ISC 2026 EXAMINATION
Sample Question Paper - 1
Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

This Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions*

EITHER *from* ***Section B OR Section C.***

Section A*: Internal choice has been provided in* ***two questions of two marks each, two questions of four marks each and two questions of six marks each.***

Section B: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

Section C: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

1. **In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.** **[15]**

(a) If |A| = 2, where A is a 2 × 2 matrix, then $|4A^{-1}|$ equals: **[1]**

a) 2　　b) 4

c) 8　　d) $\frac{1}{32}$

(b) $\int \frac{1}{(1+x^2)\sqrt{\tan^{-1} x}}$ **[1]**

a) $\frac{1}{2}\log|\tan^{-1} x| + C$　　b) $\frac{1}{2\sqrt{\tan^{-1} x}} + C$

c) $\frac{1}{4}\log|\tan^{-1} x| + C$　　d) $2\sqrt{\tan^{-1} x} + C$

(c) $\tan^{-1}3 - \tan^{-1}2 =$ **[1]**

a) $\tan^{-1}\left(\frac{1}{5}\right)$　　b) $\tan^{-1}\left(\frac{1}{7}\right)$

c) $\tan^{-1}\left(\frac{3}{2}\right)$　　d) $\tan^{-1}\left(\frac{2}{3}\right)$

(d) Find the degree of the differential equation: $\left(\frac{d^4y}{dx^4}\right)^3 - 5\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 5 = 0$ **[1]**

a) 5　　b) 3

c) 4　　d) 2

(e) A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the **[1]** probability of getting exactly one red ball is

a) $\frac{15}{56}$
b) $\frac{45}{196}$
c) $\frac{15}{29}$
d) $\frac{135}{392}$

(f) Let A = {1, 2, 3} and B = {2, 4, 6, 8}. Consider the rule f : A $\rightarrow$ B, defined f(x) = 2x, $\forall x \in$ A, then range of f is given by set **[1]**

a) {2, 4, 6}
b) {2, 4, 6, 8}
c) {1, 2, 3}
d) {6, 4, 8}

(g) The values of the constants a, b and c for which the function $f(x)=\begin{cases}(1+ax)^{1/x} & ,x<0\\ b & ,x=0\\ \frac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1} & ,x>0\end{cases}$ may be continuous at x = 0, are **[1]**

a) $a=\log_e\left(\frac{2}{3}\right), b=\frac{2}{3}, c=1$
b) $a=\log_e\left(\frac{2}{3}\right), b=-\frac{2}{3}, c=1$
c) $a=\log_e\left(\frac{2}{3}\right), b=\frac{2}{3}, c=-1$
d) $a=\log_e\left(\frac{4}{3}\right), b=-\frac{4}{3}, c=-1$

(h) If y = $\log_{10}$ x then $\frac{dy}{dx}=$? **[1]**

a) $\frac{1}{x(\log 10)}$
b) $\frac{1}{x}$
c) $\frac{1}{2x}$
d) $\frac{1}{x}$ (log 10)

(i) If A is an invertible matrix of any order, then which of the following options is NOT true? **[1]**

a) $|A^{-1}| = |A|^{-1}$
b) $(A^T)^{-1} = (A^{-1})^T$
c) $(A^2)^{-1} = (A^{-1})^2$
d) $|A| \neq 0$

(j) **Assertion (A):** If $\begin{bmatrix}x & 2\end{bmatrix}\begin{bmatrix}2 & 0\\ -4 & 0\end{bmatrix}=0$, then x = 2 **[1]**

Reason (R): If $\begin{bmatrix}x & 2\end{bmatrix}\begin{bmatrix}2 & 0\\ -4 & 0\end{bmatrix}=0$, then x = 4.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

(k) Find the domain of function given by $f(x)=\frac{1}{\sqrt{x+|x|}}$. **[1]**

(l) If A is a square matrix such that A^2 = A, then write the value of 7A - $(I + A)^3$, where I is an identity matrix. **[1]**

(m) If f : R $\rightarrow$ R is defined by f(x) = x^2, write f^{-1} (25). **[1]**

(n) Given two independent events A and B such that P (A) = 0.3 and P (B) = 0.6. Find P (A/B) **[1]**

(o) If E_1 and E_2 are independent events such that $P(E_1)$ = 0.3 and $P(E_2)$ = 0.4, find P $\left(\overline{E}_1 \cap E_2\right)$. **[1]**

2. Examine the differentiability of f, where f is defined by **[2]**

$f(x)=\begin{cases}x[x], & if\, 0\leqslant x<2\\ (x-1)x, & if\, 2\leqslant x<3\end{cases}$ at x = 2.

OR

A ladder 13 m long is leaning against a vertical wall. The bottom of the ladder is dragged away from the wall along the ground at the rate of 2 cm/sec. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

3. Evaluate: $\int \frac{\cos 2x+2\sin^2 x}{\sin^2 x}dx$ **[2]**

4. Find the interval in function 6 - 9x - x^2 is increasing or decreasing. **[2]**

5. Evaluate: $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x+3\cos x+2}dx$ **[2]**

OR

Evaluate the integral: $\int \sin^4 2x dx$

6. Prove that the relation R on the set N of all natural numbers defined by (x, y) $\in$ R $\Leftrightarrow$ x divides y, for all x ,y $\in$ N is transitive. **[2]**

7. Solve the equation for x: $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$, $x \neq 0$ **[4]**

8. Evaluate: $\int \frac{x^3}{(x^2-4)}dx$. **[4]**

9. Differentiate $\sin^{-1}(2ax\sqrt{1-a^2x^2}$ with respect to $\sqrt{1-a^2x^2}$, if $-\frac{1}{\sqrt{2}} < ax < \frac{1}{\sqrt{2}}$. **[4]**

OR

If x = 2 $\cos\theta$ - $\cos 2\theta$ and y = 2 $\sin\theta$ - $\sin 2\theta$, then prove that $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$.

10. **Read the text carefully and answer the questions:** **[4]**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(a) Find the probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics?

(b) Find the probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics?

(c) Find the probability that the selected student has passed in Mathematics, if it is known that he has failed in Economics?

(d) Find the probability that the selected student has passed in Economics, if it is known that he has failed in Mathematics?

OR

Read the text carefully and answer the questions: **[4]**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(a) Find the probability that both of them are selected.

(b) The probability that none of them is selected.

(c) Find the probability that only one of them is selected.

(d) Find the probability that atleast one of them is selected.

11. **Read the text carefully and answer the questions:** **[6]**

On her birthday, Shanti decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹ 10 more. However, if there were 16 children more, everyone would have got ₹ 10 less. Let the number of children be x and the amount distributed by Shanti for one child be y (in ₹).

(a) Find the equations related to the given problem in terms of x and y.

(b) Find the number of children. How much amount is given to each child by Shanti?

(c) Write the equations in form of matrix representation for the information given above?

12. Solve: $x\frac{dy}{dx} = y\left(\log y - \log x + 1\right)$ **[6]**

OR

Show that the differential equation of $\left(x^2 - y^2\right)dx + 2xydy = 0$ is homogeneous and solve it.

13. The perimeter of a triangle is 8 cm. If one of the sides of the triangle be 3 cm, what will be the other two sides for maximum area of the triangle? **[6]**

OR

Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 16 cm.

14. **Read the text carefully and answer the questions:** **[6]**

In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.

(a) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is NOT processed by Govind.

(b) Find the probability that Priyanka processed the form and committed an error.

(c) Find the total probability of committing an error in processing the form.

(d) Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form. The value of $\sum_{i=1}^{3} P(E_i \mid A)$?

SECTION B - 15 MARKS

15. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) A unit vector in the direction of the vector $\vec{a} = (2\hat{i} - 3\hat{j} + 6\hat{k})$ is **[1]**

a) $\left(\frac{2}{5}\hat{i} - \frac{3}{5}\hat{j} + \frac{6}{5}\hat{k}\right)$ b) $\left(\frac{1}{5}\hat{i} + \frac{2}{5}\hat{j} - \frac{6}{5}\hat{k}\right)$

c) $\left(1\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right)$ d) $\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right)$

(b) If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z+\frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$, find the relation between α and β. **[1]**

(c) Classify 20 kg weight measures as scalars and vectors. **[1]**

(d) Direction cosines of a line perpendicular to both x-axis and z-axis are: **[1]**

a) 0, 0, 1 b) 1, 1, 1

c) 1, 0, 1 d) 0, 1, 0

(e) Find the Cartesian equation of the plane $\vec{r}\,.\,[(5-2t)\hat{i} + (3-t)\hat{j} + (25+t)\hat{k}] = 15$ **[1]**

16. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$. **[2]**

OR

Find the area of the triangle whose two adjacent sides are determined by the vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$.

17. Find the angle between the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and the plane 3x + 4y + z + 5 = 0. **[4]**

OR

Find the shortest distance between the pairs of lines whose Cartesian equations are:

$\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{3} = \frac{y-2}{1}; z = 2$

18. Find the area of the region bounded by the curve $y = x^2$ and the line y = x. **[4]**

SECTION C - 15 MARKS

19. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as** **[5]**

instructed.

(a) If the demand function is p(x) = 20 - $\frac{x}{2}$, then the marginal revenue when x = 10 is **[1]**

a) ₹ 150 b) ₹ 10

c) ₹ 15 d) ₹ 5

(b) The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If Z = 30x + 24y is the objective function, then (maximum value of Z - minimum value of Z) is equal to **[1]**

a) 96 b) 136

c) 120 d) 144

(c) Find the coefficient of correlation from the regression lines: x - 2y + 3 = 0 and 4x - 5y + 1 = 0 **[1]**

(d) The demand function for a certain commodity is given by p = 1000 - 15x - x, 0 < x < 25. What is the price per unit and the total revenue from the sale of 2 units? **[1]**

(e) Find the marginal cost function (MC), if the cost function is: $C(x) = \frac{x^3}{3} + 5x^2 - 16x + 2$ **[1]**

20. A firm paid ₹ 25000 as rent of its office and ₹ 15200 as the interest of the loan taken to produce x units of a commodity. If the cost of production per unit is ₹ 8 and each item is sold at a price of ₹ 75, find the profit function. Also, find the breakeven point. **[2]**

OR

A company sells pens at ₹ 5 per unit. The fixed cost for the company is ₹ 3200 and variable cost is estimated to run 25% of the total revenue. Determine:

i. the total revenue function
ii. the total cost function
iii. the number of pens for breakeven point and
iv. the number of pens the company must sell to cover its fixed cost.

21. The correlation coefficient between x and y is 0.6. If the variance of x is 225, the variance of y is 400, mean of xis 10 and mean of y is 20, find **[4]**

i. the equations of two regression lines,
ii. the expected value of y when x = 2.

22. Solve the Linear Programming Problem graphically: **[4]**

Maximize Z = 2x + 3y Subject to

x + y $\geq$ 1

10x + y $\geq$ 5

x + 10y $\geq$ 1

x, y $\geq$ 0

OR

An automobile manufacturer makes automobiles and trucks in a factory that is divided into two shops. Shop A, which performs the basic assembly operation, must work 5 man-days on each truck but only 2 man-days on each automobile. Shop B, which performs finishing operations, must work 3 man-days for each automobile or truck that it produces. Because of men and machine limitations, shop A has 180 man-days per week available while shop B has 135 man-days per week. If the manufacturer makes a profit of ₹ 30000 on each truck and ₹ 2000 on each automobile, how many of each should he produce to maximize his profit? Formulate this as an LPP.

Solution

SECTION A - 65 MARKS

1. In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(a) **(c)** 8

Explanation:

8

(b) **(d)** $2\sqrt{\tan^{-1}x}+C$

Explanation:

Given integral is $\int\frac{1}{(1+x^2)\sqrt{\tan^{-1}x}}$

Let, $\tan^{-1}x = z^2$

$\Rightarrow \frac{1}{1+x^2}dx = 2zdz$

So,

$\int\frac{1}{(1+x^2)\sqrt{\tan^{-1}x}}dx$

$=\int\frac{2z}{z}dz$

$= 2\int dz$

= 2z + c

$=2\sqrt{\tan^{-1}x}+c$

where c is the integrating constant.

(c) **(b)** $\tan^{-1}\left(\frac{1}{7}\right)$

Explanation:

$\tan^{-1}(3) - \tan^{-1}(2)$

$\cot^{-1}\left(\frac{1}{3}\right)-\cot^{-1}\left(\frac{1}{2}\right) \quad 2.3 = 6 > 1$

$\Rightarrow \frac{\pi}{2}-\tan^{-1}\left(\frac{1}{3}\right)-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{2}\right) \qquad [as \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}]$

$\Rightarrow \left[\tan^{-1}\left(\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{3}\right)\right]$

$\Rightarrow \tan^{-1}\left[\frac{\left(\frac{1}{2}\right)-\left(\frac{1}{3}\right)}{1+\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\right] = \tan^{-1}\left(\frac{1/6}{7/6}\right) = \tan^{-1}\left(\frac{1}{7}\right)$

(d) **(b)** 3

Explanation:

Given differential equation is $\left(\frac{d^4y}{dx^4}\right)^{\frac{3}{5}}-5\left(\frac{d^3y}{dx^3}\right)+6\left(\frac{d^2y}{dx^2}\right)-3\left(\frac{dy}{dx}\right)$ + 5 = 0

Since the highest exponent of the highest derivative is called the degree of a differential equation provided exponent of each derivative and the unknown variable appearing in the differential equation is a non-negative integer.

$\therefore \left(\frac{d^4y}{dx^4}\right)^{\frac{3}{5}} = 5\left(\frac{d^3y}{dx^3}\right)-6\left(\frac{d^2y}{dx^2}\right)+8\left(\frac{dy}{dx}\right)$ - 5

$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^3$

$=\left\{5\left(\frac{d^3y}{dx^3}\right)-6\left(\frac{d^2y}{dx^2}\right)+8\left(\frac{dy}{dx}\right)-5\right\}^5$

$\therefore$ Required degree = 3

(e) **(a)** $\frac{15}{56}$

Explanation:

Probability of getting exactly one red (R) ball $= P_R\cdot P_B\cdot P_B+P_B\cdot P_R\cdot P_R+P_B\cdot P_B\cdot P_R$

$=\frac{5}{8}\cdot\frac{3}{7}\cdot\frac{2}{6}+\frac{3}{8}\cdot\frac{5}{7}\cdot\frac{2}{6}+\frac{3}{8}\cdot\frac{2}{7}\cdot\frac{5}{6}$

$=\frac{15}{4\cdot7\cdot6}+\frac{15}{4\cdot7\cdot6}+\frac{15}{4\cdot7\cdot6}$

$=\frac{5}{56}+\frac{5}{56}+\frac{5}{56}=\frac{15}{56}$.

Which is the required solution

(f) **(a)** {2, 4, 6}

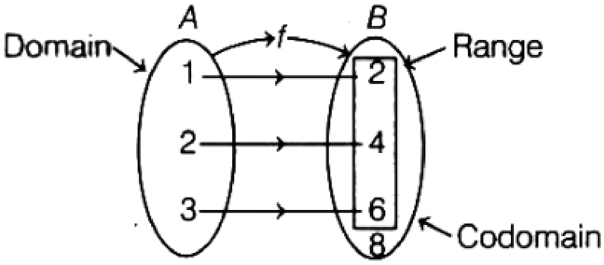
Explanation:

Given, f(x) = 2x, $\forall x \in A$

Value of function at x = 1, f(1) = 2(1) = 2

Value of function at x = 2, f(2) = 2(2) = 4

Value of function at x = 3, f(3) = 2(3) = 6



We Can write it as f = {(1, 2), (2, 4), (3, 6)}

$\therefore$ Range of f = {2, 4, 6}

(g) **(a)** $a = \log_e\left(\frac{2}{3}\right), b = \frac{2}{3}, c = 1$

Explanation:

$f(0) = \lim\limits_{x \to 0}(1+ax)^{\frac{1}{x}}$

$b = \lim\limits_{x \to 0}(1+ax)^{\frac{1}{ax}}$

b = e^a

a = $\log_e$ b

$f(0) = \lim\limits_{x \to 0^+} \frac{(x+c)^{1/3}-1}{(x+1)^{1/2}-1}$

Here, c = 1

x + 1 = y

$x \to 0 \Rightarrow y \to 1$

$f(0) = \lim\limits_{y \to 1} \frac{y^{1/3}-1}{y^{1/2}-1}$

$b = \lim\limits_{y \to 1} \frac{\frac{y^{1/3}-1}{y-1}}{\frac{y^{1/2}-1}{y-1}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$

a = log b = log $\frac{2}{3}$

(h) **(a)** $\frac{1}{x(\log 10)}$

Explanation:

Here, y = $\log_{10}$ x

Using the property that $\log_a$ b $= \frac{\log_e b}{\log_e a}$, we obtain

$y = \frac{\log_e x}{\log_e 10}$

Differentiating with respect to x, we

$\frac{dy}{dx} = \frac{1}{x \log_e 10}$

(i) **(a)** $|A^{-1}| = |A|^{-1}$

Explanation:

Since the determinant value of matrix and its reciprocal is same as the determinant value of an invertible matrix

(j) **(d)** A is false but R is true.

Explanation:

We have,

$\begin{bmatrix} x & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$

or, $\begin{bmatrix} 2x-8 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$

or 2x - 8 = 0 (By definition of equality)

or, x = 4

(k) Here we have, $f(x)=\frac{1}{\sqrt{x+|x|}}$

If x > 0, $x+|x|$ = x + x = 2x > 0

If x < 0, $x+|x|$ = x - x = 0

Clearly, x = 0 is not possible.

$\therefore$ Domain of f = R^+

(l) Since, $A^2 = A$

$7A - (I + A)^3 = 7A - I^3 - 3A^2 1 - 3AI^2 - A^3$

$= 7A - I - 3A - 3A - A^2A$

$[\because I^3 = I^2 = I$ and $A^2I = AI = A]$

= 7A - I - 3A - 3A - A.A

= 7A - I - 3A - 3A - A

= 7A - I - 7A

= -I

(m) Let $f^{-1}(25) = x$ ...(1)

Then,we have,

f(x) = 25

$\Rightarrow x^2 = 25$

$\Rightarrow x^2 - 25 = 0$

$\Rightarrow (x - 5)(x + 5) = 0$

$\Rightarrow x = \pm 5 \Rightarrow f^{-1}(25) = \{-5, 5\}$

(n) We are given that,

P(A) = 0.3

P(B) = 0.6

Since A and B are independent events, therefore,

$P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.6 = 0.18$

Therefore, required probability is given by,

$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$

$=\frac{0.18}{0.6}$

= 0.3

(o) We are given that, E_1 and E_2 are two independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$

$P\left(\overline{E_1}\cap E_2\right)=P(\overline{E_1})\times P\left(E_2\right)$

$= 0.7 \times 0.4 = 0.28$

Thus, $P\left(\overline{E_1}\cap E_2\right)$ = 0.28

2. We have $f(x)=\begin{cases} x\,[x]\,, & if\,0\leqslant x<2 \\ (x-1)\,x, & if\,2\leqslant x<3\end{cases}$

At x = 2, $Lf'(2)=\lim_{h\to 0}\frac{f(2-h)-f(2)}{-h}$

$=\lim_{h\to 0}\frac{(2-h)[2-h]-(2-1)2}{-h}$

$=\lim_{h\to 0}\frac{(2-h)(1)-2}{-h}\;[\therefore [a-h]=[a-1], where\ a\ is\ any\ positive\ integer]$

$=\lim_{h\to 0}\frac{2-h-2}{-h}=\lim_{h\to 0}\frac{-h}{-h}=1$

$Rf'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$

$=\lim_{h\to 0}\frac{(2+h-1)(2+h)-(2-1).2}{h}$

$=\lim_{h\to 0}\frac{(1+h)(2+h)-2}{h}$

$=\lim_{h\to 0}\frac{2+h+2h+h^2-2}{h}$

$=\lim_{h\to 0}\frac{h^2+3h}{h}=\lim_{h\to 0}\frac{h(h+3)}{h}=3$

$\therefore Lf'(2)\neq Rf'(2)$

So, f(x) is not differentiable at x = 2.

OR

Let y m be the height of the wall at which the ladder touches. Also, let the foot of the ladder be x m away from the wall.

Then, by Pythagoras theorem, we have:

$x^2 + y^2 = 169$ .. [length of the ladder is 13 m]

$\Rightarrow y = \sqrt{169 - x^2}$

Then, the rate of change of height (y) with respect to time (t) is given by,

$\frac{dy}{dx} = \frac{-x}{\sqrt{169-x^2}} \frac{dx}{dt}$

Now, when $x = 5\ m$, we have:

$\frac{dy}{dt} = \frac{-2\times5}{\sqrt{169-5^2}} = \frac{-10}{\sqrt{144}} = -\frac{5}{6}$

Hence, the height of the ladder on the wall is decreasing at the rate of $\frac{5}{6}\ cm/sec$.

3. $\int \frac{\cos 2x+2\sin^2 x}{\sin^2 x} dx$

$= \int \frac{1-2\sin^2 x+2\sin^2 x}{\sin^2 x} dx$

$= \int \frac{1}{\sin^2 x} dx$

$= \int \cos ec^2 x dx$

= -cot x + C

4. It is given that function f(x) = 6 - 9x - x^2

f'(x) = -9 - 2x

If f'(x) = 0,

$\Rightarrow x = \frac{-9}{2}$

So, the point x = $\frac{-9}{2}$ divides the real line two disjoint intervals, $\left(-\infty, \frac{-9}{2}\right)$ and $\left(\frac{-9}{2}, \infty\right)$

So, in interval $\left(-\infty, \frac{-9}{2}\right)$

f'(x) = -9 - 2x > 0

Therefore, the given function 'f' is strictly increasing for x < $\frac{-9}{2}$.

And in interval $\left(\frac{-9}{2}, \infty\right)$

f'(x) = -9 - 2x < 0

Therefore, the given function 'f' is strictly decreasing for $x > \frac{-9}{2}$

Thus, f is strictly decreasing for $x > \frac{-9}{2}$

5. Let I = $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x+3\cos x+2} dx$

Also let cos x = t

Differentiating w.r.t. x, we get

$-\sin x dx = dt$

Now, $x = 0 \Rightarrow t = 1$

$x = \frac{\pi}{2} \Rightarrow t = 0$

$\therefore\ \ I = \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x dx}{\cos^2 x+3\cos x+2}$

$= -\int_1^0 \frac{tdt}{t^2+3t+2}$

$= \int_0^1 \frac{tdt}{(t+2)(t+1)} \left[x - \int_a^b f(x) = \int_b^a f(x)\right]$

$= \int_0^1 \left(-\frac{1}{t+1} + \frac{2}{t+2}\right) dt$ [Applying partial fraction]

$= [-\log|1+t| + 2\log|t+2|]_0^1$

$= -\log 2 + 2\log 3 + 0 - 2\log 2$

$= 2\log 3 - 3\log 2$

$= \log \frac{9}{8}$

$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x dx}{\cos^2 x+3\cos x+2} = \log \frac{9}{8}$

OR

Let I = $\int \sin^4 2x dx$, then

$I = \int \left(\sin^2 2x\right)^2 dx$

$= \int \left[\frac{1-\cos 4x}{2}\right]^2 dx$
$= \frac{1}{4}\int (1-\cos 4x)^2$
$= \frac{1}{4}\int \left(1+\cos^2 4x - 2\cos 4x\right) dx$
$= \frac{1}{4}\int \left[1+\left(\frac{1+\cos 8x}{2}\right) - 2\cos 4x\right] dx$
$= \frac{1}{4}\int \left[\frac{3}{2}+\frac{\cos 8x}{2} - 2\cos 4x\right] dx$
$= \frac{1}{4}\left[\frac{3x}{2}+\frac{\sin 8x}{16} - \frac{2\sin 4x}{4}\right] + C$
$= \frac{3x}{8}+\frac{\sin 8x}{64} - \frac{\sin 4x}{8} + C$

6. Let x, y, z ∈ N be such that (x, y) ∈ R and (y, z) ∈ R . Then, (x, y) ∈ R and (y, z) ∈ R
⇒ x divides y and, y divides z
⇒ There exist p, q ∈ N such that y = xp and z = yq
⇒ z = (xp) q
⇒ z = x(pq)
⇒ x divides z [∵ pq ∈ N]
⇒ (x, z) ∈ R
Therefore, (x, y) ∈ R ,(y, z) ∈ R ⇒ (x, z) ∈ R for all x , y , z ∈ N.
Therefore, R is a transitive relation on N.

7. Given, $\sin^{-1}x + \sin^{-1}(1 - x) = \cos^{-1}x$
$\Rightarrow \sin^{-1}x + \sin^{-1}(1-x) = \frac{\pi}{2} - \sin^{-1}x$
$\Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} - 2\sin^{-1}x$
$\Rightarrow 1-x = \sin\left[\frac{\pi}{2} - 2\sin^{-1}x\right]$
$\Rightarrow 1 - x = \cos[2 \sin^{-1}x]$
$\Rightarrow 1 - x = \cos[\cos^{-1} (1 - 2x^2)]$
$\Rightarrow 1 - x = 1 - 2x^2$
$\Rightarrow 2x^2 - x = 0$
$\Rightarrow x(2x - 1) = 0$
$\Rightarrow x = \frac{1}{2}$ [∵ $x \neq 0$]

8. Let the given integral be,
$I = \int \frac{x^2}{x^2-4} dx$
Therefore by long division.
$I = \int x + \frac{4x}{x^2-4} dx$
$I = \int x dx + \int \frac{4x}{x^2-4} dx$
$= \frac{x^2}{2} + \int \frac{4x}{(x-2)(x+2)} dx$
Let $I_1 = \int \frac{4x}{(x-2)(x+2)} dx$
So
$I = \frac{x^2}{2} + I_1$
Therefore $I_1 = \int \frac{4x}{x^2-4} dx$
Putting $x^2 - 4 = t$
$2xdx = dt$
$I_1 = 2\int \frac{dt}{t}$
$I_1 = 2\log\left|x^2-4\right| + c$
Putting the value of I_1 in I,
$I = \frac{x^2}{2} + 2\log\left|x^2-4\right| + c$

9. Let $\mathbf{u} = \sin^{-1}(2ax\sqrt{1-a^2x^2})$
Put ax = $\sin\theta \Rightarrow \theta = \sin^{-1}(ax)$
$\therefore u = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$
$\Rightarrow u = \sin^{-1}(2 \sin\theta \cos\theta)$
$\Rightarrow u = \sin^{-1}(\sin 2\theta)$ ...(i)

And,

Let, $v=\sqrt{1-a^2x^2}$

Differentiating it with respect to x,

$\frac{dv}{dx}=\frac{1}{2\sqrt{1-a^2x^2}}\times\frac{d}{dx}\left(1-a^2x^2\right)$

$\Rightarrow\frac{dv}{dx}=\left(\frac{0-2a^2x}{2\sqrt{1-a^2x^2}}\right)$

$\Rightarrow\frac{dv}{dx}=\frac{-a^2x}{\sqrt{1-a^2x^2}}$ ...(ii)

Here,

$-\frac{1}{\sqrt{2}}<ax<\frac{1}{\sqrt{2}}$

$\Rightarrow-\frac{1}{\sqrt{2}}<\sin\theta<\frac{1}{\sqrt{2}}$

$\Rightarrow-\frac{\pi}{4}<\theta<\frac{\pi}{4}$

So, from equation (i),

$u=2\theta\left[\text{ since },\sin^{-1}(\sin\theta)=\theta,\ \text{if }\theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right]$

$\Rightarrow$ u = 2sin^{-1}x

Differentiating it with respect to x,

$\frac{du}{dx}=2\times\frac{1}{\sqrt{1-(ax)^2}}\frac{d}{dx}(ax)$

$\Rightarrow\frac{du}{dx}=\frac{2}{1-a^2x^2}(a)$

$\Rightarrow\frac{du}{dx}=\frac{2a}{1-a^2x^2}$ ...(iii)

Dividing equation (iii) by (ii),

$\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\left(\frac{2a}{\sqrt{1-a^2x^2}}\right)\left(\frac{\sqrt{1-a^2x^2}}{-a^2x}\right)$

$\therefore\frac{du}{dv}=-\frac{2}{ax}$

OR

According to the question,we are given that x = 2 cosθ - cos 2θ

and y = 2 sin θ - sin 2θ

then we have to prove that $\frac{dy}{dx}=\tan\left(\frac{3\theta}{2}\right)$.

Therefore, differentiating both sides w.r.t θ, we get,

$\frac{dx}{d\theta}=-2\sin\theta+2\sin2\theta$

and $\frac{dy}{d\theta}=2\cos\theta-2\cos2\theta$

$\therefore\quad\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{2(\cos\theta-\cos2\theta)}{2(-\sin\theta+\sin2\theta)}$

$=\frac{2\sin\left(\frac{\theta+2\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)}{2\left[\cos\left(\frac{2\theta+\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)\right]}\left[\begin{array}{l}\because\cos C-\cos D=2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)\\ \text{and }\sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\end{array}\right]$

$=\frac{\sin\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$

$=\tan\left(\frac{3\theta}{2}\right)$

10. **Read the text carefully and answer the questions:**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(a) Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Economics if it is known that he has failed in Mathematics.

Required probability = $P(\frac{E}{M})$

= $\frac{P(E \cap M)}{P(M)} = \frac{\frac{1}{4}}{\frac{7}{20}}$ = $\frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$

(b) Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Mathematics if it is known that he has failed in Economics.

Required probability = P(M/E)

$= \frac{P(M \cap E)}{P(E)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

(c) Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Mathematics if it is known that he has failed in Economics

Required probability = P(M'/E)

$\Rightarrow P(M'/E) = \frac{P(M' \cap E)}{P(E)}$

= $\frac{P(E) - P(E \cap M)}{P(E)}$

= $\frac{\frac{1}{2} - \frac{1}{4}}{\frac{1}{2}}$

$\Rightarrow P(M'/E) = \frac{1}{2}$

(d) Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Economics if it is known that he has failed in Mathematics

Required probability = P(E'/M)

$\Rightarrow P(E'/M) = \frac{P(E' \cap M)}{P(M)}$

= $\frac{P(M) - P(E \cap M)}{P(M)}$

= $\frac{\frac{7}{20} - \frac{1}{4}}{\frac{7}{20}}$

$\Rightarrow P(E'/M) = \frac{2}{7}$

OR

Read the text carefully and answer the questions:

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(a) P(A) = $\frac{1}{3}$, P(A') = $1-\frac{1}{3}=\frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1-\frac{1}{3}=\frac{1}{2}$

P(Both are selected) = $P(A\cap B)=P(A)\cdot P(B)$ = $\frac{1}{3}.\frac{1}{2}$

P(Both are selected) = $\frac{1}{6}$

(b) P(A) = $\frac{1}{3}$, P(A') = $1-\frac{1}{3}=\frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1-\frac{1}{3}=\frac{1}{2}$

P(none of them selected) = $P(A'\cap B')=P(A')\cdot P(B')$ = $\frac{2}{3}.\frac{1}{2}$

P(Both are selected) = $\frac{1}{3}$

(c) P(A) = $\frac{1}{3}$, P(A') = $1-\frac{1}{3}=\frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1-\frac{1}{3}=\frac{1}{2}$

P(none of them selected) = $P(A')\cdot P(B)+P(A)\cdot P(B')$ = $\frac{2}{3}.\frac{1}{2}+\frac{1}{3}.\frac{1}{2}$

P(Both are selected) = $\frac{3}{6}=\frac{1}{2}$

(d) P(A) = $\frac{1}{3}$, P(A') = $1-\frac{1}{3}=\frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1-\frac{1}{3}=\frac{1}{2}$

P(atleast one of them selected) = 1 - P(none selected) = $1-\frac{1}{3}$

P(atleast one of them selected) = $\frac{2}{3}$

11. **Read the text carefully and answer the questions:**

On her birthday, Shanti decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got ₹ 10 more. However, if there were 16 children more, everyone would have got ₹ 10 less. Let the number of children be x and the amount distributed by Shanti for one child be y (in ₹).

(a) Let number of children = x

Amount distributed by Shanti for one child = ₹ y

Now, Total money = xy

and Total money will remain the same.

Given that, if there were 8 children less, everyone would have got ₹ 10 more.

Total money now = Total money before

(x - 8) × (y + 10) = xy

⇒ x(y + 10) - 8(y + 10) = xy

⇒ xy + 10x - 8y - 80 = xy

$\Rightarrow$ 10x - 8y - 80 = 0

$\Rightarrow$ 10x - 8y = 80

$\Rightarrow$ 5x - 4y = 40

Also, if there were 16 children more, everyone would have got ₹ 10 less.

Total money now = Total money before

(x + 16) $\times$ (y - 10) = xy

$\Rightarrow$ x(y - 10) + 16(y - 10) = xy

$\Rightarrow$ xy - 10x + 16y - 160 = xy

$\Rightarrow$ -10x + 16y - 160 = 0

$\Rightarrow$ 10x - 16y + 160 = 0

$\Rightarrow$ 5x - 8y = -80

Thus, required equations are:

5x - 4y = 40 ...(i)

5x - 8y = - 80 ...(ii)

(b) On solving eqs. (i) & (ii), we get x = 32 and y = 30.

Hence, the number of children = 32

The amount is given to each child by Shanti = Rs. 30.

(c) Writing eq. (i) & eq. (ii) in matrix form, we get

$$\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

12. Given, $x\frac{dy}{dx} = y\left(\log y - \log x + 1\right)$

$\Rightarrow x\frac{dy}{dx} = y\left(\log \frac{y}{x} + 1\right)$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\log \frac{y}{x} + 1\right)$ ...(i)

Which is a homogeneous equation.

Put $\frac{y}{x} = v$ or y = vx

$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

On substituting these values in Eq. (i), we get

$v + x\frac{dv}{dx} = v\left(\log v + 1\right)$

$\Rightarrow x\frac{dv}{dx} = v\left(\log v + 1 - 1\right)$

$\Rightarrow x\frac{dv}{dx} = v\left(\log v\right)$

$\Rightarrow \frac{dv}{v\log v} = \frac{dx}{x}$

On integrating both sides, we get

$\int \frac{dv}{v\log v} = \int \frac{dx}{x}$

On putting $\log v = u$ in LHS integral, we get

$\frac{1}{v}.dv = du$

$\int \frac{du}{u} = \int \frac{dx}{x}$

$\Rightarrow \log u = \log x + \log C$

$\Rightarrow \log u = \log Cx$

$\Rightarrow u = Cx$

$\Rightarrow \log v = Cx$

$\Rightarrow \log\left(\frac{y}{x}\right) = Cx$

OR

We have

$2xydy = -\left(x^2 - y^2\right)dx$

$\frac{dy}{dx} = -\frac{x^2-y^2}{2xy}$

Let $f(x, y) = -\frac{x^2-y^2}{2xy}$

Here, putting x = kx and y = ky

$f(kx, ky) = -\frac{k^2x^2-k^2y^2}{2k^2xy}$

$f(kx, ky) = -\frac{k^2}{k^2} \cdot \frac{x^2-y^2}{2xy}$

= k^0.f(x,y)

Therefore, the given differential equation is homogeneous.

$\left(x^2-y^2\right)dx+2xydy=0$

$2xydy=-\left(x^2-y^2\right)dx$

$\frac{dy}{dx}=-\frac{x^2-y^2}{2xy}$

To solve it we make the substitution.

y = vx

Differentiating eq. with respect to x, we get

$\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\mathrm{v}+\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=-\frac{\mathrm{x}^2-\mathrm{v}^2\mathrm{x}^2}{2\mathrm{x}\cdot\mathrm{vx}}$

$\mathrm{v}+\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=-\frac{\mathrm{x}^2\left(1-\mathrm{v}^2\right)}{2\mathrm{vx}^2}$

$\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=\frac{\mathrm{v}^2-1}{2\mathrm{v}}-\mathrm{v}$

$x\frac{dv}{dx}=\frac{-1-v^2}{2v}$

$-\frac{2\mathrm{v}}{1+\mathrm{v}^2}\mathrm{dv}=\frac{1}{\mathrm{x}}\mathrm{dx}$

$\frac{2v}{1+v^2}dv=-\frac{1}{x}dx$

Integrating both sides, we get

$\int\frac{2v}{1+v^2}dv=-\int\frac{1}{x}dx$ ....(i)

Let $I_1=\int\frac{2v}{1+v^2}dv$

Put 1 + v^2 = t

2vdv = dt

$\mathrm{vdv}=\frac{1}{2}\mathrm{dt}$

$\Rightarrow\int\frac{2v}{1+v^2}dv=\int\frac{1}{t}dt$ = log(t)

$\therefore$ log(1 + v^2) = -logx + logC ($\therefore$ From (i) eq.)

$\log\left(1+\left(\frac{y}{x}\right)^2\right)=-\log x+\log c$

$\Rightarrow x^2+y^2=Cx$ is the required solution of the differential equation.

13. Given, the Perimeter of a triangle is 8 cm. One of the sides of the triangle is 3 cm. The area of the triangle is maximum.

Let us consider,

'x' and 'y' be the other two sides of the triangle.

Now, perimeter of the $\triangle$ABC is

8 = 3 + x + y

y = 8 - 3 - x = 5 - x

y = 5 - x ... (i)

Consider the Heron's area of the triangle,

$A=\sqrt{s(s-a)(s-b)(s-c)}$

Where $s=\frac{a+b+c}{2}$

As perimeter = a + b + c = 8

$s=\frac{8}{2}=4$

Now Area of the triangle is given by

$A=\sqrt{8(8-3)(8-x)(8-y)}$

Now substituting (i) in the area of the triangle,

$A=\sqrt{4(4-3)(4-x)(4-(5-x))}$

$A=\sqrt{4(4-x)(x-1)}$

$A=\sqrt{4\left(4x-4-x^2+x\right)}=\sqrt{4\left(5x-x^2-4\right)}$

$A=\sqrt{4\left(5x-x^2-4\right)}$

[squaring on both sides]

Z = A^2 = 4(5x - x^2 - 4) ... (ii)

For finding the maximum/ minimum of a given function, we can find it by differentiating it with x and then equating it to zero.

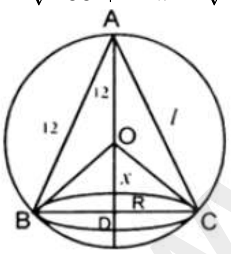
This is because if the function f(x) has a maximum/minimum at a point c then f'(c) = 0.

Differentiating both sides the equation (ii) with respect to x:

$\frac{dZ}{dx} = \frac{d}{dx}\left[4\left(5x - x^2 - 4\right)\right]$
$\frac{dZ}{dx} = 4\frac{d}{dx}(5x) - 4\frac{d}{dx}\left(x^2\right) - 4\frac{d}{dx}(4)$
[Since $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$]
$\frac{dz}{dx} = 4(5) - 4(2x) - 0$
$\frac{dZ}{dx} = 20 - 8x$ ... (iii)
To find the critical point, we need to equate equation (iii) to zero.
$\frac{dZ}{dx} = 20 - 8x = 0$
20 - 8x = 0
8x = 20
$x = \frac{5}{2}$
Now to check if this critical point will determine the maximum area of the triangle, we need to check with the second differential which needs to be negative.
Consider differentiating both sides the equation (iii) with x:
$\frac{d^2Z}{dx^2} = \frac{d}{dx}[20 - 8x]$
$\frac{d^2z}{dx^2} = -8$ ... (iv)
[Since $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$]
As $\left(\frac{d^2z}{dx^2}\right)_{x=\frac{5}{2}} = -8 < 0$, so the function A is maximum at $x = \frac{5}{2}$.
Now substituting $x = \frac{5}{2}$ in equation (i):
y = 5 - 2.5
y = 2.5
As x = y = 2.5, two sides of the triangle are equal,
Hence the given triangle is an isosceles triangle with two sides equal to 2.5 cm and the third side equal to 3cm.

OR

Let R be radius and l be the slanted height of cone.
Let OD = x, OA = OB = OC = 12, where 12 is a radius of sphere.
In right angled $\triangle ODC, R = \sqrt{144 - x^2}$
In right angled $\triangle$ADC
$l = \sqrt{R^2 + (x+12)^2} = \sqrt{144 - x^2 + x^2 + 144 + 24x}$
$= \sqrt{288 + 24x} = \sqrt{24}$ ==>$l = \sqrt{x + 12}$ ...equation (1)



Let S be the curved surface area of the cone.
$\therefore S = \pi Rl = \pi\sqrt{144 - x^2}\sqrt{24}\ \sqrt{x + 12} = \pi\sqrt{24}(x + 12)\sqrt{12 - x}$ [from equation (1)]
$\frac{dS}{dx} = \pi\sqrt{24}\left[(x + 12)\cdot\frac{-1}{2\sqrt{2 - x}} + \sqrt{12 - x}\cdot 1\right] = \pi\sqrt{24}\left[\frac{-x - 12 + 24 - 2x}{2\sqrt{12 - x}}\right]$
$= \pi\sqrt{24}\left[\frac{12 - 3x}{2\sqrt{12 - x}}\right]$
For S to be maximum or minimum, $\frac{dS}{dx} = 0$
$\therefore \quad \pi\sqrt{24}\left[\frac{12 - 3x}{2\sqrt{12 - x}}\right] = 0 \Rightarrow$ x = 4
When x < 4 (slightly), $\frac{dS}{dx} = +ve$
$\therefore$ at x = 4, $\frac{dS}{dx}$ changes from +ve to -ve
$\therefore$ S is maximum at x = 4
$\frac{dS}{dx}$ altitude AD = 4 + 12 = 16

14. **Read the text carefully and answer the questions:**
In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an

error rate of 0.04 and Tahseen has an error rate of 0.03.

(a) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5$, $P(E_2) = 0.2$, $P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06$, $P(\frac{A}{E_2}) = 0.04$, $P(\frac{A}{E_3}) = 0.03$

Using Bayes' theorem, we have

$$P(\frac{E_1}{A}) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)+P(E_3)\cdot P\left(\frac{A}{E_3}\right)}$$

$$= \frac{0.5\times 0.06}{0.5\times 0.06+0.2\times 0.04+0.3\times 0.03} = \frac{30}{47}$$

$\therefore$ Required probability = $P\left(\frac{\bar{E}_1}{A}\right)$

$= 1 - P\left(\frac{E_1}{A}\right) = 1 - \frac{30}{47} = \frac{17}{47}$

(b) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5$, $P(E_2) = 0.2$, $P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06$, $P(\frac{A}{E_2}) = 0.04$, $P(\frac{A}{E_3}) = 0.03$

$P(A \cap E_2) = P\left(\frac{A}{E_2}\right).P(E_2)$

$\Rightarrow 0.04 \times 0.2 = 0.008$

(c) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5$, $P(E_2) = 0.2$, $P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06$, $P(\frac{A}{E_2}) = 0.04$, $P(\frac{A}{E_3}) = 0.03$

$P(A) = P\left(\frac{A}{E_1}\right).P(E_1) + P\left(\frac{A}{E_2}\right).P(E_2) + P\left(\frac{A}{E_3}\right).P(E_3)$

$= 0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03 = 0.047$

(d) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5$, $P(E_2) = 0.2$, $P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06$, $P(\frac{A}{E_2}) = 0.04$, $P(\frac{A}{E_3}) = 0.03$

$$\sum_{i=1}^{3} P\left(\frac{E_i}{A}\right) = P\left(\frac{E_1}{A}\right) + P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right)$$

= 1 [$\because$ Sum of posterior probabilities is 1]

SECTION B - 15 MARKS

15. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(a) **(d)** $\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right)$

Explanation:

Given vector is $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Now, unit vector along to $\vec{a}$ is $\frac{\vec{a}}{|\vec{a}|}$

so, $|\vec{a}| = \sqrt{2^2 + (-3)^2 + 6^2} = \sqrt{49} = 7$

$\Rightarrow$ Unit vector along to $\vec{a}\ is\ \left(\frac{2}{7}\hat{i}-\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}\right)$

(b) $\cos\frac{\pi}{4}=\frac{|\alpha\cdot 1+0+\beta|}{\sqrt{\alpha^2+\beta^2+25\sqrt{2}}}\Rightarrow\frac{1}{\sqrt{2}}=\frac{|\alpha+\beta|}{\sqrt{\alpha^2+\beta^2+25\sqrt{2}}}$

Squaring both sides, we get

$\alpha^2+\beta^2+2\alpha\beta=\alpha^2+\beta^2+25$

$\Rightarrow\alpha\beta=\frac{25}{2}$

(c) 20 kg weight is a vector quantity as it involves both magnitude and direction.

(d) **(d)** 0, 1, 0

Explanation:

0, 1, 0

(e) Given equation of plane is $\overrightarrow{r}.[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}]=15$

$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}\right]=15$

(5 - 2t)x + (3 - t)y + (25 + t)z = 15

16. We have, $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+k$.

Let θ be the angle between vectors

$\vec{a}$ and $\vec{b}$. Then

$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

we have,

$\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+\hat{k}$

$\Rightarrow\vec{a}\cdot\vec{b}=1\times 3+(-2)\times(-2)+3\times 1=3+4+3=10$

$|\vec{a}|=\sqrt{1+4+9}=\sqrt{14}$ and $|\vec{b}|=\sqrt{9+4+1}=\sqrt{14}$

$\therefore\ \cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\Rightarrow\cos\theta=\frac{10}{14}=\frac{5}{7}$

Therefore, the angle between the given vectors is $\cos^{-1}\left(\frac{5}{7}\right)$

OR

Given, $\vec{a}=(3\hat{i}+4\hat{j}),\vec{b}=(-5\hat{i}+7\hat{j})$

$(\vec{a}\times\vec{b})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&4&0\\-5&7&0\end{vmatrix}=\hat{i}\begin{vmatrix}4&0\\7&0\end{vmatrix}-\hat{j}\begin{vmatrix}3&0\\-5&0\end{vmatrix}+\hat{k}\begin{vmatrix}3&4\\-5&7\end{vmatrix}$

$=\hat{k}(21+20)=41\hat{k}\Rightarrow|\vec{a}\times\vec{b}|=\sqrt{(41)^2}=41$

$\therefore$ Required area $=\frac{1}{2}|\vec{a}\times\vec{b}|=\frac{1}{2}\times 41=\frac{41}{2}$ sq. units.

17. We know that line $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ is parallel to plane $a_2x+b_2y+c_2z+d_2=0$

if $a_1a_2+b_1b_2+c_1c_2=0$ is given by

$\sin\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$ ……(1)

Now, given equation of line is

$\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$

Here, $a_1=3$, $b_1=-1$ and $c_1=2$

Equation of plane is $3x+4y+z+5=0$

So, $a_2=3$, $b_2=4$, $c_2=1$ and $d_2=-5$

$\therefore\sin\theta=\frac{3\times3+(-1)\times4+2\times(1)}{\sqrt{3^2+(-1)^2+2^2}\sqrt{3^2+4^2+(1)^2}}$

$\Rightarrow\sin\theta=\frac{9-4+2}{\sqrt{9+1+4}\sqrt{9+16+1}}$

$\Rightarrow\sin\theta=\frac{7}{\sqrt{14}\sqrt{26}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{7\sqrt{7}}{7\sqrt{52}}=\frac{\sqrt{7}}{\sqrt{52}}$

$\Rightarrow\sin\theta=\frac{\sqrt{7}}{\sqrt{52}}$

$\Rightarrow \theta = \sin^{-1}(\frac{\sqrt{7}}{\sqrt{52}})$

Hence the required angle between the plane and the line is $\sin^{-1}(\frac{\sqrt{7}}{\sqrt{52}})$

OR

Given that the equations of the given lines are

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-0}{1}$ .....(i)

$\frac{x+1}{3} = \frac{y-2}{1} = \frac{z-2}{0}$ ....(ii)

Since line (i) passes through the point (1, -1, 0) and has direction ratios proportional to 2, 3, 1, its vector equation is

$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$

Here,

$\overrightarrow{a_1} = \hat{i} - \hat{j} + 0\hat{k}$

$\overrightarrow{b_1} = 2\hat{i} + 3\hat{j} + \hat{k}$

Also, line (ii) passes through the point (-1, 2, 2) and has direction ratios proportional to 3, 1, 0. Its vector equation is

$\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$

Here,

$\overrightarrow{a_2} = -\hat{i} + 2\hat{j} + 2\hat{k}$

$\overrightarrow{b_2} = 3\hat{i} + \hat{j} + 0\hat{k}$

Now,

$\overrightarrow{a_2} - \overrightarrow{a_1} = -2\hat{i} + 3\hat{j} + 2\hat{k}$

and $|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & 1 & 0 \end{vmatrix}$

$= -\hat{i} + 3\hat{j} - 7\hat{k}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-1)^2 + 3^2 + (-7)^2}$

$= \sqrt{1 + 9 + 49}$

$= \sqrt{59}$

Also $(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2}) = (-2\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (-\hat{i} + 3\hat{j} - 7\hat{k})$

= 2 + 9 - 14 = -3

Hence the shortest distance between the lines

$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$

d = $\left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$

$= \left|\frac{-3}{\sqrt{59}}\right|$

$= \frac{3}{\sqrt{59}}$

18. The given curves are

$y = x^2$ ....(1)

$y = x$ .....(2)

From (1) and (2), we get

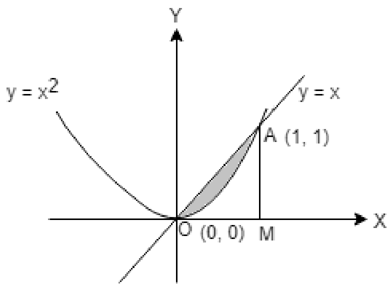
$x = x^2$

$x^2 = x$

$x(x-1) = 0$

$x = 0, 1$

Thus,curves (1) and (2) intersect in the points A(0,0) and A(1,1).



Required area = the area of the region bounded by the curve $y = x^2$ and the line y = x.

= area of the shaded region

$= \int_0^1 x\,dx - \int_0^1 x^2\,dx$

$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$

$= \left(\frac{1}{2} - 0\right) - \left(\frac{1}{3} - 0\right)$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{3-2}{6}$

$= \frac{1}{6}$ sq.units

SECTION C - 15 MARKS

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(a) **(b)** ₹ 10

Explanation:

p(x) = 20 - $\frac{x}{2}$ $\Rightarrow$ R(x) = 20x - $\frac{x^2}{2}$

MR = $\frac{d}{dx}$R(x) = 20 - x

$[MR]_{x=10}$ = 20 - 10 = ₹ 10

(b) **(d)** 144

Explanation:

Corner Points	**Z = 30x + 24y**
(0, 4)	Z = 30 $\times$ 0 + 24 $\times$ 4 = 96 $\rightarrow$ Min.
(8, 0)	Z = 30 $\times$ 8 + 24 $\times$ 0 = 240 $\rightarrow$ Max.
$\left(\frac{20}{3}, \frac{4}{3}\right)$	Z = 30 $\times$ $\frac{20}{3}$ + 24 $\times$ $\frac{4}{3}$ = 200 + 32 = 232

Then Max. Z - Min. Z = 240 - 96 = 144.

(c) Let the line of regression of y on x be

x - 2y + 3 = 0

$\Rightarrow$ 2y = x + 3

$\Rightarrow$ y = $\frac{x}{2} + \frac{3}{2}$

$\therefore$ $b_{xy} = \frac{1}{2}$

Let the line of regression of x on y be

$\Rightarrow$ 4x - 5y + 1 = 0

$\Rightarrow$ 4x = 5y - 1

$\therefore$ x = $\frac{5}{4}$x - $\frac{1}{4}$

$b_{xy} = \frac{5}{4}$

$\therefore$ $r^2 = b_{yx} \times b_{xy}$

$= \left(\frac{1}{2}\right)\left(\frac{5}{4}\right) = \frac{5}{8} < 1$

Hence, our assumption of regression equation is correct.

$\therefore$ r = $\sqrt{\frac{5}{8}}$ = 0.79

(d) Let R(x) be the revenue function. Then,

R(x) = px

$\Rightarrow$ R(x) = 1000x - $15x^2$ - x^3

When x = 2, we get

p = 1000 - 15 × 2 - 2^2 = 966 and,

R = 2000 - 15 × 4 - 8 = 1932

(e) C(x) = $\frac{x^3}{3} + 5x^2 - 16x + 2$

M.C. $= \frac{3x^2}{3} + 10x - 16$

= x^2 + 10x - 16

20. TFC = ₹(25000 + 15200) = ₹ 40200

TVC = ₹ 8x

∴ TC = TFC + TVC

⇒ C(x) = 40200 + 8x

Given p = ₹ 75

So, R(x) = px ⇒ R(x) = 75x

Profit function P(x) = R(x) - C(x)

⇒ P(x) = 75x - (40200 + 8x) = 67x - 40200

At breakeven point R(x) = C(x)

⇒ 75x = 40200 + 8x

⇒ 67x = 40200

⇒ x = 600

OR

i. Given p = ₹ 5 ⇒ R(x) = px = 5x

ii. Given TFC = ₹ 3200, TVC = 25% of total revenue = 25% of 5x

= $\frac{25}{100}$ × 5x = $\frac{5}{4}$x = 1.25x

∴ C(x) = 3200 + 1.25x

iii. **At breakeven point:** R(x) = C(x)

⇒ 5x = 3200 + 1.25x

⇒ 3.75x = 3200 ⇒ x = $\frac{3200}{3.75}$

⇒ x = 853.33

iv. **To cover fixed cost:**

R(x) =TFC

⇒ 5x = 3200

⇒ x = 640

21. Given: correlation coefficient

= 0.6

Variance of x = 225

Variance of y = 400

Mean of x = 10

Mean of y = 20

$\sigma_x = \sqrt{\text{Variance}}$

= $\sqrt{225} = \pm 15$

$\sigma_y = \sqrt{\text{Variance}}$

= $\sqrt{400} = \pm 20$

i. $b_{yx} = r\frac{\sigma_y}{\sigma_x}$

= 0.6 × $\frac{20}{15}$ = 0.8

$b_{xy} = r\frac{\sigma_x}{\sigma_y}$

= 0.6 × $\frac{15}{20}$ = 0.45

Regression line y on x

y - $\bar{y}$ = b_{yx} (x - $\bar{x}$)

y - 20 = 0.8(x - 10)

y - 20 = 0.8x - 8

y = 0.8x + 12 ...(i)

Regression line x on y

$x - \bar{x} = b_{xy}(y - \bar{y})$

x - 10 = 0.45(y - 20)

x = 0.45y - 9 + 10

x = 0.45y + 1 ...(ii)

ii. When x = 2

$y = 0.8 \times 2 + 12 = 13.6$

22. First, we will convert the given inequations into equations, we obtain the following equations and solving then we get the x + y = 1,10x + y = 5, x + 10y = 1, x = 0 and y = 0

Region represented by $x + y \geq 1$: The line x + y = 1 meets the coordinate axes at A(1,0) and B(0,1) respectively. By joining these points we obtain the line x + y = 1 Clearly (0,0) does not satisfies the inequation $x + y \geq 1$. So, the region in x y plane which does not contain the origin represents the solution set of the inequation $x + y \geq 1$ Region represented by $10x + y \geq 5$:

The line 10x + y = 5 meets the coordinate axes at $C\left(\frac{1}{2}, 0\right)$ and D(0,5) respectively. By joining these points we obtain the line

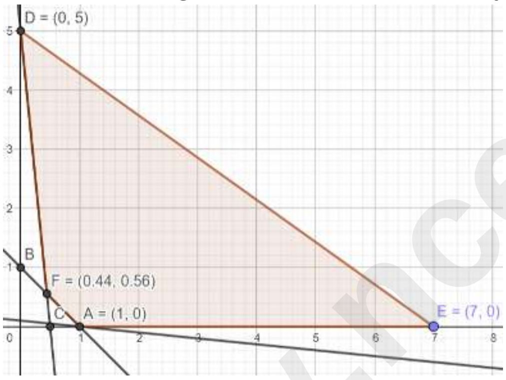
10x + y = 5 . Clearly (0,0) does not satisfies the inequation $10x + y \geq 5$. So, the region which does not contains the origin represents the solution set of the inequation $10x + y \geq 5$

Region represented by $x + 10y \geq 1$:

The line x + 10 y = 1 meets the coordinate axes at A(1,0) and $F\left(0, \frac{1}{10}\right)$ respectively. By joining these points we obtain the line

x + 10 y = 1 . Clearly (0,0) does not satisfies the inequation $x + 10y \geq 1$. So, the region which does not contains the origin represents the solution set of the inequation $x + 10y \geq 1$ Region represented by $x \geq 0$ and $y \geq 0$:

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$, and $y \geq 0$ The feasible region determined by subject to the constraints are $x + y \geq 1, 10x + y \geq 5, x + 10y \geq 1$,and the non-negative restrictions, $x \geq 0$, and $y \geq 0$, are as follows.



The feasible region is unbounded. Therefore, the maximum value of objective function is infinity i.e. the solution is unbounded.

OR

Let number of automobiles produces be x and let the number of trucks

Produced be y.

Let Z be the profit function so profit should be maximized.

Z = 2000x + 30000y

The constraints are on the man hours worked

Shop A $2x + 5y \leq 180$ (i) assembly

Shop B $3x + 3y \leq 135$ (ii) finishing

$x, y \geq 0$[non -negative restrictions]

Corner points can be obtained from

$2x = 3y + 5y = 180 \Rightarrow x = 0; y = 36$ and $x = 90; y = 0$

$3x + 3y \leq 135 \Rightarrow x = 0; y = 45$ and $x = 45; y = 0$

Solving (i) and (ii) gives x = 15 and y = 30

Corner point	Value of Z = 2000x + 30000y
0, 0	0
0, 36	10,80,000

15, 30	9,30,000
45, 0	90,000