ISC SEMESTER 2 EXAMINATION
SAMPLE PAPER - 5
MATHEMATICS

Maximum Marks: 40

Time allowed: One and a half hour

*Candidates are allowed an additional **10 minutes** for **only** reading the paper.*

*They must **Not** start writing during this time.*

The Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

Mathematical tables and graph papers are provided.

Section-A

Question 1.

Choose the correct option for the following questions.

(i) $\int \cot^2 x \, dx$ equals to :

(a) $\cot x - x + c$ (b) $\cot x + x + c$ (c) $-\cot x + x + c$ (d) $-\cot x - x + c$

(ii) $\int_0^4 (x + e^{2x})\, dx =$

(a) $\frac{15 + e^8}{2}$ (b) $\frac{15 - e^8}{2}$ (c) $\frac{e^8 - 15}{2}$ (d) $\frac{-e^8 - 15}{2}$

(iii) The degree of the differential equation $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^4 = e^x$ is :

(a) 1 (b) 2 (c) 3 (d) 4

(iv) $\int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}\, dx =$

(a) $\frac{3}{x^3} + c$ (b) $\frac{x^3}{3} + c$ (c) $\frac{x^{-3}}{3} + c$ (d) None of these

(v) A card is drawn from a well-shuffled deck of 52 cards and then a second card is drawn. Find the probability that first card is a heart and second card is a diamond if the first card is not replaced.

(a) 25/204 (b) 1/4 (c) 12/204 (d) 13/204

(vi) A speaks truth in 75% cases and B in 80% of the cases. Probability that they contradict each other in a statement is :

(a) 7/20 (b) 13/20 (c) 3/5 (d) 2/5

Question 2.

(a) Evaluate : $\int x \log x \, dx$

OR

(b) Evaluate : $\int e^x \cos x \, dx$

Question 3.

(a) Solve the differential equation : $x(1 + y^2)dx - y(1 + x^2)dy = 0$, given that $y = 0$, when $x = 1$.

OR

(b) Solve the differential equation : $\frac{dy}{dx} = \cos(x + y)$

Question 4.

Evaluate : $\int_0^2 \frac{dx}{x + 4 - x^2}$

Question 5.

(a) Three dice are thrown at the same time. Find the probability of getting three two's, if it is known that the sum of the numbers on the dice was six.

OR

(b) A bag contains 3 red and 5 black balls and a second bag contains 6 red and 4 black balls. A ball is drawn from each bag. Find the probability that one of them is red and the other one is black.

Question 6.

Evaluate : $\int_0^{\frac{\pi}{4}} \left(\frac{\sin x + \cos x}{9 + 16 \sin 2x} \right) dx$

Question 7.

A doctor claims that 60% of the patients he examines are allergic to some type of weeds. What is the probability that

(i) Exactly three of the next 4 patient he examines are allergic to weeds?

(ii) None of his next 4 patients is allergic to weeds.

Question 8.

(a) Evaluate : $\int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{2}} \left(\frac{\sin^2 x}{\sin x + \cos x} \right) dx$

Section-B

Question 9.

Choose the correct option for the following questions.

(i) Find the cartesian equation of the following plane.

$$\vec{r}.(2\hat{i} - 3\hat{j} + \hat{k}) = 5$$

(a) $2x - 3y + z = 5$ (b) $x - y + z = 5$

(c) $x - y + 5z = 1$ (d) None of these

(ii) The equation of a plane which cuts equal intercepts of unit length on the axes is :

(a) $x + y + z = 0$ (b) $x + y + z = 1$

(c) $x + y - z = 1$ (d) $x + y + z = 2$

Question 10.

Find the cartesian equation of the plane passing through the point (3, 2, 3) and perpendicular to the line with direction ratios 2, – 1, 2.

Question 11.

Find the area lying above the x-axis and under the parabola $y = 4x - x^2$.

Section-C

Question 12.

Choose the correct option for the following questions.

(i) If regression lines are $3x + 12y - 19 = 0$ and $9x + 3y - 46 = 0$, then find the coefficient of correlation.

(a) 1/12 (b) – 1/12 (c) $1/\sqrt{12}$ (d) $-1/\sqrt{12}$

(ii) If $3x + 8y - 22 = 0$ is a regression line y on x, calculate the regression coefficient of line.

(a) $\frac{8}{3}$ (b) $\frac{3}{8}$ (c) $\frac{-3}{8}$ (d) $\frac{-8}{3}$

Question 13.

The following results were obtained with respect to two variables x and y :

$\Sigma x = 15$, $\Sigma y = 25$, $\Sigma xy = 83$, $\Sigma x^2 = 55$, $\Sigma y^2 = 135$ and $n = 5$.

(i) Find the regression coefficient b_{xy}.

(ii) Find the regression equation of x on y.

Question 14.

A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs. 16 and one kg of food Y costs Rs. 20. Find the least cost of the mixture which will produce the required diet?

Answers

Section-A

Answer 1.

(i) (d) $-\cot x - x + c$

Explanation :

$$\int \cot^2 x \, dx = \int (\text{cosec}^2 x - 1)\, dx = -\cot x - x + c$$

(ii) (a) $\frac{15+e^8}{2}$

Explanation :

$$\int_0^4 (x+e^{2x})dx = \int_0^4 x\, dx + \int_0^4 e^{2x}\, dx$$

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{e^{2x}}{2}\right]_0^4$$

$$= \left(\frac{16}{2}-0\right)+\left(\frac{e^{2\times 4}}{2}-\frac{e^0}{2}\right)$$

$$= \left(8+\frac{e^8}{2}-\frac{1}{2}\right)=\frac{15+e^8}{2}$$

(iii) (a) 1

Explanation :

The order of the differential equation is 3 and the power of that term is 1, hence the degree is 1.

(iv) (b) $\frac{x^3}{3}+c$

Explanation :

$$\int \frac{e^{\log x^6}-e^{\log x^5}}{e^{\log x^4}-e^{\log x^3}}\, dx$$ [using log $(m)n = n$ log m]

$$= \int \frac{x^6-x^5}{x^4-x^3}\, dx$$ [using $e$log $f(x) = f(x)$]

$$= \int \frac{x^5(x-1)}{x^3(x-1)}\, dx = \int x^2\, dx = \frac{x^3}{3}+c$$

(v) (d) $\frac{13}{204}$

Explanation :

Probability of the first card being heart is $\frac{13}{52} = \frac{1}{4}$.

This card is not replaced.

So, the total numbers of cards in the deck becomes 51.

Since, there are 13 diamonds, probability of the second card being diamond is $\frac{13}{51}$.

Hence, the required probability is

$$\frac{1}{4} \times \frac{13}{51} = \frac{13}{204}$$

(vi) (a) $\frac{7}{20}$

Explanation :

P(A) = 75/100, $P(\overline{A})$ = 25/100

P(B) = 80/100, $P(\overline{B})$ = 20/100

$$P(\text{A and B contradicts each other}) = P\left[(A \cap \overline{B}) \cup (\overline{A} \cap B)\right]$$

$$= P(A) \times P(\overline{B}) + P(\overline{A}) \times P(B)$$

$$= (75/100 \times 20/100) + (25/100 \times 80/100)$$

$$= \frac{1500}{10000} + \frac{2000}{10000}$$

$$= \frac{3500}{10000}$$

$$= \frac{35}{100}$$

$$= \frac{7}{20}$$

Answer 2.

(a) Let, $\int x \log x \, dx = 1$

Using "ILATE", we take 'log x' as the 1st function and 'x' as the 2nd function.

$$\therefore \quad I = \log x \int x \, dx - \int \left[\frac{d}{dx}(\log x) \times \int x \, dx \right] dx$$

$$= \frac{x^2}{2} \log x - \int \left(\frac{1}{x} \times \frac{x^2}{2} \right) dx$$

$$= \frac{x^2}{2} \log x - \frac{1}{2} \left(\frac{x^2}{2} \right) + c$$

$$= \frac{x^2}{2} \log x - \frac{x^2}{4} + c$$

OR

(b) Let,

$$I = \int e^x \cos x \, dx$$
$$= e^x \int \cos x - \int e^x (\int \cos x \, dx) \, dx \quad \text{(By uv Rule-Product rule)}$$
$$= e^x \sin x - \int e^x \sin x \, dx + c'$$
$$= e^x \sin x - [e^x \int \sin x - \int e^x (\int \sin x \, dx) \, dx] + c'$$
$$= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx + c'$$
$$= e^x \sin x + e^x \cos x - I + c'$$
$$\Rightarrow \quad 2I = e^x \sin x + e^x \cos x + c'$$
$$\Rightarrow \quad I = 1/2 \, e^x (\sin x + \cos x) + c \quad \text{where, } c = c'/2$$

Answer 3.

(a)

$$x(1 + y^2)dx - y(1 + x^2)dy = 0$$
$$\Rightarrow \quad 2x(1 + y^2)dx = 2y(1 + x^2)dy$$
$$\Rightarrow \quad \int \frac{2x \, dx}{(1+x^2)} = \int \frac{2y \, dy}{(1+y^2)}$$
$$\Rightarrow \quad \log |1 + y^2| = \log |1 + x^2| + \log c$$
$$\Rightarrow \quad (1 + y)^2 = c\,(1 + x^2)$$

By substituting $y = 0$ and $x = 1$, we get $c = 1/2$

$$\therefore \quad (1 + y^2) = (1/2)(1 + x^2)$$
$$\Rightarrow \quad 2(1 + y^2) = (1 + x^2)$$

OR

(b) Let,

$$v = x + y$$
$$\therefore \quad \frac{dv}{dx} = 1 + \frac{dy}{dx}$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{dv}{dx} - 1 = \cos(x + y)$$
$$\Rightarrow \quad \frac{dv}{dx} - 1 = \cos v$$
$$\Rightarrow \quad 1 + \cos v = \frac{dv}{dx}$$
$$\Rightarrow \quad \int \frac{dv}{1+\cos v} = \int dx$$
$$\Rightarrow \quad \int \frac{dv}{2\cos^2(v/2)} = \int dx$$
$$\Rightarrow \quad \int \frac{\sec^2(v/2)}{2} dv = \int dx$$
$$\Rightarrow \quad \frac{1}{2} \times \frac{\tan(v/2)}{\frac{1}{2}} = x + c$$
$$\Rightarrow \quad \tan\left(\frac{x+y}{2}\right) = x + c$$

Answer 4.

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{-dx}{x^2-x+\frac{1}{4}-4-\frac{1}{4}}$$

$$= \int_0^2 \frac{-dx}{\left(x-\frac{1}{2}\right)^2-\frac{17}{4}}$$

Let $\left(x-\frac{1}{2}\right)=t \Rightarrow dx=dt$

for $x=0$, $t=-1/2$

for $x=2$, $t=3/2$

$$\therefore \quad \int_0^2 \frac{-dx}{\left(x-\frac{1}{2}\right)^2-\frac{17}{4}} = \int_{-1/2}^{3/2} \frac{-dx}{(t)^2-\frac{17}{4}}$$

$$= \int_{-1/2}^{3/2} \frac{-dx}{(t)^2-\frac{17}{4}}$$

$$= \frac{-1}{\frac{2\sqrt{17}}{2}} \log\left[\frac{t-\frac{\sqrt{17}}{2}}{t+\frac{\sqrt{17}}{2}}\right]_{-1/2}^{3/2}$$

$$= \frac{-1}{\sqrt{17}}\left[\log\left|\frac{\frac{3}{2}-\frac{\sqrt{17}}{2}}{\frac{3}{2}+\frac{\sqrt{17}}{2}}\right| - \log\left|\frac{-\frac{1}{2}-\frac{\sqrt{17}}{2}}{-\frac{1}{2}+\frac{\sqrt{17}}{2}}\right|\right]$$

$$= \frac{-1}{\sqrt{17}}\left[\log\left|\frac{3-\sqrt{17}}{3+\sqrt{17}}\right| - \log\left|\frac{1+\sqrt{17}}{1-\sqrt{17}}\right|\right]$$

Answer 5.

(a) Three dice are thrown simultaneously.

There are 6 × 6 × 6 = 216 element in the total sample space.

Let us define an element A by assuming the sum of the numbers on the dice is 6.

∴ A = (2, 2, 2), (2, 1, 3), (2, 3, 1), (1, 3, 2), (1, 2, 3), (1, 1,4), (4, 1, 1), (1, 4, 1), (3, 1, 2,), (3, 2, 1)

B = getting a number 2 in all 3 dice = (2, 2, 2)

Since, it is given that the event A has already occurred.

i.e., the sum of the numbers on each die is 6 we have 10 cases out of which only one (2, 2, 2) is favorable to us.

Therefore, P(E) = 1/10

OR

(b) Probability of getting a red ball from bag 1 = $\frac{3}{8}$

Probability of getting a black ball from bag 1 = $\frac{5}{8}$

Probability of getting a red ball from bag 2 = $\frac{6}{10}$

Probability of getting a black ball from bag 2 = $\frac{4}{10}$

Probability of getting a red ball from bag 1 and black ball from bag 2 = (3/8) × (4/10)

= 12/80

= 3/20

Probability of getting a black ball from bag 1 and red ball from bag 2 = (5/8) × (6/10)

= 30/80

= 3/8

Hence, the required probability $= \frac{3}{20}+\frac{3}{8}$

$= \frac{24+60}{160}$

$= \frac{84}{60} = \frac{21}{40}$

Answer 6.

Let, $I = \int_0^{\frac{\pi}{4}} \left(\frac{\sin x + \cos x}{9+16\sin 2x}\right) dx$

Let, $\sin x - \cos x = t$

$\Rightarrow (\cos x + \sin x)dx = dt$

and $(\sin x - \cos x)^2 = t^2$

$\Rightarrow \sin^2 x + \cos^2 x - 2\sin x \cos x = t^2$

$\Rightarrow 1 - \sin 2x = t^2$

$\Rightarrow \sin 2x = 1 - t^2$

When $x = 0$, $t = \sin 0 - \cos 0 = -1$

When $x = \pi/4$, $t = \sin \pi/4 - \cos \pi/4 = 0$

$$\therefore \quad I = \int_{-1}^{0}\left(\frac{1}{9+16(1-t^2)}\right)dt = \frac{1}{16}\int_{-1}^{0}\left(\frac{1}{\left(\frac{25}{16}-t^2\right)}\right)dt$$

$$= \frac{1}{16}\left(\frac{1}{2\times\frac{5}{4}}\log\left[\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right]\right)_{-1}^{0}$$

$$= \frac{1}{4}\left(\frac{1}{10}\log\left[\frac{5+4t}{5-4t}\right]\right)_{-1}^{0}$$

$$= \frac{1}{40}\left(\log\left[\frac{5+4\times 0}{5-4\times 0}\right]-\log\left[\frac{5+4(-1)}{5-4(-1)}\right]\right)$$

$$= \frac{1}{40}\left(\log\left[\frac{5+0}{5-0}\right]-\log\left[\frac{5-4}{5+4}\right]\right)$$

$$= \frac{1}{40}\ (\log 1 - \log (1/9))$$

$$= \frac{1}{40}(\log 1 - \log 1 + \log 9)$$

$$= \frac{\log 9}{40}$$

Answer 7.

The probability of a person being allergic = 6/10

Probability of a person not being allergic = 4/10

Probability of first person being allergic = 6/10

Probability of second person being allergic = 6/10

Probability of third person being allergic = 6/10

Probability of fourth person not being allergic = 4/10

(i) Probabiliy that exactly 3 of the next 4 people are allergic = (6/10) × (6/10) × (6/10) × (4/10) × 4

= 3456/10000 = 0.3456

We multiply by 4 because the non allergic person could be first, second or third. So we need to account for all sequences AAAN, AANA, ANAA, NAAA

(ii) Probability that none of the next 4 people are allergic = (4/10) × (4/10) × (4/ 10) × (4/ 10)

= 256/ 10000 = 0.0256

Answer 8.

(a)

$$\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx = \int e^x\left(\frac{1+2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)}\right)dx$$

$$= \int e^x\left(\frac{\sin^2(x/2)+\cos^2(x/2)+2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)}\right)dx$$

$$= \int e^x\left(\frac{[\sin(x/2)+\cos(x/2)]^2}{2\cos^2(x/2)}\right)dx$$

$$= \frac{1}{2}\int e^x\left(\frac{\sin(x/2)+\cos(x/2)^2}{\cos(x/2)}\right)dx$$

$$= \frac{1}{2}\int e^x\,[\tan(x/2)+1]^2\,dx$$

$$= \frac{1}{2}\int e^x\,[\tan^2(x/2)+1+2\tan(x/2)]\,dx$$

$$= \frac{1}{2}\int e^x\,[2\tan(x/2)+\sec^2(x/2)]\,dx$$

$$= e^x\tan(x/2)+c \qquad (\because \int e^x(f(x)+f'(x))dx = e^x f(x)+c)$$

OR

(b) Let,

$$I = \int_0^{\frac{\pi}{2}}\left(\frac{\sin^2 x}{\sin x+\cos x}\right)dx \qquad ...(1)$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}}\left(\frac{\sin^2(\pi/2-x)}{\sin(\pi/2-x)+\cos(\pi/2-x)}\right)dx \qquad \left(\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}}\left(\frac{\cos^2 x}{\cos x+\sin x}\right)dx \qquad ...(2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\frac{\pi}{2}}\left(\frac{\sin^2 x+\cos^2 x}{\sin x+\cos x}\right)dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{\sin x + \cos x} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2}\left(\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x \right)} dx$$

$$= \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{1}{\cos x . \cos \frac{\pi}{4} + \sin x . \sin \frac{\pi}{4}} dx$$

$$= \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \left(\frac{1}{\cos \left(x - \frac{\pi}{4} \right)} \right) dx$$

$$= \frac{1}{\sqrt{2}} \int_0^{\frac{\pi}{2}} \left(\sec \left(x - \frac{\pi}{4} \right) \right) dx$$

$$= \frac{1}{\sqrt{2}} \left(\log \left[\sec \left(x - \frac{\pi}{4} \right) + \tan \left(x - \frac{\pi}{4} \right) \right] \right)_0^{\frac{\pi}{2}}$$

$$= \frac{1}{\sqrt{2}} \left(\log \left[\sec \frac{\pi}{4} + \tan \frac{\pi}{4} \right] - \log \left[\sec \frac{-\pi}{4} + \tan \frac{-\pi}{4} \right] \right)$$

$$= \frac{1}{\sqrt{2}} \left(\log [\sqrt{2} + 1] - \log [\sqrt{2} - 1] \right)$$

$$= \frac{1}{\sqrt{2}} \log \frac{(\sqrt{2} + 1)}{(\sqrt{2} - 1)} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$= \frac{1}{\sqrt{2}} \log \left[\frac{\sqrt{2} + 1}{1} \right]^2$$

$$\Rightarrow \quad 2I = \frac{2}{\sqrt{2}} \log(\sqrt{2} + 1)$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$$

Section-B

Answer 9.

(i) (a) $2x - 3y + z = 5$

Explanation :

$$\vec{r} . (2\hat{i} - 3\hat{j} + \hat{k}) = 5$$

$$\because \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\Rightarrow \quad (x\hat{i} + y\hat{j} + 2\hat{k}) . (2\hat{i} - 3\hat{j} + \hat{k}) = 5$$

$$\Rightarrow \quad 2x - 3y + z = 5$$

(ii) (b) $x + y + z = 1$

Explanation :

The plane makes intercepts at $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ with the x, y and z - axes respectively is given by

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$$

Given, plane cuts equal intercepts of unit length

$\therefore$ $a = 1$, $b = 1$ and $c = 1$.

Hence the equation is $x + y + z = 1$

Answer 10.

The plane passes through the point $(x_1, y_1, z_1) = (3, 2, 3)$

Normal vector $\vec{n}$ perpendicular to the plane is

$$\vec{n} = 2\hat{i}-\hat{j}+2\hat{k} = a\hat{i}+b\hat{j}+c\hat{k}$$

$\therefore$ Required equation is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$\Rightarrow \quad 2(x - 3) - (y - 2) + 2(z - 3) = 0$$

$$\Rightarrow \quad 2x - 6 - y + 2 + 2z - 6 = 0$$

$$\Rightarrow \quad 2x - y + 2z - 10 = 0$$

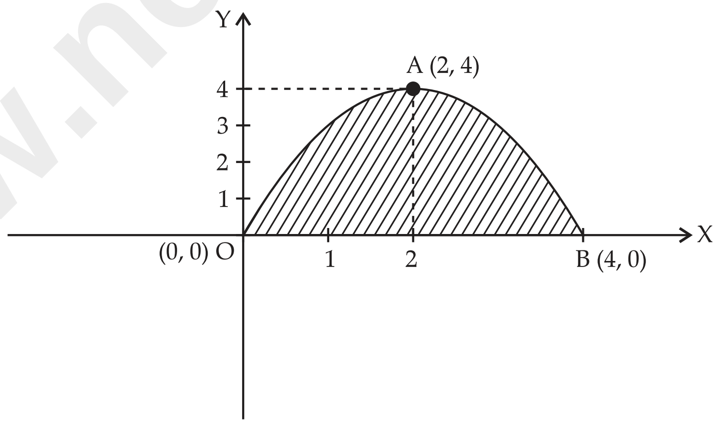
Answer 11.

$$y = 4x - x^2 \quad \text{...(1)}$$

$$\Rightarrow \quad y - 4 = -(x^2 - 4x + 4) \quad \text{(Subtracting 4 on both sides)}$$

$$\Rightarrow \quad -(y - 4) = (x - 2)^2$$

Equation (1) represents a downward parabola with vertex at (2, 4) and passing through (0, 0) and (4, 0) on the x – axis



$$\text{Required Area} = \int_0^4 y\,dx$$

$$= \int_0^4 (4x - x^2)\,dx$$

$$= \left(\frac{4x^2}{2}-\frac{x^3}{3}\right)_0^4$$

$$= \left(\frac{4 \times 4^2}{2} - \frac{4^3}{3}\right)$$

$$= 32 - \frac{64}{3}$$

$$= \frac{32}{3} \text{ sq. units.}$$

Section-C

Answer 12.

(i) (d) $\frac{-1}{\sqrt{12}}$

Explanation :

Given lines of regression are :

$$3x + 12y - 19 = 0 \quad ...(1)$$

$$9x + 3y - 46 = 0 \quad ...(2)$$

From equation (1), the regression line of y on x is

$$12y = -3x + 19$$

$\Rightarrow$ $y = (-3/12)x + (19/12)$

$\Rightarrow$ $y = (-1/4)x + (19/12)$

$\therefore$ $b_{yx} = (-1/4)$

From equation (2), the regression line of x on y is.

$$9x + 3y - 46 = 0$$

$\Rightarrow$ $x = (-3/9)y + (46/9)$

$\Rightarrow$ $x = (-1/3)y + (46/9)$

$\therefore$ $b_{xy} = (-1/3)$

$\therefore$ $r = \sqrt{b_{yx} \cdot b_{xy}}$

$$= \sqrt{\left(\frac{-1}{4}\right)\left(\frac{-1}{3}\right)}$$

$$= \frac{-1}{\sqrt{12}}$$ ($\because$ b_{xy} and b_{yx} are negative, therefore r will be negative)

(ii) (c) $\frac{-3}{8}$

Explanation :

Given, regression equation of y on x

$$3x + 8y = 22$$

$\Rightarrow$ $8y = -3x + 22$

$\Rightarrow$ $y = \frac{-3}{8}x + \frac{22}{8}$

$\therefore$ $b_{yx} = \frac{-3}{8}$

Answer 13.

(i) Given $\Sigma x = 15$, $\Sigma y = 25$, $\Sigma xy = 83$, $\Sigma x^2 = 55$, $\Sigma y^2 = 135$ and $n = 5$

$$b_{xy} = \frac{\Sigma xy - n\bar{x}\,\bar{y}}{\Sigma y^2 - n(\bar{y})^2}$$

$$= \frac{83 - 5 \times 3 \times 5}{135 - 5(25)} \text{ (as } \bar{x} = \Sigma x/n = 15/5 = 3 \text{ and } \bar{y} = \Sigma y/n = 25/5 = 5)$$

$$= \frac{83 - 75}{135 - 125} = \frac{8}{10} = 0.8$$

(ii) $(x - \bar{x}) = b_{xy}(y - \bar{y})$

$\Rightarrow$ $x - 3 = 0.8\ (y - 5)$

$\Rightarrow$ $x = 0.8y - 4 + 3$

$\Rightarrow$ $x = 0.8y - 1$

Answer 14.

Let's assume that mixture contains x kg of food X and y kg of food Y.

Cost of 1 kg of food X = ₹ 16

Cost of 1 kg of food Y = ₹ 20

So, Total cost (C) = $16x + 20y$ Rs ...(1)

Now, food X contains 1 units and food Y contains 2 units of Vitamin A.

So, Total Vitamin A = $x + 2y$ units

Since, minimum requirement of Vitamin A is 10 units.

$\therefore$ $x + 2y \geq 10$ ...(2)

Again, food X contains 2 units and food Y contains 2 units of Vitamin B.

So, Total Vitamin B = $2x + 2y$ units

Since, minimum requirement of Vitamin B is 12 units.

$\therefore$ $2x + 2y \geq 12$

or $x + y \geq 6$ ...(3)

Also, food X contains 3 units and food Y contains 1 units of Vitamin C.

So, Total Vitamin C = $3x + y$ units

Since, minimum requirement of Vitamin C is 8 units.

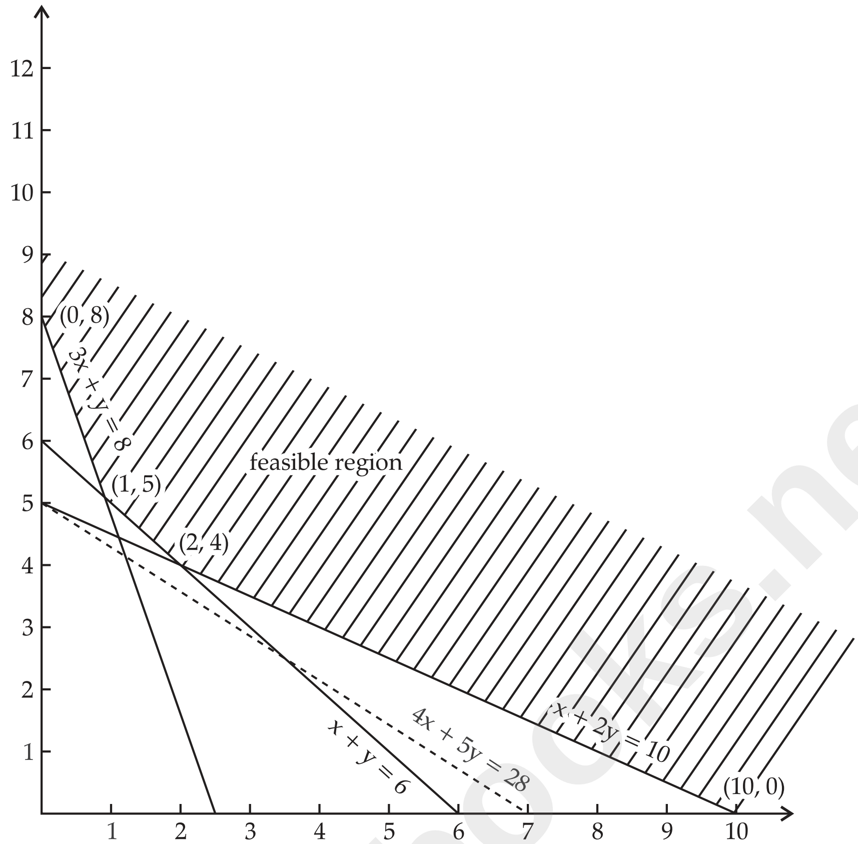
$\therefore$ $3x + y \geq 8$ ...(4)

Since, amount of food can never be negative.

So, $x \geq 0,\ y \geq 0$ ...(5)

We have to minimise the cost of mixture given in equation (1) subject to the constraints given in (2), (3), (4) and (5).

After plotting all the constraints, we get the feasible region as shown in the image.



Corner points	Value of $Z = 16x + 20y$
A (0, 8)	160
B (1, 5)	116
C (2, 4)	112 (minimum)
D (10, 0)	160

Now, since region is unbounded, Hence we need to confirm that minimum value obtained through corner points is true or not.

Now, plot the region $Z < 112$ to check if there exist some points in feasible region whre value can be less than 112.

$$\Rightarrow \quad 16x + 20y < 112$$

$$\Rightarrow \quad 4x + 5y < 28.$$

Since there is no common points between the feasible region and the region which contains $Z < 112$ (See the image). So 112 Rs is the minimum cost.

❑❑