region are examined for the maximum value of d.
At $B(25,0), d = 25 + 0 = 25$
At $\mathrm{E}\left(\frac{50}{3}, \frac{40}{3}\right), d = \frac{50}{3} + \frac{40}{3} = 30$

At $C(0, 20), d = 0 + 20 = 20$

d is maximum, when youngman travel $\frac{50}{3}$ km at a speed of 25 km/hour and $\frac{40}{3}$ km at a speed of 40 km/hour.

Total distance covered = 30 km.