ISC 2026 EXAMINATION
Sample Question Paper - 2
Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

This Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions*

EITHER** from **Section B OR Section C.

Section A**: Internal choice has been provided in **two questions of two marks each, two questions of four marks each and two questions of six marks each.

Section B:** Internal choice has been provided in **one question of two marks and one question of four marks.

Section C:** Internal choice has been provided in **one question of two marks and one question of four marks.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

1. **In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.** **[15]**

(a) If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to: **[1]**

a) I + A　　b) I

c) 0　　d) I - A

(b) $\int$ cosec xdx=? **[1]**

a) log |cosec x - cot x| + C　　b) log |cosec x + cot x| + C

c) - log |cosec x - cot x| + C　　d) 2 log |cosec x - cot x| + C

(c) $\cot(\cos^{-1}x)$ is equal to **[1]**

a) $\frac{x}{\sqrt{1+x^2}}$　　b) $\frac{|x|}{\sqrt{1-x^2}}$

c) $\frac{x}{\sqrt{1-x^2}}$　　d) $\frac{-x}{\sqrt{1-x^2}}$

(d) General solution of $\frac{dy}{dx}$ + y tan x = sec x is: **[1]**

a) tan x = y tan x + c　　b) y tan x = sec x + c

c) x sec x = tan y + c　　d) y sec x = tan x + c

(e) Two events E and F are independent. If P(E) = 0.3, P(E ∪ F)=0.5 then P(E|F) - P(F|E) equals **[1]**

a) $\frac{1}{70}$
b) $\frac{3}{35}$
c) $\frac{2}{7}$
d) $\frac{1}{7}$

(f) Let A = {1,2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Based on the given information, f is best defined as: **[1]**

a) not bijective
b) Bijective function
c) Surjective function
d) Injective function

(g) If f (x) = $\tan^{-1}x$ and g (x) = $\tan^{-1}\left(\frac{x+1}{1-x}\right)$, then **[1]**

a) f (x) = g(x)
b) $D_f = D_g$
c) f (x) = 2g(x)
d) f ‘ (x) = g ‘ (x)

(h) If f(x) = $\begin{cases} \frac{\sin 5x}{x^2+2x}, & x \neq 0 \\ k+\frac{1}{2}, & x=0 \end{cases}$ is continuous at x = 0, then the value of k is **[1]**

a) 2
b) -2
c) $\frac{1}{2}$
d) 1

(i) Which of the following is correct **[1]**

a) Determinant is a number associated to a matrix.
b) Determinant is a square matrix
c) Determinant is a number associated to a square matrix.
d) Determinant is a number not associated to a square matrix.

(j) **Assertion (A):** If A = $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$, then A^{-1} does not exist. **[1]**

Reason (R): On using elementary column operations $C_2 \to C_2 - 2C_1$ in the following matrix equation $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, we have $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

(k) Consider f : {1, 2, 3} $\to$ {a, b, c} given by f(1) = a, f(2) = b and f(3) = c. Find f^{-1} and show that $(f^{-1})^{-1}$ = f. **[1]**

(l) Give an example of a skew symmetric matrix of order 3. **[1]**

(m) Let A = {x $\in$ R : - 4 $\leq$ x $\leq$ 4 and x $\neq$ 0} and f : A $\to$ R be defined by $f(x) = \frac{|x|}{x}$. Write the range of f. **[1]**

(n) If A and B are two events such that $P(A) = \frac{7}{13}, P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, find $P(\bar{A}/B)$. **[1]**

(o) Let A and B be the events such that P(A) = $\frac{5}{11}$, P(B) = $\frac{6}{11}$ and P(A $\cup$ B) = $\frac{7}{11}$, find P($\frac{B}{A}$). **[1]**

2. Find $\frac{dy}{dx}$ for $(x + y)^2 = 2axy$ **[2]**

OR

Find the interval of the function f(x) = $x^3 - 12x^2 + 36x + 17$ is increasing or decreasing.

3. Evaluate: $\int e^x \left(\frac{\cos x+\sin x}{\cos^2 x}\right) dx$. **[2]**

4. The edge of a variable cube is increasing at the rate of 10 cm/sec. How fast is the volume of the cube increasing **[2]**

when the edge is 5 cm long?

5. Integrate the function: $\frac{(\log x)^2}{x}$ **[2]**

OR

Evaluate: $\int \frac{x^3-x^2+x-1}{x-1} dx$

6. Show the relation R in the set A = {x ∈ Z : 0 ≤ x ≤ 12}, given by **[2]**

R = {(a, b) : |a – b| is a multiple of 4} is an equivalence relation.

Find the set of all elements related to 1 in each case.

7. Prove that $\tan^{-1} y + \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{3y-y^3}{1-3y^2}\right)$. **[4]**

8. Evaluate: $\int \frac{(3-2x)}{\sqrt{2+x-x^2}} dx$ **[4]**

9. If x= $\tan\left(\frac{1}{a}\log y\right)$, prove that $\left(1+x^2\right)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$ **[4]**

OR

At what points on the curve, is the tangent parallel to the x-axis? If y = 12(x + 1) (x – 2) on [–1, 2]

10. **Read the text carefully and answer the questions:** **[4]**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

(a) Find the probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics?

(b) Find the probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics?

(c) Find the probability that the selected student has passed in Mathematics, if it is known that he has failed in Economics?

(d) Find the probability that the selected student has passed in Economics, if it is known that he has failed in Mathematics?

OR

Read the text carefully and answer the questions: **[4]**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

(a) How is Bayes' theorem different from conditional probability?

(b) Write the rule of Total Probability.

(c) What is the probability that the shell fired from exactly one of them hit the plane?

(d) If it is know that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

11. **Read the text carefully and answer the questions:** **[6]**

Two farmers Shyam and Balwan Singh cultivate only three varieties of pulses namely Urad, Masoor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.

September sales (in ₹)

$$A = \begin{array}{c} \begin{array}{ccc} \mathit{Urad} & \mathit{Masoor} & \mathit{Mung} \end{array} \\ \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ Shyam \\ Balwan\ Singh \end{array}$$

October sales (in ₹)

$$B = \begin{array}{c} \begin{array}{ccc} \mathit{Urad} & \mathit{Masoor} & \mathit{Mung} \end{array} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ Shyam \\ Balwan\ Singh \end{array}$$

(a) What were the combined sales of Masoor for farmer Balwan Singh in September and October?

(b) What were the combined sales of Urad for farmer Shyam in September and October?

(c) How much did the sales of Mung decrease from September to October for farmer Shyam?

12. Solve the following differential equation : $x^2dy + (xy + y^2)dx = 0$, when x = 1 and y = 1. **[6]**

OR

Show that the differential equation $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$ is homogeneous and solve it.

13. Show that the altitude of a right circular cone of maximum that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. **[6]**

OR

Find the coordinates of a point on the parabola $y = x^2 + 7x + 2$ which is closest to the straight line $y = 3x - 3$.

14. **Read the text carefully and answer the questions:** **[6]**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(a) Find the probability that she gets grade A in all subjects.

(b) Find the probability that she gets grade A in no subjects.

(c) Find the probability that she gets grade A in two subjects.

(d) Find the probability that she gets grade A in at least one subject.

SECTION B - 15 MARKS

15. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ Then **[1]**

a) $|\vec{a}|=|\vec{b}|$

b) $\vec{a}\perp\vec{b}$

c) $\vec{a}\|\vec{b}$

d) $|\vec{a}|\neq|\vec{b}|$

(b) Write the equation of the plane parallel to YZ-plane and passing through the point (-3, 2, 0). **[1]**

(c) Find λ andμ, if **[1]**

$(\hat{i}+3\hat{j}+9\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k})=\vec{0}$.

(d) The line $\frac{x-1}{2}=\frac{y-2}{4}=\frac{z-3}{4}$ meets the plane 2x + 3y - z = 14 in the point. **[1]**

a) (6, 5, 3)

b) (5, 7, 3)

c) (3, 5, 7)

d) (2, 5, 7)

(e) Find a unit vector normal to the plane x - 2y + 2z = 6. **[1]**

16. Show that the vectors a $\vec{a}=\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k}),\vec{b}=\frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k}),\vec{c}=\frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})$ are mutually perpendicular unit vectors. **[2]**

OR

Write a vector of magnitude of 18 units in the direction of the vector $\hat{i}-2\hat{j}-2\hat{k}$.

17. Find the angle between the lines $\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and $\vec{r}=7\hat{i}-6\hat{j}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$. **[4]**

OR

Find the direction cosines of the perpendicular from the origin to the plane $\vec{r}\cdot(6\hat{i}-3\hat{j}-2\hat{k})+3=0$.

18. Find the equation of the plane that contains the point A(2, 1, -1) and is perpendicular to the line of intersection of the planes 2x + y - z = 3 and x + 2y + z = 2. Also, find the angle between the plane thus obtained and the y-axis. **[4]**

SECTION C - 15 MARKS

19. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) Profit function is the difference of ________ and ________. **[1]**

a) R(x) and C(x)

b) P(X) and C(X)

c) P(x) and R(x)

d) Both P(X) and R(X) and R(X) and C(X)

(b) The linear programming problem minimize Z = 3x + 2y subject to constraints x + y $\geq$ 8, 3x + 5y $\leq$15, x $\geq$ 0 and y $\geq$ 0, has **[1]**

a) one solution

b) infinitely many solutions

c) no feasible solution

d) two solutions

(c) For the lines of regression 4x - 2y = 4 and 2x - 3y + 6 = 0, find the mean of **x** and the mean of **y**. **[1]**

(d) A company finds that total revenue from sale of q units of its product is 2q rupees whereas the cost is $500+\frac{1}{2}\left(\frac{q}{20}\right)^2$. Find the rate of change of profit when q = 500. **[1]**

(e) Find the maximum profit that a company can make, if the profit function is given P(x) = 41 + 24x - $18x^2$ **[1]**

20. A company has fixed cost of ₹10000 and cost of producing one unit of its product is ₹50. If each unit sells for **[2]**

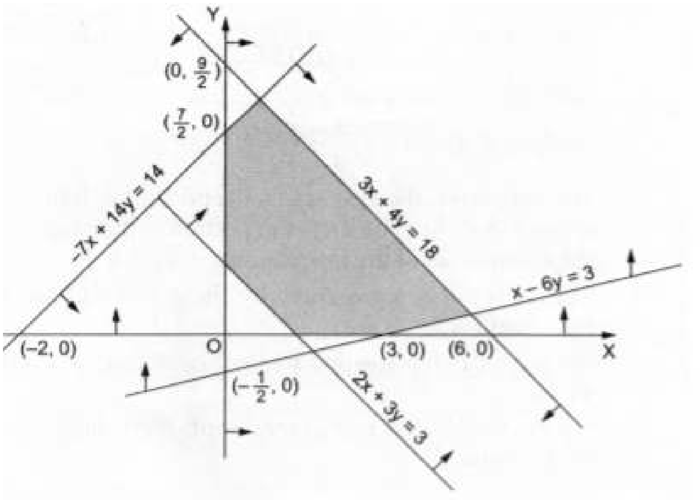
₹75, find the breakeven value. Also, find the values of x for which the company always results in profit.

OR

A company has produced x items and the total cost C and total revenue R are given by the equations C = 100 + $0.015x^2$ and R = 3x. Find how many items should be produced to maximize the profit. What is this profit?

21. The two lines of regression are x + 2y - 5 = 0 and 2x + 3y - 8 = 0 and the variance of x is 12. Find the variance of y and the coefficient of correlation. **[4]**

22. Find the linear constraints for which the shaded area in the figure below is the solution set **[4]**



OR

A resourceful home decorator manufactures two types of lamps say A and B. Both lamps go through two technicians, first a cutter, second a finisher. Lamp A requires 2 hours of the cutter's time and 1 hour of the finisher's time. Lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 104 hours and finisher has 76 hours of time available each month. Profit on one lamp A is Rs.6.00 and on one lamp B is Rs.11.00. Assuming that he can sell all that he produces. Develop a mathematical formulation to obtain the best return.

Solution

SECTION A - 65 MARKS

1. In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(a) **(b)** I

Explanation:

Given that $A^2 = A$

Calculating value of $(I - A)^3 + A$:

$(I - A)^3 + A = I^3 - 3I^2A + 3IA^2 - A^3 + A$

$= I - A^2A - 3A + 3A^2 + A$ ($\because I^n = I$ and $IA = A$)

$= I - AA - 3A + 3A + A$ ($\because A^2 = A$)

$= I + A^2 - 3A + 3A + A$

$= I$

Hence, $(I - A)^3 + A = I$

(b) **(a)** log |cosec x - cot x| + C

Explanation:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\sin(\alpha+b) = \sin\alpha \cos b + \cos\alpha \sin b$

$\int \cot x = \log(\sin x) + c$

Therefore ,

$\int cosecx \frac{\csc x - \cot x}{\csc x - \cot x} dx$

$\int \frac{cosec^2 x - \csc x \cot x}{cosecx - \cot x} dx$

Put cosecx - cot x = t $(cosec^2 x - cosec\, x \cot x)dx = dt$

$\Rightarrow \int \frac{dt}{t}$

= log t + c

= log |cosec x - cot x| + c

(c) **(c)** $\frac{x}{\sqrt{1-x^2}}$

Explanation:

$cos^{-1}x = \theta \Rightarrow x = \cos\theta$

$\Rightarrow \cos\theta = \frac{x}{1} = \frac{Base.}{Hyp.}$

$cot(cos^{-1}x) = \cot\theta = \frac{Base.}{Perp.} = \frac{x}{\sqrt{1-x^2}}.$

(d) **(d)** y sec x = tan x + c

Explanation:

We have, $\frac{dy}{dx}$ + y tan x = sec x

which is a linear differential equation

Here, P = tan x, Q = sec x,

$\therefore$ I.F = $e^{\int \tan x dx} = e^{\log \sec x} = \sec x$

$\therefore$ The general solution is

y sec x = $\int \sec x \cdot \sec x$ + C

$\Rightarrow$ y sec x = $\int \sec^2 x dx$ + C

$\Rightarrow$ y sec x = tan x + C

(e) **(a)** $\frac{1}{70}$

Explanation:

Here P(E) = 0.3, $P(E \cup F) = 0.5$

Let P(F) = x

$\therefore P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= P(E) + P(F) - P(E) \cdot P(F)$

$\Rightarrow$ 0.5 = 0.3 + x - 0.3x

$\Rightarrow x = \frac{0.5-0.3}{0.7} = \frac{2}{7} = P(F)$

$\therefore P(E/F) - P(F/E) = \frac{P(E\cap F)}{P(F)} - \frac{P(F\cap E)}{P(E)}$

$= \frac{P(E\cap F)\cdot P(E) - P(F\cap E)\cdot P(F)}{P(E)\cdot P(F)}$

$= \frac{P(E\cap F)[P(E)-P(F)]}{P(E\cap F)} = P(E) - P(F)$

$= \frac{3}{10} - \frac{2}{7} = \frac{21-20}{70} = \frac{1}{70}$

(f) **(d)** Injective function

Explanation:

f is injective since every element in set B has atmost one pre-image in set A.

(g) **(d)** f ‘ (x) = g ‘ (x)

Explanation:

$g(x) = \tan^{-1}\left(\frac{1+x}{1-x}\right) \Rightarrow g'(x) = \frac{1}{1+\left(\frac{1+x}{1-x}\right)^2} \frac{(1-x).1-(1+x).(-1)}{(1-x)^2} = \frac{1}{(1+x^2)}$

(h) **(a)** 2

Explanation:

LHL = $\lim\limits_{h\to 0}$ f(0 - h) = $\lim\limits_{h\to 0} \frac{\sin 5(0-h)}{(0-h)^2+2(0-h)}$

= $-\lim\limits_{h\to 0} \frac{\frac{\sin 5h}{5h}}{\frac{1}{5}(h-2)} = \frac{5}{2} \quad \left[\because \lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right]$

Since, it is continuous at x = 0.

Therefore, LHL = f(0).

$\Rightarrow \frac{5}{2} = k + \frac{1}{2}$

$\Rightarrow$ k = 2

(i) **(c)** Determinant is a number associated to a square matrix.

Explanation:

The determinant is an operation that we perform on arranged numbers. A square matrix is a set of arranged numbers. We perform some operations on a matrix and we get a value that value is called as a determinant of that matrix hence a determinant is a number associated to a square matrix.

(j) **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation:

Assertion: We have, A = IA

i.e., $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{5} \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & 0 \\ 0 & 1 \end{bmatrix} A$ [applying $R_1 \to \frac{1}{10} R_1$]

$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{5} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$ [applying $R_2 \to R_2 + 5R$]

We have all zeroes in the second row of the left hand side matrix of above equation. Therefore, A^{-1} does not exist.

Reason: The given matrix equation is $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

$\because$ The column transformation $C_2 \to C_2 - 2C_1$ is applied.

$\therefore$ This transformation is applied on LHS and on second matrix of RHS.

Thus, we have $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$.

(k) f = {(1, a), (2, b), (3, c)}, then it is clear that f is 1 - 1 and onto and therefore f^{-1} exists.

Also, f^{-1} = {(1, a), (b, 2), (c, 3)} and $(f^{-1})^{-1}$ = {(1, a), (2, b), (3, c)} = f

Hence, $(f^{-1})^{-1}$ = f

(l) $\begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$

(m)We are given that, $A = \{x \in R : -4 \leq x \leq 4 \text{ and } x \neq 0\}$

f : A $\rightarrow$ R defined by $f(x) = \frac{|x|}{x}$

Clearly $f(x) = \begin{cases} 1 & ; x > 0 \\ -1; & x < 0 \end{cases}$

Therefore,Range of f = {-1, 1}

(n) P(A) = $\frac{7}{13}$, P(B) = $\frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$

Therefore, required probability is given by,

$P\left(\bar{A}/B\right) = \frac{P\left(\bar{A} \cap B\right)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$= \frac{\frac{9}{13} - \frac{4}{13}}{\frac{9}{13}} = \frac{\frac{5}{13}}{\frac{9}{13}} = \frac{5}{9}$

(o) A and B be the events such that P(A) $= \frac{5}{11}, P(B) = \frac{6}{11}$ and

$\mathrm{P(A \cap B) = P(A) + P(B) - P(A \cup B)}$

$= \frac{5}{11} + \frac{6}{11} - \frac{7}{11} = \frac{4}{11}$

$\mathrm{P(B/A)} = \frac{P(A \cap B)}{P(A)}$

$= \frac{4}{11} \div \frac{5}{11} = \frac{4}{5}$

2. We have, $(x + y)^2 = 2axy$

Differentiating both sides with respect to x, we get

$\Rightarrow \frac{d}{dx}(x+y)^2 = \frac{d}{dx}(2axy)$

$\Rightarrow 2(x+y)\frac{d}{dx}(x+y) = 2a\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right]$ [using chain rule and product rule]

$\Rightarrow 2(x+y)\left[1 + \frac{dy}{dx}\right] = 2a\left[x\frac{dy}{dx} + y(1)\right]$

$\Rightarrow 2(x+y) + 2(x+y)\frac{dy}{dx} = 2ax\frac{dy}{dx} + 2ay$

$\Rightarrow \frac{dy}{dx}$[2(x + y) - 2ax] = 2ay - 2(x + y)

$\Rightarrow \frac{dy}{dx} = \frac{2[ay-x-y]}{2[x+y-ax]}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{ay-x-y}{x+y-ax}\right)$

OR

Given: $f(x) = x^3 - 12x^2 + 36x + 17$

$\Rightarrow f(x) = \frac{d}{dx}\left(x^3 - 12x^2 + 36x + 17\right)$

$\Rightarrow f'(x) = 3x^2 - 24x + 36$

For f(x) lets find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 3x^2 - 24x + 36 = 0$

$\Rightarrow 3(x^2 - 8x + 12) = 0$

$\Rightarrow 3(x^2 - 6x - 2x + 12) = 0$

$\Rightarrow x^2 - 6x - 2x + 12 = 0$

$\Rightarrow (x - 6)(x - 2) = 0$

$\Rightarrow$ x = 2, 6

clearly, f'(x) > 0 if x < 2 and x > 6

and f'(x) < 0 if 2 < x < 6

Thus, the given function f(x) increases on $(-\infty, 2) \cup (6, \infty)$ and f(x) is decreasing on interval $x \in (2, 6)$.

3. $\int e^x \left(\frac{\cos x + \sin x}{\cos^2 x}\right) dx$

$= \int e^x(\sec x + \sec x \tan x)\, dx$

$= \int e^x \sec x\, dx \int e^x \sec x, \tan x\, dx$

Taking $f_1(x) = \sec x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact, Using intehration by parts,

$\int e^x \sec x dx + \int e^x \sec x \tan x\, dx$

$= \sec x \int e^x dx - \int \left[\frac{d}{dx}(\sec x) \int e^x dx\right] dx + \int e^x \sec x \tan x dx$

= e^x sec x-$\int$ e^x sec x tan x dx +$\int$$e^x$sec x tan x dx + c

= e^x sec x+c , where c is the integrating constant

4. Let the edge of cube be x cm

$\therefore$ $\frac{dx}{dt}$ = 10 cm/s (given)

Volume of cube (V) = x^3

$\frac{dV}{dt} = \frac{d}{dt}x^3 = 3x^2\frac{dx}{dt}$

$\left(\frac{dV}{dt}\right)_{x=5}$ = $3(5)^2$ (10)

= 750 cm^3/sec.

$\therefore$ Volume of cube increasing by 750 cm^3/s when edge of cube is 5 cm.

5. Putting log x = t

$\Rightarrow \frac{dx}{x} = dt$

$\therefore \int \frac{(\log x)^2}{x}dx = \int t^2 dt$

$= \frac{t^3}{3} + c$

$= \frac{1}{3}(\log x)^3 + c$

OR

We have,

$\int \frac{x^3-x^2+x-1}{x-1}dx$ = $\int \frac{[x^2(x-1)+1(x-1)]}{x-1}dx$

= $\frac{(x-1)(x^2+1)}{(x-1)}dx$

= $\int (x^2+1) dx$ = $\int x^2 dx + \int dx$

= $\frac{x^3}{3} + x + C$

6. The relation R on the set A = {x $\in$ Z : 0 $\leq$ x $\leq$ 12}, is given by

R = {(a, b) : |a – b| is a multiple of 4}

For any element a $\in$ A, we have (a,a) $\in$ R as |a-a|=0 is a multiple of 4.

Therefore, R is reflexive.

Now, Let (a,b) $\in$ R

$\Rightarrow$ |a – b| is a multiple of 4

$\Rightarrow$ |b – a| = |a – b| is a multiple of 4

$\Rightarrow$ (b,a) $\in$ R

Therefore, R is symmetric.

Finally, Let (a,b), (b,c) $\in$ R

$\Rightarrow$ |a – b| is a multiple of 4 and |b - c| is a multiple of 4

$\Rightarrow$ $a - b$ is a multiple of 4 and $b - c$ is a multiple of 4

$\Rightarrow$ $a - c = a - b + b - c$, is a multiple of 4

$\Rightarrow$ $|a - c|$ is a multiple of 4

$\Rightarrow$ (a,c) $\in$ R

Therefore, R is transitive.

Hence, R is an equivalence relation.

The set of elements related to 1 is {1,5,9}

|1-1| = 0 is multiple of 4

|5-1| = 4 is multiple of 4

|9-1| = 8 is multiple of 4.

7. **To Prove -** $\tan^{-1} y + \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{3y-y^3}{1-3y^2}\right)$

LHS = $\tan^{-1} y + \tan^{-1}\left(\frac{2y}{1-y^2}\right)$

$[tan^{-1}x + tan^{-1}y = tan^{-1}\frac{x+y}{1-xy}]$

$= \tan^{-1}\left[\frac{y+\frac{2y}{1-y^2}}{1-y\left(\frac{2y}{1-y^2}\right)}\right]$

$= \tan^{-1}\left(\frac{y-y^3+2y}{1-y^2-2y^2}\right)$

$= \tan^{-1}\left(\frac{3y-y^3}{1-3y^2}\right)$

= **RHS**

8. Formula to be used $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$\therefore \int \frac{(3-2x)}{\sqrt{2+x-x^2}} dx$

$= \int \frac{(1-2x)+2}{\sqrt{2+x-x^2}} dx$

$= \int \frac{(1-2\mathrm{x})}{\sqrt{2+\mathrm{x}-\mathrm{x}^2}} \mathrm{dx} + \int \frac{2}{\sqrt{2+x-x^2}} dx$

Now for 1st integral Assuming 2 + x - x^2 = a^2, (1 - 2x)dx = 2ada

$\therefore \int \frac{(1-2x)}{\sqrt{2+x-x^2}} dx$

$= \int \frac{2ada}{a}$

= 2a + c_1

$= 2\sqrt{2+x-x^2} + c_1$

For 2nd integral.

$\therefore \int \frac{2}{\sqrt{2+x-x^2}} dx$

$= 2\int \frac{dx}{\sqrt{\left(\frac{3}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2}}$

$= 2\sin^{-1}\frac{\left(x-\frac{1}{2}\right)}{\left(\frac{3}{2}\right)} + c_2$

$= 2\sin^{-1}\left(\frac{2x-1}{3}\right) + c_2$

$\therefore \int \frac{(1-2x)}{\sqrt{2+x-x^2}} dx + \int \frac{2}{\sqrt{2+x-x^2}} dx$

$= 2\sqrt{2+x-x^2} + 2\sin^{-1}\left(\frac{2x-1}{3}\right) + c$, c is the integrating constant

9. Given $x = \tan\left(\frac{1}{a}\log y\right)$

$\Rightarrow \frac{1}{a}\log y$ = $\tan^{-1}$ x

Differentiating w.r.t x, we get

$\frac{1}{y} \cdot \frac{dy}{dx} = a \cdot \frac{1}{1+x^2}$

$\Rightarrow \left(1+x^2\right)\frac{dy}{dx} = a \cdot y$

Differentiating again w.r.t. x, we get

$\left(1+x^2\right)\frac{d^2y}{dx^2} + 2x \cdot \frac{dy}{dx} = a\frac{dy}{dx}$

$\left(1+x^2\right)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$

Hence Proved

OR

Given function is:

⇒ y = 12(x + 1)(x – 2) on [– 1,2]

We know that polynomials are continuous and differentiable over R

check the values of y at the extremums

⇒ y(– 1) = 12(– 1 + 1)(– 1 – 2)

⇒ y(– 1) = 12(0)(– 3)

⇒ y(– 1) = 0

⇒ y(2) = 12(2 + 1)(2 – 2)

⇒ y(2) = 12(3)(0)

⇒ y(2) = 0

We got y(– 1) = y(2). So, there exists a c such that f’(c) = 0.

For a curve g to have a tangent parallel to the x – axis at point r, the criteria to be satisfied is g’(r) = 0.

⇒ y’(x) = 0

$\Rightarrow \frac{d(12(x+1)(x-2))}{dx} = 0$

$\Rightarrow 12\left((x+1)\frac{d(x-2)}{dx}+(x-2)\frac{d(x+1)}{dx}\right)=0$

$\Rightarrow$ ((x + 1)×1) + ((x – 2)×1) = 0

$\Rightarrow$ x + 1 + x – 2 = 0

$\Rightarrow$ 2x – 1 = 0

$\Rightarrow$ 2x = 1

$\Rightarrow x=\frac{1}{2}$

The value of y is

$\Rightarrow y=12\left(\frac{1}{2}+1\right)\left(\frac{1}{2}-2\right)$

$\Rightarrow y=12\left(\frac{3}{2}\right)\left(-\frac{3}{2}\right)$

$\Rightarrow$ y = – 27

The point at which the curve has tangent parallel to x – axis is $\left(\frac{1}{2},-27\right)$

10. **Read the text carefully and answer the questions:**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

(a) Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$, P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$

The probability that the selected student has failed in Economics if it is known that he has failed in Mathematics.

Required probability = $P(\frac{E}{M})$

= $\frac{P(E\cap M)}{P(M)}=\frac{\frac{1}{4}}{\frac{7}{20}}=\frac{1}{4}\times\frac{20}{7}=\frac{5}{7}$

(b) Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$, P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$

The probability that the selected student has failed in Mathematics if it is known that he has failed in Economics.

Required probability = P(M/E)

$=\frac{P(M\cap E)}{P(E)}=\frac{\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}$

(c) Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$, P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$

The probability that the selected student has passed in Mathematics if it is known that he has failed in Economics

Required probability = P(M'/E)

$\Rightarrow P\left(M'/E\right)=\frac{P(M'\cap E)}{P(E)}$

= $\frac{P(E)-P(E\cap M)}{P(E)}$

= $\frac{\frac{1}{2}-\frac{1}{4}}{\frac{1}{2}}$

$\Rightarrow P\left(M'/E\right)=\frac{1}{2}$

(d) Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E)=\frac{50}{100}=\frac{1}{2}$, P(M) = $\frac{35}{100}=\frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100}=\frac{1}{4}$

The probability that the selected student has passed in Economics if it is known that he has failed in Mathematics

Required probability = P(E'/M)

$\Rightarrow P(E'/M) = \frac{P(E' \cap M)}{P(M)}$

$= \frac{P(M) - P(E \cap M)}{P(M)}$

$= \frac{\frac{7}{20} - \frac{1}{4}}{\frac{7}{20}}$

$\Rightarrow P(E'/M) = \frac{2}{7}$

OR

Read the text carefully and answer the questions:

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

(a) Bayes' theorem defines the probability of an event based on the prior knowledge of the conditions related to the event whereas in case of the condition probability, we find the reverse probabilities using Bayes' theorem.

(b) Consider on event E which occurs via two different events A and B. The probability of E is given by the value of total probability as:

P(E) = P(A ∩ E) + P(B ∩ E)

P(E) = P(A) P$\left(\frac{E}{A}\right)$ + P(B)$\left(\frac{E}{B}\right)$

(c) Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

E_1 = PQ, E_2 = $\bar{P}\bar{Q}$, E_3 = $\bar{P}$Q, E_4 = P$\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

P(E_1) = 0.3 × 0.2 = 0.06,

P(E_2) = 0.7 × 0.8 = 0.56,

P(E_3) = 0.7 × 0.2 = 0.14,

P(E_4) = 0.3 × 0.8 = 0.24

P$\left(\frac{E}{E_1}\right)$ = 0, P$\left(\frac{E}{E_2}\right)$+0, P$\left(\frac{E}{E_3}\right)$ = 1, P$\left(\frac{E}{E_4}\right)$ = 1

P(E) = P(E_1) · P$\left(\frac{E}{E_1}\right)$ + P(E_2) · P$\left(\frac{E}{E_2}\right)$ + P(E_3).

P$\left(\frac{E}{E_3}\right)$ + P(E_4) · P$\left(\frac{E}{E_4}\right)$

= 0.14 + 0.24 = 0.38

(d) By Bayes' Theorem,

$$P\left(\frac{E_3}{E}\right) = \frac{P(E_3) \cdot P\left(\frac{E}{E_3}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) + P(E_3) P\left(\frac{E}{E_3}\right) + P(E_4) \cdot P\left(\frac{E}{E_4}\right)}$$

$= \frac{0.14}{0.38} = \frac{7}{19}$

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses E_1 and E_2 are actually eliminated as P$\left(\frac{E}{E_1}\right)$ = P$\left(\frac{E}{E_2}\right)$ = 0

Alternative way of writing the solution:

i. P(Shell fired from exactly one of them hits the plane)

= P[(Shell from A hits the plane and Shell from B does not hit the plane) or (Shell from A does not hit the plane and Shell from B hits the plane)]

= 0.3 × 0.8 + 0.7 × 0.2 = 0.38

ii. $\frac{\text{P(Shell fired from B hit the plane} \cap \text{Exactly one of them hit the plane)}}{\text{P(Exactly one of them hit the plane)}}$

$= \frac{P\text{ (Shell from only } B \text{ hit the plane)}}{P\text{ (Exactly one of them hit the plane)}}$

$= \frac{0.14}{0.38} = \frac{7}{19}$

11. **Read the text carefully and answer the questions:**

Two farmers Shyam and Balwan Singh cultivate only three varieties of pulses namely Urad, Masoor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.

September sales (in ₹)

$$A = \begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

October sales (in ₹)

$$B = \begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

(a) Combined sales in September and October for each farmer in each variety is given by

$$A + B = \begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

Combined sales of Masoor in September and October for farmer Balwan Singh = ₹ 40000

(b) Combined sales in September and October for each farmer in each variety is given by

$$A + B = \begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

Combined sales of Urad in September and October for farmer Shyam = ₹ 15000

(c) Change in sales from September to October is given by

$$A - B = \begin{array}{c} \begin{array}{ccc} \text{Urad} & \text{Masoor} & \text{Mung} \end{array} \\ \begin{bmatrix} 5000 & 10000 & 24000 \\ 30000 & 20000 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

$\therefore$ Decrease in sales of Mung from September to October for farmer Shyam = ₹ 24000.

12. $\frac{dy}{dx} = -\frac{(xy+y^2)}{x^2} = \frac{-y}{x} - \left(\frac{y}{x}\right)^2$

Put y = vx

$\frac{dy}{dx} = v + x\frac{du}{dx}$

$\therefore v + x\frac{dv}{dx} = -v - v^2$

$x\frac{dv}{dx} = -2v - v^2$

$\int \frac{dv}{(v^2+2v')} = \int -\frac{dx}{x}$

$\frac{1}{2}\log\left|\frac{v}{v+2}\right| = -\log x + \log c$

$\frac{v}{(v+2)} = \frac{c^2}{x^2}$

$\frac{y}{(y+2x)} = \frac{c^2}{x^2}$

Given when x = 1, y = 1

$\therefore c = \frac{1}{\sqrt{3}}$

Hence, solution is $3x^2 y = y + 2x$

OR

We have

$$\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$$

$$\left(1+e^{x/y}\right)dx=-e^{x/y}\left(1-\frac{x}{y}\right)dy$$

$$\frac{dy}{dx}=\frac{-e^{x/y}\left(1-\frac{x}{y}\right)}{\left(1+e^{x/y}\right)}$$

Let $f(x,y)=\frac{-e^{x/y}\left(1-\frac{x}{y}\right)}{\left(1+e^{x/y}\right)}$

Here, putting x = kx and y = ky

$$f(kx,ky)=\frac{-e^{kx/ky}\left(1-\frac{kx}{ky}\right)}{\left(1+e^{kx/ky}\right)}$$

$$=\frac{-e^{x/y}\left(1-\frac{x}{y}\right)}{\left(1+e^{x/y}\right)}$$

= $k^0 f(x,y)$

Therefore, the given differential equation is homogeneous.

$$\left(1+e^{x/y}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$

$$\left(1+e^{x/y}\right)dx=-e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy$$

$$\frac{dx}{dy}=\frac{-e^{x/y}\left(1-\frac{x}{y}\right)}{\left(1+e^{x/y}\right)}$$

To solve it we make the substitution.

x = vy

Differentiation above equation with respect to x, we get

$$\frac{dx}{dy}=v+y\frac{dv}{dy}$$

$$v+y\frac{dv}{dy}=\frac{-e^{vy/y}\left(1-\frac{vy}{y}\right)}{(1+ev/y)}$$

$$y\frac{dv}{dy}=\frac{-e^{v}(1-v)}{1+e^{v}}$$

$$\frac{1+e^{v}}{e^{v}+v}dv=-\frac{1}{y}dy$$

Integrating both sides, we get

$\int\frac{1+e^{v}}{e^{v}+v}dv=\int\frac{1}{y}dy$ ......(i)

Let $I_1=\int\frac{1+e^{v}}{e^{v}+v}dv$

Put $e^v + v = t$

$(e^v + 1)dv = dt$

$$e^{v}+1=\frac{dt}{dv}$$

$$dv=\frac{dt}{e^{v}+1}$$

$$\int\frac{1}{t}dt$$

log t

$\log(e^v + v)$

$\therefore \log(e^v + v) = -\log y + \log C$ ($\therefore$ From (i) eq.)

$$\log\left(e^{x/y}+\frac{x}{y}\right)=-\log y+\log C$$

$$\log\left(e^{x/y}+\frac{x}{y}\right)=\log\frac{C}{y}$$

$$e^{x/y}+\frac{x}{y}=\frac{C}{y}$$

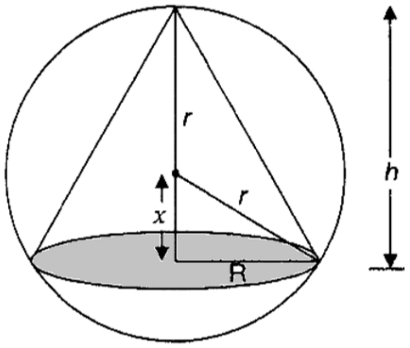
Multiply by y on both side, we get

$ye^{x/y} + x = C$

$x + ye^{x/y} = C$

The required solution of the differential equation.

13. Let R be the radius of base and h be the height of the cone, let x be the distance between the centre of sphere and centre of base of cone.



$\therefore$ h = r + x

and $R^2 = r^2 - x^2$

Volume of cone V(x) = $\frac{1}{3}\pi R^2 h$

$= \frac{1}{3}\pi\left(r^2 - x^2\right)(r + x)$

$\therefore$ V'(x) = $\frac{1}{3}\pi$ {-2x (r + x) + ($r^2 - x^2$)}

= $\frac{1}{3}\pi$(r + x) (-2x + r - x)

= $\frac{1}{3}\pi$(r + x) (r - 3x)

For max. and min. volume

V'(x) = 0

$\Rightarrow$ (r + x)(r - 3x) = 0

$\Rightarrow$ r = -x (not possible) or r = 3x

$\Rightarrow x = \frac{r}{3}$

Volume of cone V(x) = $\frac{1}{3}\pi R^2 h$

Now, V''(x) = $\frac{1}{3}\pi$ {(r - 3x) -3 (r + x)}

at x = $\frac{r}{3}$, $V''\left(\frac{r}{3}\right) < 0$

$\therefore$ Volume is maximum when x = $\frac{r}{3}$

$\therefore$ h = r + x = r = $\frac{r}{3}$ = $\frac{4r}{3}$

i.e., height of cone is $\frac{4r}{3}$

OR

Given equation of curve is, y = x^2 + 7x + 2 and equation of straight line is y=3x - 3.

Let P(x,y) be any point on the parabola y=x^2+7x+2

Let D be the distance of point P from straight line

then

$D = \frac{|3x-y-3|}{\sqrt{3^2+(-1)^2}} = \frac{|3x-y-3|}{\sqrt{9+1}}$

$= \frac{|3x-y-3|}{\sqrt{10}}$

$= \frac{\left|3x-\left(x^2+7x+2\right)-3\right|}{\sqrt{10}}$

$= \frac{\left|-\left(x^2+4x+5\right)\right|}{\sqrt{10}}$

$= \frac{x^2+2\cdot 2x+2^2+1}{\sqrt{10}}$

$\Rightarrow \quad D = \frac{(x+2)^2+1}{\sqrt{10}}$ .....(i)

On differentiating both sides of Eq. (i) w.r.t. x, we get

$\frac{dD}{dx} = \frac{2(x+2)+0}{\sqrt{10}}$

For Extremum value of D, put $\frac{dD}{dx} = 0$

$\Rightarrow$ 2(x+2)=0

$\Rightarrow$x= -2

Now, $\frac{d^2D}{dx^2} = \frac{2}{\sqrt{10}} > 0$

Thus, D is minimum when x= -2

Now, y = x^2 + 7x + 2= $(-2)^2$ + 7 (- 2) + 2

$=4 - 14 + 2 = - 8$

Hence, point (-2, -8) is on the prabola, which is closet to the given straight line.

14. **Read the text carefully and answer the questions:**

Shama is studying in class XII. She wants do graduate in chemical engineering. Her main subjects are mathematics, physics, and chemistry. In the examination, her probabilities of getting grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(a) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in all subjects) = P(M $\cap P \cap$ C)

= P(M) $\times$ P(P) $\times$ P(C)

= 0.2 $\times$ 0.3 $\times$ 0.5 = 0.03

(b) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in on subjects) = $P(\overline{M} \cap \overline{P} \cap \overline{C})$

= $P(\overline{M}) \times P(\overline{P}) \times P(\overline{C})$

= 0.8 $\times$ 0.7 $\times$ 0.5 = 0.280

(c) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in 2 subjects)

$\Rightarrow$ P(grade A in M and P not in C) + P(grade A in P & C not in M) + P(grade A in M & C not in P)

$\Rightarrow$ P$\left(M \cap P \cap \overline{C}\right)$ + P$\left(P \cap C \cap \overline{M}\right)$ + P$(M \cap C \cap \overline{P})$

$\Rightarrow$ 0.2 $\times 0.3 \times$ 0.5 + 0.3 $\times 0.5 \times$ 0.8 + 0.2 $\times 0.5 \times$ 0.7 = 0.03 + 0.12 + 0.07

P(getting grade A in 2 subjects) = 0.22

(d) P (Grade A in Maths) = P(M) = 0.2

P (Grade A in Physics) = P(P) = 0.3

P (Grade A in Chemistry) = P(C) = 0.5

P(not A garde in Maths) = P($\overline{M}$) = 1 - 0.2 = 0.8

P(not A garde in Physics) = P($\overline{P}$) = 1 - 0.3 = 0.7

P(not A garde in Chemistry) = P($\overline{C}$) = 1 - 0.5 = 0.5

P(getting grade A in 1 subjects)

$\Rightarrow$ P(grade A in M not in P and C) + P(grade A in P not in M and C) + P(grade A in C not in P and M)

$\Rightarrow$ P$\left(M \cap \overline{P} \cap \overline{C}\right)$ + P$\left(P \cap \overline{C} \cap \overline{M}\right)$ + P$(C \cap \overline{M} \cap \overline{P})$

$\Rightarrow$ 0.2 $\times 0.7 \times$ 0.5 + 0.3 $\times 0.5 \times$ 0.8 + 0.5 $\times 0.8 \times$ 0.7 = 0.07 + 0.12 + 0.028

P(getting grade A in 1 subjects) = 0.47

SECTION B - 15 MARKS

15. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(a) **(b)** $\vec{a} \perp \vec{b}$

Explanation:

Here $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$

$\Rightarrow |\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2$

$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})=(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$

$\Rightarrow |a|^2+2\vec{a}\cdot\vec{b}+|b|^2 = |a|^2-2\vec{a}\cdot\vec{b}+|b|^2$

$\Rightarrow \quad 2\vec{a}\cdot\vec{b}=-2\vec{a}\cdot\vec{b}$

$\Rightarrow 4\vec{a}\cdot\vec{b}=0$

$\Rightarrow \quad \vec{a}\cdot\vec{b}=0$

$\Rightarrow \quad \vec{a}\perp\vec{b}$

(b) Any plane parallel to YZ plane is x = k

since it passes through (-3, 2, 0) we have -3 = k

Hence the required equation of the plane is x = -3

(c) In order to find the two unknowns, we evaluate and simplify the cross product.

Given: $(\hat{i}+3\hat{j}+9\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k})=\vec{0}$

$$\therefore \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 9 \\ 3 & -\lambda & \mu \end{vmatrix}=\vec{0}$$

$\Rightarrow \hat{i}(3\mu+9\lambda)-\hat{j}(\mu-27)+\hat{k}(-\lambda-9)=0\hat{i}+0\hat{j}+0\hat{k}$

On comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$,we get

$3\mu+9\lambda=0, -\mu+27=0$ and $-\lambda-9=0$

$\Rightarrow \quad \mu=27$ and $-\lambda=9$

$\Rightarrow \quad \mu=27$ and $\lambda=-9$

(d) **(c)** (3, 5, 7)

Explanation:

Let $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda(\text{ say })$

A general point on this line is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$.

For some value of λ, let the given line meet the plane 2x + 3y - z = 14 at a point $P(2\lambda+1, 3\lambda+2, 4\lambda+3)$.

Then, $2(2\lambda+1)+3(3\lambda+2)-(4\lambda+3)=14$

$\Rightarrow 9\lambda=9 \Rightarrow \lambda=1$

So, the required point is P(2 + 1, 3 + 2, 4 + 3),

i.e., P(3, 5, 7).

(e) We have,equation of plane is x - 2y + 2z = 6

Here direction ratios normal to the plane are 1, -2, 2

Thus,A vector normal to the plane $\vec{n}=\hat{i}-2\hat{j}+2\hat{k}$

$\Rightarrow |n|=\sqrt{1^2+(-2)^2+2^2}=3$

Now, $\hat{n}=\frac{\bar{n}}{|\bar{n}|}=\frac{1}{3}\hat{i}-\frac{2}{3}j+\frac{2}{3}\hat{k}$.

16. Here,it is given

$\bar{a}=\frac{1}{7}(2\hat{i}+3\hat{j}+6\bar{k})$

$\bar{b}=\frac{1}{7}(3\hat{i}-6\hat{j}+2\bar{k})$

$\vec{c}=\frac{1}{7}(6\hat{i}+2\hat{j}-3\vec{k})$

$\bar{a}\cdot\bar{b}=\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k})\times\frac{1}{7}(3\hat{i}-6\hat{j}+2k)$

$=\frac{1}{49}(6-18+12)=0$

similarly,

$\bar{b}\cdot\vec{c}=\bar{a}\cdot\vec{c}=0$

Therefore a, b, c, are mutually perpendicular.

OR

$\vec{a}=\hat{i}-2\hat{j}-2\hat{k}$

$|\vec{a}|=\left|\sqrt{1^2+(-2)^2+(-2)^2}\right|$

= $|\sqrt{1+4+4}|$ = 3

So, its Unit Vector, $\vec{a}=\frac{\vec{a}}{|\vec{a}|}=\frac{\hat{i}-2\hat{j}-2\hat{k}}{3}$

$18\vec{a}=18\times\frac{\hat{i}-2\hat{j}-2\hat{k}}{3}$

= $6(\hat{i}-2\hat{j}-2\hat{k})$

Required vector = $6\hat{i}-12\hat{j}-12\hat{k}$

17. If vector form of lines are $\vec{r}=\overrightarrow{a_1}+\lambda\overrightarrow{b_1}$ and $\vec{r}=\overrightarrow{a_2}+\lambda\overrightarrow{b_2}$ then angle between them is

$\cos\theta=\frac{\left|\overrightarrow{b_1}.\overrightarrow{b_2}\right|}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}$

Given equations of lines are

$\vec{r}=(2\hat{i}-5\hat{j}+\hat{k})+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ ......(i)

and $\vec{r}=(7\hat{i}-6\hat{j}-6\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$ ....(ii)

On comparing Equations , (i) and (ii) with vector form of equation of line i.e.

$\vec{r}=\vec{a}+\lambda\vec{b},$ we get,

$\overrightarrow{a_1}=2\hat{i}-5\hat{j}+\hat{k},\overrightarrow{b_1}=\hat{3}\hat{i}+2\hat{j}+6\hat{k}$

and $\overrightarrow{a_2}=\hat{7}\hat{i}-6\hat{j}-6\hat{k},\vec{b}_2=\hat{i}+2\hat{j}+2\hat{k}$

We know that, the angle between two lines is given by

$\cos\theta=\frac{\left|\overrightarrow{b_1}\cdot\overrightarrow{b_2}\right|}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}$

$\cos\theta=\frac{(3\hat{i}+2\hat{j}+6\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{(3)^2+(2)^2+(6)^2}\cdot\sqrt{(1)^2+(2)^2+(2)^2}}$

$\Rightarrow\cos\theta=\left|\frac{4+12}{3\sqrt{49}\times\sqrt{9}}\right|$

$\Rightarrow\cos\theta=\left|\frac{19}{7\times3}\right|\Rightarrow\cos\theta=\frac{19}{21}$

Hence, the angle between given two lies is

$\theta=\cos^{-1}\left(\frac{19}{21}\right)$

OR

Clearly, we have to find the direction cosines of the normal to the given plane.

The given equation may be written as

$\vec{r}\cdot(6\hat{i}-3\hat{j}-2\hat{k})=-3\Rightarrow\vec{r}\cdot(-6\hat{i}+3\hat{j}+2\hat{k})=3$

$\Rightarrow\vec{r}\cdot\vec{n}=3$, where $\vec{n}=(-6\hat{i}+3\hat{j}+2\hat{k})$

$\Rightarrow\vec{r}\cdot\frac{\vec{n}}{|\vec{n}|}=\frac{3}{|\vec{n}|}$ where $|\vec{n}|=\sqrt{(-6)^2+3^2+2^2}=7$

$\Rightarrow\vec{r}\cdot\frac{(-6\hat{i}+3\hat{j}+2\hat{k})}{7}=\frac{3}{7}\Rightarrow\vec{r}\cdot\left(-\frac{6}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}\right)=\frac{3}{7}$

Hence, the direction cosines of the normal to the given plane are

$\left(-\frac{6}{7},\frac{3}{7},\frac{2}{7}\right).$

18. Equation of plane that contains the point $A(2,1,-1)$

$a(x-2)+b(y-1)+c(z+1)=0$

Given that plane is perpendicular to the line of intersection of $2x+y-z=3$

and $x+2y+z=2$

$\Rightarrow 2.a+b.1+c.(-1)=02a+b-c=0$ ...(1)

$1.a+2.b+1.c=0a+2b+c=0$ ...(2)

Adding $(1)+(2)$

$3a+3b=0\rightarrow a=-b$

Substituting $a=-b$ in (1)

$2a-a-c=0\Rightarrow a=c$

$a=-b=c=k$

Equation of plane $=k(x-2)-k(y-1)+k(z+1)=0$

$\Rightarrow x-y+z=0$

Angle between line and plane is given by

$\sin\theta=\frac{1a+mb+nc}{\sqrt{l^2+m^2+n^2}\sqrt{a^2+b^2+c^2}}$

Angle between y -axis(D.R's $(0,1,0)$) and $x-y+z=0$ is

$\sin\theta=\left|\frac{0-1+0}{\sqrt{1^2+1^2+1^2}\sqrt{1}}\right|$

$=\left|\frac{-1}{\sqrt{3}}\right|$

$\theta=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

SECTION C - 15 MARKS

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(a) **(d)** Both P(X) and R(X) and R(X) and C(X)

Explanation:

Both P(X) and R(X) and R(X) and C(X)

(b) **(c)** no feasible solution

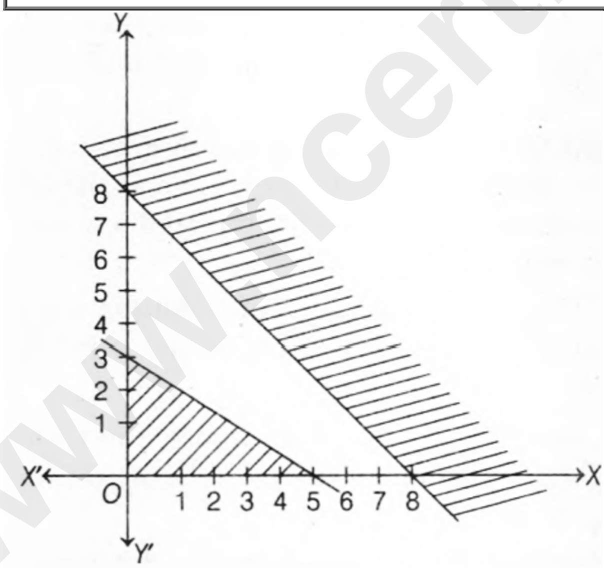
Explanation:

Table for equation x + y = 8 is

x	0	8
y = 8 - x	8	0

Table for equation 3x + 5y = 15 is

x	0	5
y = $\frac{15-3x}{5}$	3	0



It can be concluded from the graph, that there is no point, which can satisfy all the constraints simultaneously. Therefore, the problem has no feasible solution.

(c) 4x - 2y - 4 = 0

2x - 3y + 6 = 0

$\frac{x}{-12-12}=\frac{y}{-8-24}-\frac{1}{12+4}$

$\frac{x}{24}=\frac{y}{-32}=\frac{1}{-8}$

x = $\frac{-24}{-8}=3$

y =$\frac{-32}{-8}=4$

$\therefore$ $\bar{x}$ = 3 and $\bar{y}$ = 4

(d) Given R = 2q and C = 500 + $\frac{1}{2}\left(\frac{q}{20}\right)^2$

So, profit function P = R - C

$\Rightarrow$ P = 2q - 500 - $\left(\frac{q^2}{800}\right)$

$\therefore \frac{dp}{dq} = 2 - \frac{2q}{800} \Rightarrow \frac{dp}{dq} = 2 - \frac{q}{400}$

$\left[\frac{dP}{dq}\right]_{q=500} = 2 - \frac{500}{400} = 2 - 1.25 = 0.75$

(e) We have, P(x) = 41 + 24x -18x^2

$\Rightarrow \frac{d}{dx}$(P(x)) = 24 - 36x and $\frac{d^2}{dx^2}$(p(x)) = -36

For maximum or minimum, we must have

$\frac{d}{dx}$(P(x)) = 0 $\Rightarrow$ 24 - 36x = 0 $\Rightarrow$ x = $\frac{2}{3}$

Also,

$\left\{\frac{d^2}{dx^2}(P(x))\right\}_{x=2/3} = -36 < 0$

So profit is maximum when x = $\frac{2}{3}$

Maximum profit = (Value of P(x) at x = $\frac{2}{3}$) = 41 + 24 $\times$ ($\frac{2}{3}$) - 18 ($\frac{2}{3}$)^2 = 49

20. Given TFC = ₹ 10000, TVC = ₹ 50x

$\Rightarrow$ C(x) = 10000 + 50x

Also, P = ₹ 75 $\Rightarrow$ R(x) = 75x

At breakeven point: R(x) = C(x)

$\Rightarrow$ 75x = 10000 + 50x

$\Rightarrow$ 25x = 10000 $\Rightarrow$ x = 400

For profit: R(x) > C(x)

$\Rightarrow$ 75x > 10000 + 50x

$\Rightarrow$ x > 400

Hence, the breakeven value is 400 units. The company will always remain in profit if it produces and sells more than 400 units.

OR

Let P be the profit function. Then,

P = R - C = 3x - (100 + 0.15x^2) = 3x -100 - 0.015x^2

$\Rightarrow \frac{dP}{dx}$ = 3 - 0.03x and $\frac{d^2P}{dx^2}$ = -0.03

For P to be maximum, we must have

$\frac{dP}{dx}$ = 0 $\Rightarrow$ 3 - 0.03x = 0

$\Rightarrow$ x = 100

Also, $\frac{d^2P}{dx^2}$ = -0.030 < 0 for all x. Thus, P is maximum when x = 100.

Putting x = 100 in P = 3x - 100 - 0.015x^2, w e get: P = 300 - 100 - 0.015 $\times$ 10000 = 50

Hence, P is maximum when x = 100 and the maximum profit = ₹ 50

21. Given lines of regression are

x + 2y - 5 = 0

and 2x - 3y - 8 = 0

i.e., y = $-\frac{x}{2} + \frac{5}{2}$

and x = $-\frac{3}{2}y + \frac{8}{2}$

$\therefore$ b_{xy} = r $\frac{\sigma_y}{\sigma_x}$ = -0.5 (regression line of y on x)

and $b_{xy} = r\frac{\sigma_x}{\sigma_y}$ = -1.5 (regression line of x on y)

$r^2 = b_{xy} \times b_{yx}$ = (-1.5) $\times$ (-0.5)

= 0.75

$\therefore$ r = $\pm\sqrt{0.75}$ = $\pm$0.866

As b_{xy} and b_{yx} are both negative, r is also -ve.

$\therefore$ Correlation coefficient, r = -0.866

Given, σ_x^2 = 12

We have, $r\frac{\sigma_y}{\sigma_x}$ = -0.5

$\Rightarrow \sigma_y = \frac{(-0.5)\times\sigma_x}{r}$

$\sigma_y^2 = \frac{0.25\times\sigma_x^2}{r^2}$

= $\frac{0.25\times 12}{0.75}$ = 4

$\therefore$ variance of y = σ_y^2 = 4

22. Consider the line 3x + 4y = 18.

Clearly, 0(0, 0) satisfies 3x + 4y < 18 Clearly, the shaded area and (0, 0) lie on the same side of the line 3x + 4 y = 18.

Therefore, we must have 3x + 4y < 18

Consider the line x - 6y = 3

We note that (0, 0) satisfies the inequation x - 6y < 3 Also, the shaded area and (0, 0) lie on the same side of the line x - 6y = 3.

Therefore, we must have x - 6y < 3

Consider the line 2x + 3y = 3

Clearly, (0, 0) satisfies the inequation 2x + 3y < 3

But, the shaded region and the point (0, 0) lie on the opposite sides of the line 2x + 3y - 3 .

Clearly, (0, 0) satisfies the inequation -7x + 14y < 14 Also, the shaded region and the point (0, 0) lie on the same side of the line -7 x + 14y = 14.

Therefore, we must have -7 x + 14y < 14 The shaded region is above the x-axis and on the right-hand side of the y-axis, Therefore, we have y > 0 and x > 0.

Therefore, the linear constraints for which the shaded area in the given figure is the solution set, are

$3x + 4y \leq 18, x - 6y \leq 3, 2x + 3y \geq 3$

$-7x + 14y \leq 14, x \geq 0$ and $y \geq 0$

OR

The above information can be put in the following tabular form:

Lamp	**Cutter's time**	**Finisher's time**	**Profit in Rs.**
A	2	1	6
B	1	2	11
Maximum time available	**104**	**76**	

Let the decorator manufacture x lamps of type A and y lamps of type B.

$\therefore$ Total profit = Rs.(6x + 11y)

Total time taken by the cutter in preparing x lamps of type A and y lamps of type B is (2x + y) hours. But, the cutter has 104 hours only for each month.

$\therefore$ 2x + y $\leq$ 104

Similarly, the total time taken by the finisher in preparing x lamps of type A and y lamps of type B is (x + 2y) hours. But, the cutter has 76 hours only for each month.

$\therefore$ x + 2y $\leq$ 76

Since the number of lamps cannot be negative.

$\therefore$ x $\geq$ 0 and y $\geq$ 0

Let Z denote the total profit. Then, Z = 6x + 11y.

Since the profit is to be maximized. So, the mathematical formulation of the given LPP is as follows:

Maximize Z = 6x + 11y

Subject to

2x + y $\leq$ 104

x + 2y $\leq$ 76

and, x $\geq$ 0, y $\geq$ 0