ISC SEMESTER 2 EXAMINATION
SPECIMEN QUESTION PAPER
MATHEMATICS

Maximum Marks: 40

Time allowed: One and a half hour

Candidates are allowed an additional ***10 minutes*** *for* ***only*** *reading the paper.*

They must ***Not*** *start writing during this time.*

The Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions* ***EITHER*** *from* ***Section B OR Section C***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 32 MARKS

Question 1

Choose the correct option for the following questions.

(i) If $\int \frac{(\log x)^2}{x}\,dx = \frac{(\log x)^k}{k} + c$, then the value of k is: **[1]**

(a) 3

(b) 2

(c) 1

(d) None of the above options

(ii) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(k-x)\,dx$, then the value of k is: **[1]**

(a) a

(b) $2a$

(c) Independent of a

(d) None of the above options

(iii) The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2\left(\frac{d^2y}{dx^2}\right)^2$ is: **[1]**

(a) 1

(b) 2

(c) 3

(d) 4

(iv) Given $\int e^x\left(\frac{x-1}{x^2}\right)dx = e^x f(x) + c$. Then $f(x)$ satisfying the equation is: **[1]**

(a) x

(b) x^2

(c) $\frac{1}{x}$

(d) None of the above options

(v) Two cards are drawn out randomly from a pack of 52 cards one after the other, without replacement. The probability of first card being a king and second card not being a king is: **[1]**

(a) $\frac{48}{663}$

(b) $\frac{24}{663}$

(c) $\frac{12}{663}$

(d) $\frac{4}{663}$

(vi) If two balls are drawn from a bag containing 3 white, 4 black and 5 red balls. Then, the probability that the drawn balls are of different colours is: **[1]**

(a) $\frac{1}{66}$

(b) $\frac{3}{66}$

(c) $\frac{19}{66}$

(d) $\frac{47}{66}$

Question 2 **[2]**

(a) Evaluate : $\int \frac{x^3-x^2+x-1}{x-1}\, dx$

OR

(b) Evaluate : $\int \log_{10} x\, dx$

Question 3 **[2]**

(a) Solve the differential equation :
$cosec^3 x dy - \operatorname{cosec} y\, dx = 0$

OR

(b) Solve the differential equation :
$\frac{dy}{dx} = 2^{-y}$

Question 4 **[2]**

Evaluate : $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx$

Question 5 **[4]**

(a) A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn from the first bag and without noticing its colour is placed in the second bag. If a ball is drawn from the second bag, then find the probability that the drawn ball is red in colour.

OR

(b) A bag contains 3 red and 4 white balls and another bag contains 2 red and 3 white balls. If one ball is drawn from the first bag and 2 balls are drawn from the second bag, then find the probability that all three balls are of the same colour.

Question 6 **[4]**

Evaluate : $\int \frac{1+\sin x}{1-sin\, x} dx$

Question 7 **[6]**

In a bolt factory, machines X, Y and Z manufacture 20%, 35% and 45% respectively of the total output. Of their output 8%, 6% and 5% respectively are defective bolts. One bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured in the machine Y?

Question 8 **[6]**

(a) Evaluate: $\int \frac{dx}{\sin x + \sin 2x}$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{4}} \log(1 + tanx)\, dx$

SECTION B - 8 MARKS

Question 9 **[2]**

Choose the correct option for the following questions.

(i) The equation of the plane which is parallel to $2x - 3y + z = 0$ and which passes through $(1, -1, 2)$ is:

(a) $2x - 3y + z - 7 = 0$

(b) $2x - 3y + z + 7 = 0$

(c) $2x - 3y + z - 8 = 0$

(d) $2x - 3y + z + 6 = 0$

(ii) The intercepts made on the coordinate axes by the plane $2x + y - 2z = 3$ are:

(a) $\frac{-3}{2}, -3, \frac{-3}{2}$

(b) $\frac{3}{2}, 3, \frac{-3}{2}$

(c) $\frac{3}{2}, -3, \frac{-3}{2}$

(d) $\frac{3}{2}, 3, \frac{3}{2}$

Question 10 **[2]**

Find the equation of the plane passing through the point $(1, 1, 1)$ and is perpendicular to the line $\frac{x-1}{3} = \frac{y-2}{0} = \frac{z-3}{4}$. Also, find the distance of this plane from the origin.

Question 11 **[4]**

Using integration, find the area of the region bounded between the line $x = 4$ and the parabola $y^2 = 16x$.

SECTION C - 8 MARKS

Question 12 **[2]**

Choose the correct option for the following questions.

(i) If the regression line of $x \text{ on } y$ is, $9x + 3y - 46 = 0$ and $y \text{ on } x$ is, $3x + 12y - 7 = 0$, then the correlation coefficient 'r' is equal to:

(a) $\frac{-1}{12}$

(b) $\frac{1}{12}$

(c) $\frac{-1}{2\sqrt{3}}$

(d) $\frac{1}{2\sqrt{3}}$

(ii) If $\bar{X} = 40, \bar{Y} = 6, \sigma_x = 10, \sigma_y = 1.5 \text{ and } r = 0.9$ for the two sets of data X and Y, then the regression line of X on Y will be :

(a) $x - 6y - 4 = 0$

(b) $x + 6y - 4 = 0$

(c) $x - 6y + 4 = 0$

(d) $x + 6y + 4 = 0$

Question 13 **[2]**

For 5 observations of pairs (x, y) of variables X and Y, the following results are obtained:
$\sum x = 15, \sum y = 25, \sum x^2 = 55, \sum y^2 = 135, \sum xy = 83.$

Calculate the value of $b_{xy} \text{ and } b_{yx}$.

Question 14 **[4]**

A manufacturer wishes to produce two commodities A and B. The number of units of material, labour and equipment needed to produce one unit of each commodity is shown in the table given below. Also shown is the available number of units of each item, material, labour, and equipment.

Items	Commodity A	Commodity B	Available no. of Units
Material	1	2	8
Labour	3	2	12
Equipment	1	1	10

Find the maximum profit if each unit of commodity A earns a profit of ₹ 2 and each unit of B earns a profit of ₹ 3.

Answers

Section-A

Answer 1.

(i) (a) 3

Explanation :

Given, $\int \frac{(\log x)^2}{x} dx = \frac{(\log x)^k}{k} + c$

Taking L.H.S. $= \int \frac{(\log x)^2}{x} dx$

Let, $\log x = t$

$\therefore$ $\frac{1}{x} dx = dt$

$= \int t^2 dt = \frac{t^3}{3} + c$

Substituting the value of t,

$= \frac{(\log x)^3}{3} + c$

On comparing with R.H.S. we get

$k = 3$

(ii) (b) $2a$

Explanation :

Given, $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(k - x) dx$ ...(i)

We know, from properties of integrals

$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a - x) dx$ ...(ii)

From equations (i) and (ii), we get

$k = 2a$

(iii) (b) 2

Explanation :

Given differential equation is

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2\left(\frac{d^2y}{dx^2}\right)^2$$

The degree of a differential equation is the degree of the highest order occuring in it.

so, order of the differential equtaion is 2 and the degree of the differential equation is also 2.

(iv) (c) $\frac{1}{x}$

Explanation :

Given, $\int e^x \left(\frac{x-1}{x^2}\right) dx = e^x f(x) + c$

Taking L.H.S. $= \int e^x \left(\frac{x-1}{x^2}\right) dx$

$$= \int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

$$= \int e^x . \frac{1}{x} dx - \int e^x . \frac{1}{x^2} dx$$

Integrating the first integral by parts taking $\frac{1}{x}$ as the first function,

$$= \frac{1}{x}.e^x + \int \frac{1}{x^2}.e^x dx - \int e^x . \frac{1}{x^2} dx + c$$

$$= \frac{1}{x}.e^x + c$$

On comparing with the R.H.S., we get

$$f(x) = \frac{1}{x}$$

(v) (a) $\frac{48}{663}$

Explanation :

In a pack of cards there are 52 cards.

The number of kings in a pack = 4

Number of cards without kings = 48

So, the probability that the first card is a king and second is not a king

$$= \frac{4}{52} \times \frac{48}{51} \quad \text{(without replacement)}$$

$$= \frac{48}{663}$$

(vi) (d) $\frac{47}{66}$

Explanation :

Given, Number of white balls = 3

Number of Black balls = 4

Number of Red balls = 5

Total no. of balls (ways) = 12

Total no. of ways of drawing 2 balls out of 12 balls

$$= {}^{12}C_2 = \frac{12!}{10!2!} = 66$$

Total no. of ways of drawing 2 balls of different colours

= 1 white 1 black + 1 black 1 red + 1 red 1 white

$= {}^3C_1 \times {}^4C_1 + {}^4C_1 \times {}^5C_1 + {}^5C_1 \times {}^3C_1$

$= 3 \times 4 + 4 \times 5 + 5 \times 3$

= 47

So, probabiltiy of drawing 2 balls of different colours

$$= \frac{47}{66}$$

Answer 2.

(a) Given : $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

$$= \int \frac{x^2(x-1) + 1(x-1)}{(x-1)} dx$$

$$= \int \frac{(x-1)(x^2+1)}{(x-1)} dx$$

$$= \int (x^2+1) dx$$

$$= \int x^2 dx + \int 1 dx$$

$$= \frac{x^3}{3} + x + c$$

OR

(b) Given : $\int \log_{10} x \, dx = \int \frac{\log_e x}{\log_e 10} dx$

$$= \frac{1}{\log_e 10} \int \log_e x . 1 dx$$

On integrating by parts, we get

$$= \frac{1}{\log_e 10}\left[\log_e x . x - \int \frac{1}{x} . x \, dx\right] + c$$

$$= \frac{1}{\log_e 10}(x \log_e x - x) + c$$

$= x(\log_e x - 1). \log_{10} e + c$

Answer 3.

(a) Given, differential equation is

$\operatorname{cosec}^3 x \, dy - \operatorname{cosec} y \, dx = 0$

$\Rightarrow \quad \operatorname{cosec}^3 x \, dy = \operatorname{cosec} y \, dx$

$\Rightarrow \quad \int \frac{dy}{\operatorname{cosec} y} = \int \frac{dx}{\operatorname{cosec}^3 x}$

$\Rightarrow \quad \int \sin y \, dy = \int \sin^3 x \, dx$

$\Rightarrow \quad -\cos y = \int \sin^2 x . \sin x \, dx$

$$= \int (1 - \cos^2 x) . \sin x \, dx$$

Let $\cos x = t$,

$\Rightarrow \quad -\sin x \, dx = dt$

$\therefore \quad -\cos y = -\int (1 - t^2) dt$

$$\Rightarrow \quad \cos y = t - \frac{t^3}{3} + c$$

$$\therefore \quad \cos y = \cos x - \frac{\cos^3 x}{3} + c$$

OR

(b) Given differential equation is

$$\frac{dy}{dx} = 2^{-y}$$

On separating the variables, we get

$$\frac{dy}{2^{-y}} = dx$$

$$\Rightarrow \quad 2^y \, dy = dx$$

On integrating both sides, we get

$$\int 2^y dy = \int dx$$

$$\Rightarrow \quad \frac{2^y}{\log 2} = x + c_1$$

$$\Rightarrow \quad 2^y = x \log 2 + c_1 \log 2$$

$$\therefore \quad 2^y = x \log 2 + c$$

where $c = c_1 \log 2$

Answer 4.

Given, $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx$

Let $\quad I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx \quad$ ...(i)

Then using property :

$$\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$$

$$I = \int_2^8 \frac{\sqrt{10-(2+8-x)}}{\sqrt{2+8-x}+\sqrt{10-(2+8-x)}} dx$$

$$= \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}} dx \quad \text{...(ii)}$$

Adding equation (i) and (ii), we get

$$2I = \int_2^8 \frac{\sqrt{10-x}+\sqrt{x}}{\sqrt{x}+\sqrt{10-x}} dx$$

$$\Rightarrow \quad 2I = \int_2^8 1.dx$$

$$\Rightarrow \quad 2I = [x]_2^8$$

$$\Rightarrow \quad 2I = 8 - 2 = 6$$

$$\therefore \quad I = 3$$

Answer 5.

(a) Let P(A) = Probability of getting a red ball from the second bag

P(E_1) = Probability of getting a red ball from the first bag

$P(E_2)$ = Probability of getting a blue ball from the first bag

$$\therefore \quad P(E_1) = \frac{6}{11}$$

$$\Rightarrow \quad P(A/E_1) = \frac{6}{14}$$

and $$P(E_2) = \frac{5}{11}$$

$$\Rightarrow \quad P(A/E_2) = \frac{5}{14}$$

$$\therefore \ P(A) = P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)$$

$$\Rightarrow P(A) = \frac{6}{11}\times\frac{6}{14}+\frac{5}{11}\times\frac{5}{14}$$

$$\therefore \ P(A) = \frac{61}{154}$$

OR

(b) Possible selections are as follows :

1 red ball from bag-I, 2 red balls from bag-II

or 1 white ball from bag-I, 2 white balls from bag-II

$\therefore$ Probability of drawing three balls of same colour

i.e., $$P(E) = \frac{{}^3C_1}{{}^7C_1}\times\frac{{}^2C_2}{{}^5C_2}+\frac{{}^4C_1}{{}^7C_1}\times\frac{{}^3C_2}{{}^5C_2}$$

$$= \frac{3}{7}\times\frac{1}{10}+\frac{4}{7}\times\frac{3}{10}$$

$$= \frac{3}{70}+\frac{12}{70}=\frac{15}{70}$$

$$\therefore \quad P(E) = \frac{3}{14}$$

Answer 6.

Let $$I = \int\frac{1+\sin x}{1-\sin x}dx$$

$$= \int\frac{1+\sin x}{1-\sin x}\times\frac{1+\sin x}{1+\sin x}dx$$

$$= \int\frac{(1+\sin x)^2}{1-\sin^2 x}\,dx$$

$$= \int\frac{1+\sin^2 x+2\sin x}{\cos^2 x}\,dx$$

$$= \int\left(\frac{1}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}+2\frac{\sin x}{\cos^2 x}\right)dx$$

$$\Rightarrow \quad I = \int\sec^2 x.dx+\int\tan^2 x\,dx+2\int\frac{\sin x}{\cos^2 x}dx$$

$$= \int\sec^2 x\,dx+\int(\sec^2 x-1)dx+2\int\frac{\sin x}{\cos^2 x}dx$$

Let $\cos x = t$,

$\therefore \quad -\sin x \, dx = dt$

$$= 2\int \sec^2 x \, dx - \int 1.dx - 2\int \frac{1}{t^2} dt$$

$$= 2\tan x - x + \frac{2}{t} + c$$

$$\therefore \quad I = 2\tan x + \frac{2}{\cos x} - x + c$$

Answer 7.

Let E_1, E_2, E_3 be the events of drawing of bolt produced by machine X, Y and Z respectively.

Let D be the event of drawing a defective bolt.

$$\therefore \quad P(E_1) = \frac{20}{100}$$

$$P(E_2) = \frac{35}{100}$$

$$P(E_3) = \frac{45}{100}$$

and $$P(D/E_1) = \frac{8}{100}$$

$$P(D/E_2) = \frac{6}{100}$$

$$P(D/E_3) = \frac{5}{100}$$

By Bayes theorem

P(defective bolt is produced by machine Y)

$$P(E_2/D) = \frac{P(E_2)P(D/E_2)}{P(E_1)P(D/E_1) + P(E_2)P(D/E_2) + P(E_3)P(D/E_3)}$$

$$= \frac{\frac{35}{100} \times \frac{6}{100}}{\frac{20}{100} \times \frac{8}{100} + \frac{35}{100} \times \frac{6}{100} + \frac{45}{100} \times \frac{5}{100}}$$

$$= \frac{210}{160 + 210 + 225}$$

$$\therefore \quad P(E_2/D) = \frac{210}{595} = \frac{6}{17}$$

Answer 8.

(a) Let, $$I = \int \frac{dx}{\sin x + \sin 2x}$$

$$= \int \frac{dx}{\sin x + 2\sin x \cos x}$$

$$= \int \frac{dx}{\sin x(1 + 2\cos x)}$$

$$= \int \frac{\sin x \, dx}{\sin^2 x(1 + 2\cos x)}$$

$$= \int \frac{\sin x\, dx}{(1-\cos^2 x)(1+2\cos x)}$$

$$= \int \frac{\sin x\, dx}{(1+\cos x)(1-\cos x)(1+2\cos x)}$$

Let $\cos x = t$,

$\therefore\ -\sin x\, dx = dt$

$$\Rightarrow \quad I = -\int \frac{dt}{(1-t)(1+t)(1+2t)}$$

Now, let

$$\frac{-1}{(1-t)(1+t)(1+2t)} = \frac{A}{(1-t)} + \frac{B}{(1+t)} + \frac{C}{(1+2t)}$$

$-1 = A(1+t)(1+2t) + B(1-t)(1+2t) + C(1-t^2)$

$-1 = A(1+3t+2t^2) + B(1+t-2t^2) + C(1-t^2)$

$-1 = (2A-2B-C)t^2 + (3A+B)t + A + B + C$

On comparing the coefficient of t^2, t and constants on both sides

$$2A - 2B - C = 0 \quad \text{...(i)}$$

$$3A + B = 0 \quad \text{...(ii)}$$

$$A + B + C = -1 \quad \text{...(iii)}$$

From equations (i), (ii) and (iii), we get

$$A = -\frac{1}{6}$$

$$B = \frac{1}{2}$$

and $$C = -\frac{4}{3}$$

$$\therefore \quad I = \int \left[\frac{-\frac{1}{6}}{(1-t)} + \frac{\frac{1}{2}}{(1+t)} + \frac{-\frac{4}{3}}{(1+2t)} \right] dt$$

$$= -\frac{1}{6}\log(1-t) + \frac{1}{2}\log(1+t) - \frac{4}{3\times 2}\log(1+2t) + c$$

$$= -\frac{1}{6}\log(1-t) + \frac{1}{2}\log(1+t) - \frac{2}{3}\log(1+2t) + c$$

$$\therefore \quad I = -\frac{1}{6}\log(1-\cos x) + \frac{1}{2}\log(1+\cos x) - \frac{2}{3}\log(1+2\cos x) + c$$

OR

(b) Let, $$I = \int_0^{\pi/4} \log(1+\tan x)\, dx \quad \text{...(i)}$$

Using property :

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$\Rightarrow \quad I = \int_0^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right] dx$$

$$= \int_0^{\pi/4} \log\left[1+\frac{1-\tan x}{1+\tan x}\right]dx$$

$$= \int_0^{\pi/4} \log\left[\frac{1+\tan x+1-\tan x}{1+\tan x}\right]dx$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} [\log 2-\log(1+\tan x)]dx$$

$$\Rightarrow \quad I = \int_0^{\pi/4} \log 2\, dx-\int_0^{\pi/4} \log(1+\tan x)\, dx \quad \text{...(ii)}$$

Adding equation (i) and (ii), we get

$$2I = \int_0^{\pi/4} \log 2\, dx$$

$$= \log 2\int_0^{\pi/4} 1.dx$$

$$\Rightarrow \quad 2I = \log 2[x]_0^{\pi/4}$$

$$\Rightarrow \quad I = \frac{\log 2}{2}\left(\frac{\pi}{4}-0\right)$$

$$\therefore \quad I = \frac{\pi}{8}\log 2$$

Section-B

Answer 9.

(i) (a) $2x - 3y + z - 7 = 0$

Explanation :

Given, $2x - 3y + z = 0$ ...(i)

Equation of plane parallel to (i) is,

$2x - 3y + z = d$ ...(ii)

Which is passes through (1, – 1, 2), then

$2 \times 1 - 3 \times (-1) + 2 = d$

$\Rightarrow \quad d = 7$

Now from (ii), we get

$2x - 3y + z = 7$

or $2x - 3y + z - 7 = 0$

(ii) (b) $\frac{3}{2}, 3, -\frac{3}{2}$

Explanation :

Given equation of plane is

$2x + y - 2z = 3$

Divide by 3 on both sides

$$\frac{2x}{3}+\frac{y}{3}-\frac{2z}{3} = 1$$

or $$\frac{x}{\frac{3}{2}}+\frac{y}{3}+\frac{z}{-\frac{3}{2}} = 1$$

Hence, the intercepts made on coordinates axes are $\frac{3}{2}, 3, -\frac{3}{2}$.

Answer 10.

Since, the plane is perpendicular to the given line, its direction ratios are proportional to 3, 0, 4.

So, the required equation of the plane is

$$3x + 0y + 4z + d = 0 \quad \text{...(i)}$$

where d is a constant.

Since this plane passes through (1, 1, 1)

$$3 + 0 + 4 + d = 0$$

$$\Rightarrow \quad d = -7$$

From equation (i), we get

$$3x + 4z - 7 = 0 \quad \text{...(ii)}$$

This is the required equation of the plane.

Perpendicular distance of (ii) from the origin is given by,

$$d = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \quad d = \frac{|3\times0+4\times0-7|}{\sqrt{3^2+0^2+4^2}}$$

$$\Rightarrow \quad d = \frac{|-7|}{\sqrt{9+16}} = \frac{7}{\sqrt{25}}$$

$$= \frac{7}{5}\text{ units}$$

Answer 11.

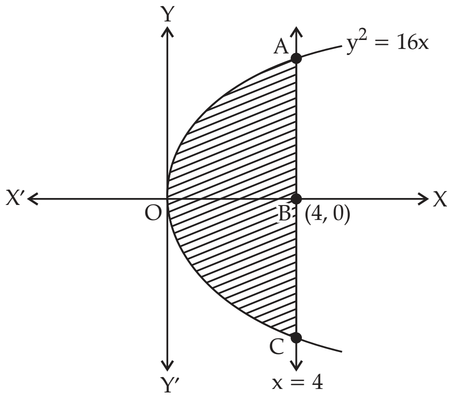
The shaded region OACO is the region bounded by the parabola $y^2 = 16x$ and the line $x = 4$.

The area OACO is symmetrical about X-axis.

Area of OACO = 2(Area of OAB)

$$= 2\int_0^4 y\,dx$$

$$= 2\int_0^4 4\sqrt{x}\,dx$$



$$= 8\times\frac{2}{3}\left[x^{3/2}\right]_0^4$$

$$= \frac{16}{3}(4)^{3/2}$$

$$= \frac{16}{3}\times 8 = \frac{128}{3}$$

So, the required area is $\frac{128}{3}$ sq. units.

Section-C

Answer 12.

(i) (c) $\frac{-1}{2\sqrt{3}}$

Explanation :

Given lines of regression are :

$$9x + 3y - 46 = 0 \quad (x \text{ on } y)$$

$$9x = -3y + 46$$

$$\Rightarrow \quad x = -\frac{3}{9}y + \frac{46}{9}$$

$$\Rightarrow \quad x = -\frac{1}{3}y + \frac{46}{9}$$

$$\therefore \quad b_{xy} = -\frac{1}{3}$$

and $3x + 12y - 7 = 0$ $(y \text{ on } x)$

$$\Rightarrow \quad 12y = -3x + 7$$

$$\Rightarrow \quad y = -\frac{3}{12}x + \frac{7}{12}$$

$$\Rightarrow \quad y = -\frac{1}{4}x + \frac{7}{12}$$

$$\therefore \quad b_{yx} = -\frac{1}{4}$$

Now, $r = \sqrt{b_{xy}.b_{yx}}$

$$= \sqrt{\left(-\frac{1}{3}\right)\left(-\frac{1}{4}\right)}$$

$\Rightarrow$ $r = \frac{1}{\sqrt{12}} = \frac{1}{2\sqrt{3}}$

$\therefore$ $r = -\frac{1}{2\sqrt{3}}$ (Since b_{xy} and b_{yx} are negative $\therefore$ r is negative)

(ii) (a) $x - 6y - 4 = 0$

Explanation :

The regression line of X on Y is given by

$$(x - \overline{X}) = b_{xy}(y - \overline{Y})$$

or, $(x - \overline{X}) = r.\frac{\sigma_x}{\sigma_y}(y - \overline{Y})$

So, $x - 40 = 0.9 \times \frac{10}{1.5}(y - 6)$

$\Rightarrow$ $x - 40 = 6(y - 6)$

$\Rightarrow$ $x - 40 = 6y - 36$

$\therefore$ $x - 6y - 4 = 0$

Answer 13.

We have,

$n = 5$, $\Sigma x = 15$, $\Sigma y = 25$, $\Sigma x^2 = 55$, $\Sigma y^2 = 135$, $\Sigma xy = 83$

We know that,

$$b_{xy} = \frac{\Sigma xy - \frac{\Sigma x.\Sigma y}{n}}{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}$$

or $b_{xy} = \frac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$

$$= \frac{5 \times 83 - 15 \times 25}{5 \times 135 - (25)^2}$$

$$= \frac{415 - 375}{675 - 625}$$

$\Rightarrow$ $b_{xy} = \frac{40}{50} = \frac{4}{5} = 0.8$

Now, $b_{yx} = \frac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$

$$= \frac{5 \times 83 - 15 \times 25}{5 \times 55 - (15)^2}$$

$$= \frac{415 - 375}{275 - 225}$$

$$= \frac{40}{50} = \frac{4}{5} = 0.8$$

Answer 14.

Let x be the number of units commodity A produced and y be the number of units commodity B produced. The values of x and y must satisfy the following

$x + 2y \leq 8$ (material constraint) ...(i)

$3x + 2y \leq 12$ (labour constraint) ...(ii)

$x + y \leq 10$ (equipment constraint) ...(iii)

Also, $x \geq 0$, $y \geq 0$

Let Z be the profit, then

$$Z = 2x + 3y$$

Plotting the graph using these constraints :

From equation (i),

$$x + 2y = 8$$

x	8	0
y	0	4

From equation (ii),

$$3x + 2y = 12$$

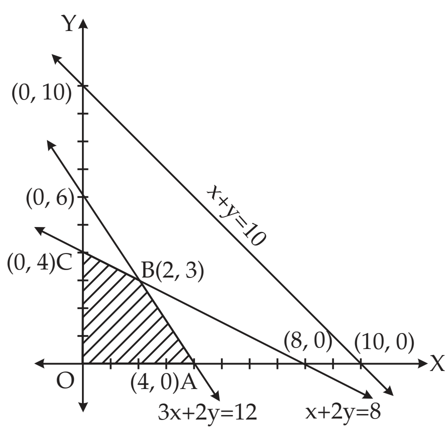
x	4	0
y	0	6

From equation (iii),

$$x + y = 10$$

x	10	0
y	0	10

Clearly the coordinates of the vertices of shade region OABC are O(0, 0), A(4, 0), B(2, 3) and C(0, 4).



Now we can determine the value of Z by evaluating Z at these four points :

Vertices	Z = 2x + 3y
O(0, 0)	0
A(4, 0)	8
B(2, 3)	13 (max)
C(0, 4)	12

So, the maximum profit is ₹ 13.

❑❑