Paper
Class – XII Subject
–MATHEMATICS

[Time – 3 Hrs] **[Full Marks – 100]**

Answer question 1 (compulsory)and five other questions from section A

and answer any two questions from either section B or section C.

<u>Section – A</u>

Question – 1.

(i) Construct a 2x2 matrix A = [aij], whose elements are given by :
aij = |2i – 3j|. [3]

(ii) Show that $\sin^{-1} \sqrt{3}/2 + 2 \tan^{-1} 1/\sqrt{3} = 2\pi/3$. [3]

`` (iii) Find the locus of the points such that the difference from the points (4, 0) and (– 4, 0) is always equal to 2. Name the curve. [3]

(iv) Evaluate the limit : $Lt_{x\to 1} \log [x/(x – 1)]$ [3]

(v) Evaluate the integral : $\int \sin 2x/(a^2 \cos^2 x + b^2 \sin^2 x).dx$. [3]

(vi) Differentiate w. r. t. x : $3^x/(2 + \sin x)$ [3]

(vii) For a post three person A, B and C appear in the interview. The probability of A being selected is twice that of B and the probability of B being selected is thrice that of C. If the post is filled, what are the probabilities of A, B and C being selected ? [3]

(viii) Fit a straight line to the following data, treating y as the depending variable :

x	14	12	13	14	12
y	22	23	22	24	24

Hence, predict the value of y when x = 16. [3]

(ix) Find the real values of x and y satisfying the equality :

$\{x + 2 + (y - 3)i\}/(1 + i) = 1 - 3i.$ [3]

(x) Solve : $dy/dx = y \sin 2x$, given that $y(0) = 1$. [3]

Question – 2.

(a) Solve the following system of linear equations by Cramer's rule :

$6x + y - 3z = 5$

$x + 3y - 2z = 5$

$2x + y + 4z = 8.$ [5]

(b) Using Martin's rule solve the following equations :

$5x + 3y + z = 16$

$2x + y + 3z = 19$

$x + 2y + 4z = 25.$ [5]

Question – 3.

(a) Find a point on the graph of $y = x^3$, where the tangent is parallel to the chord joining (1, 1) and (3, 27). [5]

(b) The two lines $ty = x + t^2$ and $y + tx = 2t + t^3$ intersect at the point P. Show that P lies on the curve whose equation is $y^2 = 4x$. [5]

Question – 4.

(a) Prove that : $\sin^{-1}(12/13) + \cos^{-1}(4/5) + \tan^{-1}(63/16) = \pi.$ [5]

(b) A, B and C represent switches in an ON position and A', B' and C' represent switches in OFF position. Construct a switching circuit representing the polynomial [B (B + A)] [C (B' + C)]. Use Boolean algebra to that the above circuit is equivalent to a switching circuit in which when B and C are on, the light is on. Construct an equivalent switching circuit. [5]

Question – 5,

(a) Differentiate $\tan^{-1}[x/\{1 + \sqrt{(1 - x^2)}\}]$ w. r. t. $\sec^{-1}\{1/(2x^2 - 1)\}$. [5]

(b) Find the dimensions of the right circular cone of minimum volume that can be circumscribed about a sphere of radius 8 cm. [5]

Question – 6.

(a) Evaluate the integrals : $\int\{(1 - \sin 2x)/(x + \cos^2 x)\}\,dx$ [5]

(b) Draw a rough sketch of the curve $y^2 + 1 = x$, $x < 2$. Find the area enclosed by the curve and the line $x = 2$. [5]

Question – 7.

(a) Spearman's coefficient of rank correlation of the marks obtained by 10 students in History and Mathematics was found to be 0.2. Later it was discovered that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 9 instead of 7. Find the correct value of the correlation coefficient. [5]

(b) Fit a straight line to the following data, treating y as the dependant variable :

x	14	12	13	14	12
y	22	23	22	24	24

Hence predict the value of y when x = 16. [5]

Question – 8.

(a) Two coins are tossed once. Determine P(E/F) if

(i) E : tail appears at least on one coin, F : at least one coin shows head.

(ii) E : no tail appears, F : no head appears. [5]

(b) A bag contains n + 1 coins. It is known that one of these coins shows head on both sides, whereas all other coins are fair. One coin is selected at random and tossed. If the probability that the toss results in head is 7/12, find the value of n. [5]

Question – 9.

(a) If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, prove that

$x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$. [5]

(b) If $z = x + iy$, $w = (2 - iz)/(2z - i)$ and $|w| = 1$, find the locus of z and illustrate it in the complex plane. [5]

Section – B

Question – 10.

(a) Find the shortest distance between the lines :

$(x + 1)/1 = (y + 1)/-6 = (z + 1)/1$

$(x - 3)/1 = (y - 5)/-2 = (z - 7)/1$. [5]

(b) Show that the equation $by + cz + d = 0$ represents a plane parallel to the axis OX. Find the equation to a plane through the points (2, 3, 1) and (4, – 5, 3) and parallel to OX. [5]

Question – 11.

(a) Find the volume of a parallelepiped whose edges are represented by :
$\vec{a} = 2i - 3j + 4k$, $\vec{b} = i + 2j - k$ and $\vec{c} = 3i - j - 2k$. [5]

(b) Prove by vector method that in a right-angled triangle, the mid-point of the hypotenuse is equidistant from the vertices. [5]

Question – 12.

(a) Bag I contains 3 white and 4 black balls and Bag II contains 4 white and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn at random from Bag II. If the ball so drawn is white, find the probability that the transferred ball is black. [5]

(b) Four defective oranges are accidently mixed with sixteen good ones. Three oranges are drawn from the mixed lot. Find the probability distribution of X, the number of defective oranges. Now find the mean of the number of defective oranges. [5]

Section – C

Question – 13.

(a) Solve the following problem graphically :
Minimize $Z = 5x + 2y$ subject to the constraints
$3x + 5y \leq 15$, $5x + 2y \leq 10$, $x, y \geq 0$. [5]

(b) An annuity of equal payments at the beginning of every year for 16 years is calculated to be Rs 60000. If the interest is 9 percent, how much is each payment ? [5]

Question – 14.

(a) The cost of manufacturing x units of a commodity is $27 + 15x + 3x2$.
(i) Find the output for which AC is decreasing and for which AC is increasing.
(ii) Find the output where AC = MC. [5]

(b) Find the actual rate of interest which a banker earns when it discounts a bill nominally due after 70 days at 5.5% p.a. [5]

Question – 15.

(a) The price relatives and weights of a set of commodities are given below :

Comodity	A	B	C	D
Price relative	125	120	127	119
Weight	x	2x	y	y+3

If the sum of weights is 40 and the index for the set is 122, find the numerical values of x and y. [5]

(b) The following table gives the number of failure of commercial industries in a country during the year 1995 to 2010.

Year	1995	1996	1997	1998	1999	2000	2001	2002
Number of failures	23	26	28	32	20	12	12	10
Year	2003	2004	2005	2006	2007	2008	2009	2010
Number of failures	9	13	11	14	12	9	3	1

Draw a graph illustrating these figure.

Calculate the 4 yearly moving average and plot them on the same graph. [5]