ISC SEMESTER 2 EXAMINATION
SAMPLE PAPER - 2
MATHEMATICS

Maximum Marks: 40

Time allowed: One and a half hour

*Candidates are allowed an additional **10 minutes** for **only** reading the paper.*

*They must **Not** start writing during this time.*

The Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B** **OR** **Section C***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

Mathematical tables and graph papers are provided.

Section-A

Question 1.

Choose the correct option for the following questions.

(i) If $\int (ax+b)^n \, dx = \frac{1}{k} \cdot \frac{(ax+b)^{n+1}}{n+1} + c$, then the value of k is :

(a) b (b) a (c) n (d) 1

(ii) If $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, then $f(x)$ is an :

(a) even function (b) odd function (c) identity function (d) None of these

(iii) If P(A) = 0.4, P(B) = 0.8 and P(B/A) = 0.6. then $P(A \cup B)$ is :

(a) 0.24 (b) 0.30 (c) 0.48 (d) 0.96

(iv) $\int e^x \sec x(1 + \tan x)\, dx =$

(a) $e^x \tan x + c$ (b) $e^x (\sec x . \tan x) + c$ (c) $e^x \sec x + c$ (d) None of these

(v) Write the order of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/4} + x^{1/5} = 0.$

(a) 1 (b) $\frac{1}{4}$ (c) 2 (d) not defined

(vi) P speaks truth in 70% of the cases and Q in 80% of the cases. In what percent of cases are they likely to agree in stating the same fact ?

(a) 50 % (b) 10 % (c) 62 % (d) 38 %

Question 2.

Evaluate : $\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx.$

OR

Evaluate : $\int_0^1 \frac{x}{(x+1)^2} dx.$

Question 3.

Solve the differential equation $\frac{dy}{dx} - e^{y+x} = e^{x-y}$.

OR

Solve : $\frac{dy}{dx} = \frac{1}{y^2 + \sin y}$

Question 4.

Evaluate : $\int_2^5 [\,|x-2| + |x-3| + |x-5|\,]\,dx$.

Question 5.

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and found to be both clubs. Find the probability of the lost card being a club.

OR

A bag contains 8 red and 5 white balls. Two successive draws of 3 balls are made at random from the bag without replacement. Find the probability that the first draw yield 3 white balls and the second draw 3 red balls.

Question 6.

Evaluate : $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2\, dx$.

Question 7.

An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into the urn. Otherwise it is replaced with the another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black.

Question 8.

Evaluate : $\int \tan^{-1} \sqrt{x}\, dx$.

OR

Prove that : $\int_1^2 \frac{\log x}{(1+x)^2}\, dx = \frac{5}{3} \log 2 - \log 3$

Section-B

Question 9.

Choose the correct option for the following questions.

(i) The equation of the plane through the points A(2, 2, – 1), B(3, 4, 2) and C(7, 0, 6) :

(a) $5x + 2y - 3z = 17$ (b) $5x + 2y - 3z = 0$ (c) $5x + 3y - 2z = 17$ (d) None of these

(ii) The equation of the plane whose intercepts on the coordinate axes are – 4, 2 and 3 respectively :

(a) $3x + 6y + 4z = 12$ (b) $-3x + 6y + 4z = 12$ (c) $3x + 4y + 6z = 12$ (d) $6x + 4y + 3z - 12 = 0$

Question 10.

Find the equation of the plane which bisects the line segment joining the points (2, 3, 4) and (4, 5, 8) at right angle.

Question 11.

Find the area of the region bounded by $y = x^2 + 1$, $y = x$, $x = 0$ and $y = 2$.

Section-C

Question 12.

Choose the correct option for the following questions.

(i) If $\Sigma x = 55$, $\Sigma x^2 = 385$, $\Sigma y = 88$, $\Sigma y^2 = 1114$, $n = 10$, $\Sigma xy = 586$ then b_{yx} is :

(a) less than 1 (b) greater than 1 (c) equal to 1 (d) None of these

(ii) Find the mean of x-series and mean of y-series, if $y = \frac{x}{2} + \frac{3}{2}$ and $x = \frac{5}{4}y - \frac{1}{4}$:

(a) 13, 11 (b) $\frac{13}{3}, \frac{11}{3}$ (c) $\frac{13}{3}, \frac{1}{3}$ (d) None of these

Question 13.

Compute b_{yx} for the following data :
$\{(x, y) : (5, 2); (7, 4), (8, 3), (4, 2), (6, 4)\}$.

Question 14.

A carpenter has 90, 80 and 50 running feet respectively of teak wood, plywood and rosewood which is used to produce product A and product B. Each unit of product A requires 2, 1 and 1 running feet and each unit of product B requires 1, 2 and 1 running feet of teak wood, plywood and rosewood respectively. If product A is sold for ₹ 48 per unit and product B is sold for ₹ 40 per unit, how many units of product A and product B should be produced and sold by carpenter, in order to obtain the maximum gross income ? Formulate the above as a linear programming problem and solve it.

Answers

Section-A

Answer 1.

(i) (b) a

Explanation :

$$\int (ax+b)^n \, dx$$

Let $u = ax + b$

$\Rightarrow \frac{du}{dx} = a$

$\Rightarrow \frac{du}{a} = dx$

$\therefore \int (ax+b)^n \, dx = \int u^n \frac{du}{a} = \frac{1}{a}\int u^n \, du$

$= \frac{1}{a}\frac{u^{n+1}}{n+1} + c$

(ii) (a) even function

Explanation :

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is an even function} \\ 0, & \text{if } f(x) \text{ is an odd function} \end{cases}$$

(iii) (d) 0.96

Explanation :

We know, $P(B/A) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow 0.6 = \frac{P(A \cap B)}{0.4}$

$\therefore P(A \cap B) = 0.24$

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.8 - 0.24$

$= 0.96$

(iv) (c) $e^x \sec x + c$

Explanation :

$$\int e^x \sec x(1+\tan x)\,dx = \int e^x (\sec x + \sec x \tan x)\,dx$$

$$\quad\quad\quad\quad\downarrow f(x) \quad \downarrow f'(x)$$

$$= e^x \sec x + c \qquad [\because \int e^x [f(x)+f'(x)]\,dx = e^x f(x) + c]$$

(v) (c) 2

Explanation :

Since, the highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$, so its order is 2.

(vi) (c) 62%

Explanation :

Let P speaks truth is event A.

$\therefore$ $P(A) = 70\% = 0{\cdot}7$

Also, Q speaks truth is event B.

$\therefore$ $P(B) = 80\% = 0{\cdot}8$

Both speak truth $= P(A).P(B) = 0{\cdot}7 \times 0{\cdot}8 = 0{\cdot}56$

Both are lying $= P(\bar{A}).P(\bar{B}) = 0{\cdot}3 \times 0{\cdot}2 = 0{\cdot}06$

Required probability $= 0{\cdot}56 + 0{\cdot}06 = 0{\cdot}62 = 62\%$.

Answer 2.

$$\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x}\,dx = \int \frac{2 \sin x \cos x}{a^2 \sin^2 x + b^2 \cos^2 x}\,dx$$

Let $a^2 \sin^2 x + b^2 \cos^2 x = t$

$\Rightarrow$ $(2a^2 \sin x \cos x - 2b^2 \sin x \cos x)\,dx = dt$

$$2 \sin x \cos x\,dx = \frac{dt}{a^2 - b^2}$$

$\therefore$ $$\int \frac{2\sin x \cos x}{a^2 \sin^2 x + b^2 \cos^2 x}\,dx = \int \frac{dt}{(a^2-b^2)t} = \frac{1}{a^2-b^2} \log|t| + c$$

$\therefore$ $$\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x}\,dx = \frac{1}{a^2-b^2} \log|a^2 \sin^2 x + b^2 \cos^2 x| + c$$ **Ans.**

OR

Let $$I = \int_0^1 \frac{x}{(x+1)^2}\,dx$$

$$= \int_0^1 \frac{x+1-1}{(x+1)^2}\,dx$$

$$= \int_0^1 \frac{(x+1)}{(x+1)^2}\,dx - \int_0^1 \frac{dx}{(x+1)^2}$$

$$= \int_0^1 \frac{1}{x+1}\,dx - \int_0^1 \frac{dx}{(x+1)^2}$$

$$= [\log|x+1|]_0^1 + \left[\frac{1}{x+1}\right]_0^1$$

$$= [\log 2 - \log 1] + \frac{1}{1+1} - \frac{1}{0+1}$$

$$= \log 2 - \frac{1}{2}.$$ **Ans.**

Answer 3.

Given : $\frac{dy}{dx} - e^{x+y} = e^{x-y}$

$\Rightarrow \quad \frac{dy}{dx} = e^{x-y} + e^{x+y} = e^x\,(e^{-y} + e^y)$

$\Rightarrow \quad \frac{dy}{dx} = e^x \left(\frac{1}{e^y} + e^y\right) = \frac{e^x(1+e^{2y})}{e^y}$

$\Rightarrow \quad \frac{e^y}{e^{2y}+1}\,dy = e^x\,dx$

Integrating both sides,

$$\int \frac{e^y}{e^{2y}+1}\,dy = \int e^x\,dx$$

Let $e^y = t$

$\Rightarrow e^y\,dy = dt$

$\therefore \quad \int \frac{dt}{t^2+1} = e^x + c$

$\Rightarrow \quad \tan^{-1} t = e^x + c$

$\Rightarrow \quad \tan^{-1}(e^y) = e^x + c.$ **Ans.**

OR

We have, $\frac{dy}{dx} = \frac{1}{y^2 + \sin y}$

$\Rightarrow \quad \frac{dx}{dy} = y^2 + \sin y$

$\Rightarrow \quad dx = (y^2 + \sin y)\,dy$

Integrating both sides

$$\int dx = \int y^2\,dy + \int \sin y\,dy$$

$\Rightarrow \quad x = \frac{y^3}{3} + (-\cos y) + c$

$\Rightarrow \quad x = \frac{y^3}{3} - \cos y + c$ **Ans.**

Which is required Answer.

Answer 4.

Let $\mathrm{I} = \int_2^5 [\,|x-2| + |x-3| + |x-5|\,]\,dx$

$\mathrm{I} = \int_2^3 [(x-2) - (x-3) - (x-5)]\,dx + \int_3^5 [(x-2) + (x-3) - (x-5)]\,dx$

$\Rightarrow \quad \mathrm{I} = \int_2^3 (6-x)\,dx + \int_3^5 x\,dx$

$\Rightarrow \quad \mathrm{I} = \left[6x - \frac{x^2}{2}\right]_2^3 + \left[\frac{x^2}{2}\right]_3^5$

$\Rightarrow \quad \mathrm{I} = \left[18 - \frac{9}{2} - 12 + 2\right] + \frac{1}{2}[25 - 9]$

$\therefore \quad \mathrm{I} = \frac{7}{2} + 8 = \frac{23}{2}.$ **Ans.**

Answer 5.

Let E_1, E_2 and A be the events defined as follows :

E_1 = Lost card is club

E_2 = Lost card is not a club

A = Two cards drawn are both club

$$\therefore \quad P(E_1) = \frac{13}{52} = \frac{1}{4}$$

$$P(E_2) = \frac{39}{52} = \frac{3}{4}$$

Probability of getting two club cards if lost card is club

$$P(A/E_1) = \frac{^{12}C_2}{^{51}C_2}$$

$$= \frac{12 \times 11}{51 \times 50} = \frac{22}{425}$$

Probability of getting two club cards if lost card is not a club

$$P(A/E_2) = \frac{^{13}C_2}{^{51}C_2}$$

$$= \frac{13 \times 12}{51 \times 50} = \frac{26}{425}$$

Probability of the lost card being a club

$$= \frac{P(E_1) \times P(A/E_1)}{P(E_1) \times P(A/E_1) + P(E_2)\, P(A/E_2)}$$

$$= \frac{\frac{1}{4} \times \frac{22}{425}}{\frac{1}{4} \times \frac{22}{425} + \frac{3}{4} \times \frac{26}{425}}$$

$$= \frac{22}{22 + 78} = \frac{22}{100}$$

Hence, required probability = 0·22. **Ans.**

OR

Given :

8 red balls + 5 white balls

Let E : Event that 3 balls in the first draw are all white.

$$\therefore \quad P(E) = \frac{^{5}C_3}{^{13}C_3} = \frac{5 \times 4 \times 3}{13 \times 12 \times 11} = \frac{5}{143}$$

F : Event that 3 balls in the second draw are all red.

$$\therefore \quad P\left(\frac{F}{E}\right) = \frac{^{8}C_3}{^{10}C_3} = \frac{8 \times 7 \times 6}{10 \times 9 \times 8} = \frac{7}{15}$$

$$\text{Required probability} = P(E \cap F) = P(E).P(F/E)$$

$$= \frac{5}{143} \cdot \frac{7}{15} = \frac{7}{429}$$

Hence probability that the first draw yield 3 white balls and the second draw 3 red balls

$$= \frac{7}{429}.$$ **Ans.**

Answer 6.

Let $$I = \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2 \, dx$$

$$I = \int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx - 2\cos ax \sin bx)\, dx$$

Suppose, $f(x) = \cos^2 ax$,

$\therefore$ $f(-x) = [\cos(-ax)]^2 = \cos^2 ax$

$\therefore$ It is an even function.

Similarly, $\phi(x) = \sin^2 bx$

$\therefore$ $\phi(-x) = [\sin(-ax)]^2 = \sin^2 ax$

It is an even function.

And $\theta(x) = 2\cos ax \sin bx$

$$\theta(-x) = 2\cos(-ax)\sin(-bx)$$
$$\theta(x) = -2\cos ax \sin bx = -\theta(x)$$

$\therefore$ It is an odd function.

$$\because \quad \int_{-a}^{a} f(x)\,dx = \begin{cases} 0, & \text{if it is an odd function} \\ 2\int_0^a f(x)\,dx, & \text{if it is an even function} \end{cases}$$

$$\int_{-\pi}^{\pi} \cos^2 ax\, dx = 2\int_0^{\pi} \cos^2 ax\, dx$$

$$\int_{-\pi}^{\pi} \sin^2 bx\, dx = 2\int_0^{\pi} \sin^2 bx\, dx$$

$$\int_{-\pi}^{\pi} 2\cos ax \sin bx = 0$$

$$\therefore \quad \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2\, dx = 2\int_0^{\pi} \cos^2 ax\, dx + 2\int_0^{\pi} \sin^2 bx - 0$$

$$= 2\int_0^{\pi} \frac{1+\cos 2ax}{2}\, dx + 2\int_0^{\pi} \frac{1-\cos 2bx}{2}\, dx$$

$$= \int_0^{\pi} (1 + \cos 2ax + 1 - \cos 2bx)\, dx$$

$$= \int_0^{\pi} 2\, dx + \int_0^{\pi} \cos 2ax\, dx - \int_0^{\pi} \cos 2bx\, dx$$

$$= [2x]_0^{\pi} + \left[\frac{\sin 2ax}{2a}\right]_0^{\pi} - \left[\frac{\sin 2bx}{2b}\right]_0^{\pi}$$

$$= 2\pi + \frac{1}{2a}\sin 2a\pi - \frac{1}{2b}\sin 2b\pi.$$ **Ans.**

Answer 7.

Given :

In a bag 2 white and 2 black balls. When the black ball is drawn it is replaced along with another black ball and when the white ball is drawn, then white ball is not replaced.

Four possibilities :

	First ball	Second ball	Event
(a)	White	White	E_1
(b)	White	Black	E_2
(c)	Black	White	E_3
(d)	Black	Black	E_4

Let E denote the event of 3^{rd} ball drawn is black.

Case (a) : When first two balls are white.

$$P(E_1) = \frac{2}{4}\cdot\frac{1}{3} = \frac{1}{6}$$

When first two balls are white and 3^{rd} balls are black.

$$\therefore \quad P(E/E_1) = \frac{2}{2} = 1$$

Case (b) : When first ball is white and second ball is black.

$$\therefore \qquad P(E_2) = \frac{2}{4} \cdot \frac{2}{3} = \frac{1}{3}$$

When first ball is white, second and third balls are black.

$$P(E/E_2) = \frac{3}{4}$$

Case (c) : When first ball is black and second ball is white.

$$P(E_3) = \frac{2}{4} \cdot \frac{2}{5} = \frac{1}{5}$$

When first ball is black, second is white and third ball is black.

$$P(E/E_3) = \frac{3}{4}$$

Case (d) : When first two balls drawn are black.

$$P(E_4) = \frac{2}{4} \cdot \frac{3}{5} = \frac{3}{10}$$

When all three balls are black.

$$P(E/E_4) = \frac{4}{6} = \frac{2}{3}$$

$$\begin{aligned}\text{Required probability} &= P(E_1).P(E/E_1) + P(E_2).P(E/E_2) + P(E_3).P(E/E_3) + P(E_4).P(E/E_4) \\ &= \frac{1}{6} \times 1 + \frac{1}{3} \times \frac{3}{4} + \frac{1}{5} \times \frac{3}{4} + \frac{3}{10} \times \frac{2}{3} \\ &= \frac{1}{6} + \frac{1}{4} + \frac{3}{20} + \frac{1}{5} \\ &= \frac{10 + 15 + 9 + 12}{60} = \frac{46}{60} = \frac{23}{30}\end{aligned}$$

Hence, the probability that the third ball drawn is black, is $\frac{23}{30}$. **Ans.**

Answer 8.

$$\int \tan^{-1} \sqrt{x}\, dx$$

Let $\quad I = \int \underset{II}{\tan^{-1}} \sqrt{x}.\underset{I}{1}\, dx$

We know, $\quad \int uv\, dx = u\int v\, dx - \int \left[\frac{du}{dx} \int v\, dx \right] dx$

$$\therefore \qquad I = \tan^{-1} \sqrt{x} \int 1\, dx - \int \left[\frac{d}{dx} (\tan^{-1} \sqrt{x}) \int 1\, dx \right] dx$$

$$\Rightarrow \qquad = x \tan^{-1} \sqrt{x} - \int \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} \cdot x\, dx$$

$$\Rightarrow \qquad = x \tan^{-1} \sqrt{x} - \frac{1}{2} \int \frac{\sqrt{x}}{1+x} dx$$

$$\therefore \qquad I = x \tan^{-1} \sqrt{x} - \frac{1}{2} I_1 \qquad \ldots(i)$$

Now, $\quad I_1 = \int \frac{\sqrt{x}}{1+x} dx$

Let $x = \tan^2 \theta$

$\Rightarrow dx = 2 \tan \theta \sec^2 \theta\, d\theta$

$\therefore \quad I_1 = \int \frac{\tan\theta . 2\tan\theta \sec^2\theta \, d\theta}{1+\tan^2\theta}$

$\Rightarrow \quad I_1 = \int \frac{2\tan^2\theta \sec^2\theta}{\sec^2\theta} d\theta$

$\Rightarrow \quad I_1 = 2\int \tan^2\theta \, d\theta = 2\int (\sec^2\theta - 1)\, d\theta$

$\therefore \quad I_1 = 2(\tan\theta - \theta) = 2(\sqrt{x} - \tan^{-1}\sqrt{x}) + c$ ...(ii)

From (i) and (ii),

$$I = x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x}$$

$\therefore \quad I = (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$ **Ans.**

OR

We have,

$$\text{L.H.S.} = \int_1^2 \frac{\log x}{(1+x)^2} dx$$

Integrating by parts

$$= \int_1^2 \log x \frac{1}{(1+x)^2} dx$$

On integrating by parts,

$$= \left[\log x\left(\frac{-1}{1+x}\right) - \int \frac{(-1)}{x(1+x)} dx\right]_1^2$$

$$= \left[-\frac{\log x}{1+x} + \int \frac{1}{x(1+x)} dx\right]_1^2$$

$$= \left[-\frac{\log x}{1+x} + \int \frac{1+x}{x(1+x)} dx - \int \frac{x}{x(1+x)} dx\right]_1^2$$

$$= \left[-\frac{\log x}{1+x} + \int \frac{1}{x} dx - \int \frac{1}{1+x} dx\right]_1^2$$

$$= \left[-\frac{\log x}{1+x} + \log x - \log(1+x)\right]_1^2$$

$$= \left(-\frac{\log 2}{3} + \log 2 - \log 3\right) - \left(-\frac{\log 1}{2} + \log 1 - \log 2\right)$$

$= \frac{2}{3}\log 2 - \log 3 + \log 2$ [$\because$ log 1 = 0]

$$= \frac{5}{3}\log 2 - \log 3$$

= R.H.S. **Hence Proved.**

Section-B

Answer 9.

(i) (a) $5x + 2y - 3z = 17$

Explanation :

The equation of the plane passing through (2, 2, – 1), (3, 4, 2) and (7, 0, 6) is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

or $$\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$$

$$\Rightarrow (x-2)(14+6) - (y-2)(7-15) + (z+1)(-2-10) = 0$$

$$\Rightarrow \quad 20(x-2) + 8(y-2) - 12(z+1) = 0$$

$$\Rightarrow \quad 20x + 8y - 12z - 68 = 0$$

or $$5x + 2y - 3z = 17$$

(ii) (b) $-3x + 6y + 4z = 12$

Explanation :

Equation of intercept form the plane is given by : $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$

Here, $a = -4$, $b = 2$, and $c = 3$

So, $$\frac{x}{-4}+\frac{y}{2}+\frac{z}{3} = 1$$

or $$-3x + 6y + 4z = 12$$

Answer 10.

Let C be the mid-point of the line segment joining A(2, 3, 4) and B(4, 5, 8) then the coordinates of the mid point C are (3, 4, 6).

$$\vec{A} = 2\hat{i}+3\hat{j}+4\hat{k}$$

$$\vec{B} = 4\hat{i}+5\hat{j}+8\hat{k}$$

$$\vec{AB} = \vec{B}-\vec{A} = 4\hat{i}+5\hat{j}+8\hat{k}-2\hat{i}-3\hat{j}-4\hat{k}$$

$$= 2\hat{i}+2\hat{j}+4\hat{k}$$

We observe that the required plane passes through mid-point of AB *i.e.* $\vec{C} = 3\hat{i}+4\hat{j}+6\hat{k}$

$\therefore$ $\vec{a} = 3\hat{i}+4\hat{j}+6\hat{k}$ and is normal to the plane $\vec{n} = \vec{AB} = 2\hat{i}+2\hat{j}+4\hat{k}$

So, the vector equation of the plane is $\vec{r}.\vec{n} = \vec{a}.\vec{n}$

$$\Rightarrow \quad \vec{r}.(2\hat{i}+2\hat{j}+4\hat{k}) = (3\hat{i}+4\hat{j}+6\hat{k}).(2\hat{i}+2\hat{j}+4\hat{k})$$

$$\Rightarrow \quad \vec{r}.(2\hat{i}+2\hat{j}+4\hat{k}) = 3\times 2 + 4\times 2 + 6\times 4$$

$$\Rightarrow \quad \vec{r}.(2\hat{i}+2\hat{j}+4\hat{k}) = 6 + 8 + 24 = 38$$

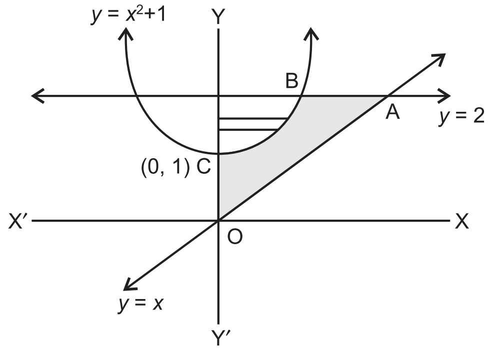
$$\Rightarrow \quad \vec{r}.(\hat{i}+\hat{j}+2\hat{k}) = 19$$ [Vector equation of the plane]

$$\Rightarrow \quad (x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}+\hat{j}+2\hat{k}) = 19$$

$$\Rightarrow \quad x + y + 2z = 19$$ [Cartesion equation of the plane]

Answer 11.

Area bounded by $y = x^2 + 1$, $y = x$, $y = 2$ and $x = 0$ is OABCO



$$\text{Required area} = \int_0^2 y\,dy - \int_1^2 \sqrt{y-1}\,dy$$

$$= \left[\frac{y^2}{2}\right]_0^2 - \frac{2}{3}\left[(y-1)^{3/2}\right]_1^2$$

$$= (2-0) - \frac{2}{3}(1-0)$$

$$= 2 - \frac{2}{3} = \frac{4}{3}\ \text{unit}^2.$$

Ans.

Section-C

Answer 12.

(i) (b) greater than 1

Explanation :

Since we know that

$$b_{yx} = \frac{\Sigma xy - \frac{\Sigma x\,\Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} = \frac{586 - \frac{55 \times 88}{10}}{385 - \frac{(55)^2}{10}}$$

$$= \frac{586 - 484}{385 - 302.5} = \frac{102}{82.5} = 1.24$$

(ii) (b) $\frac{13}{3}, \frac{11}{3}$

Explanation :

Given : $y = \frac{x}{2} + \frac{3}{2}$ and $x = \frac{5}{4}y - \frac{1}{4}$

$\Rightarrow$ $2y = x + 3$ and $4x = 5y - 1$

$\Rightarrow$ $2y - 3 = x$ and $4x = 5y - 1$

$\therefore$ $4(2y - 3) = 5y - 1$

$\Rightarrow$ $8y - 12 = 5y - 1$

$\Rightarrow$ $3y = 11$

$\Rightarrow$ $y = \frac{11}{3}$

$\therefore$ $x = 2y - 3 = \frac{22}{3} - 3 = \frac{13}{3}$

$\therefore$ Mean of x-series $= \frac{13}{3}$

and, Mean of y-series $= \frac{11}{3}$

Answer 13.

x	y	xy	x^2
5	2	10	25
7	4	28	49
8	3	24	64
4	2	8	16
6	4	24	36
$\Sigma x = 30$	$\Sigma y = 15$	$\Sigma xy = 94$	$\Sigma x^2 = 190$

We know that

$$b_{yx} = \frac{\Sigma xy - \frac{\Sigma x \, \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} = \frac{94 - \frac{30 \times 15}{5}}{190 - \frac{(30)^2}{5}}$$

$$\Rightarrow \qquad b_{yx} = \frac{470 - 450}{950 - 900} = \frac{20}{50} = 0.4$$

Answer 14.

	Teak wood	Plywood	Rosewood	Profit
Product A	2	1	1	₹ 48
Product B	1	2	1	₹ 40
Available Length	90	80	50	—

Let he produce x units of product A and y units of product B.

Object : To get maximum income

$$Z = 48x + 40y$$

Subject to constraints :

$$2x + y \leq 90$$
$$x + 2y \leq 80$$
$$x + y \leq 50$$
$$x \geq 0, \; y \geq 0$$

Consider, $2x + y = 90$

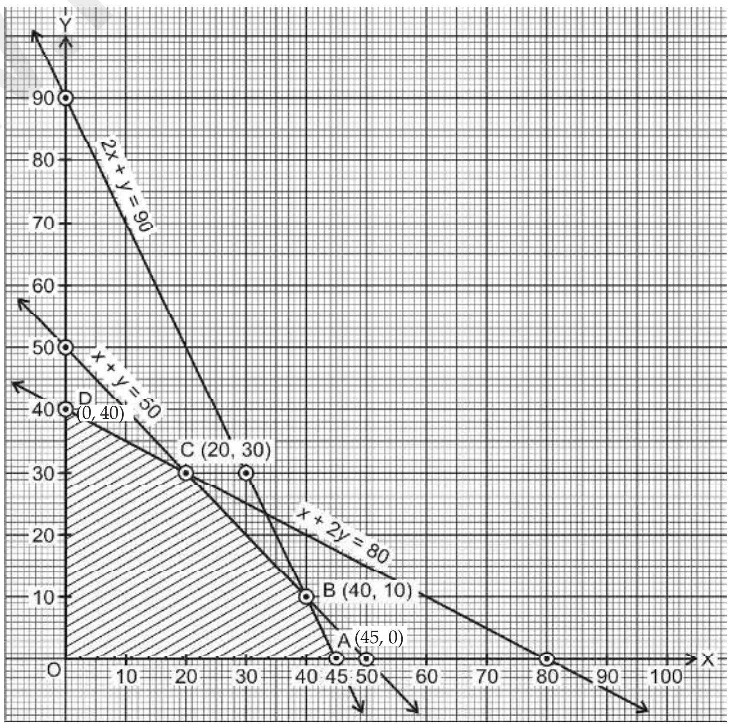
x	0	45	30
y	90	0	30

$x + 2y = 80$

x	0	80	20
y	40	0	30

$x + y = 50$

x	0	50	25
y	50	0	25



The shaded region OABCD is the feasible region.

At A(45, 0),	$Z = 48 \times 45 + 40 \times 0 = ₹\ 2160$
At B(40, 10),	$Z = 48 \times 40 + 40 \times 10 = 1920 + 400 = ₹\ 2320$
At C(20, 30),	$Z = 48 \times 20 + 40 \times 30 = 960 + 1200 = ₹\ 2160$
At D(0, 40),	$Z = 48 \times 0 + 40 \times 40 = 0 + 1600 = ₹\ 1600$
At O(0, 0),	$Z = 48 \times 0 + 40 \times 0 = ₹\ 0$

So, he should produce 40 units of product A and 10 units of product B to get maximum income of ₹ 2320. **Ans.**

❑❑