ISC 2025 EXAMINATION

Sample Question Paper - 5

Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

This Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions*

EITHER *from* ***Section B OR Section C.***

Section A*: Internal choice has been provided in* ***two questions of two marks each, two questions of four marks each and two questions of six marks each.***

Section B: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

Section C: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

1. **In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.** **[15]**

(a) The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is **[1]**

a) a symmetric matrix b) a unit matrix

c) a diagonal matrix d) a skew- symmetric matrix

(b) $\int_a^b \frac{\log x}{x} dx$ is equal to **[1]**

a) $\frac{log\ (b-a)}{b-a}$ b) log (a+b).log (b–a)

c) $\log(ab).log(\frac{b}{a})$ d) $\frac{1}{2}\log(ab).log\left(\frac{b}{a}\right)$

(c) The principal value of $\sin^{-1}\frac{1}{2}$ is **[1]**

a) Both $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ b) $\frac{\pi}{6}$

c) $\frac{5\pi}{6}$ d) $\frac{-\pi}{6}$

(d) What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$? **[1]**

a) 6 b) not defined

c) 2   d) 3

(e) A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is **[1]**

a) $\frac{1}{28}$

b) $\frac{3}{28}$

c) $\frac{167}{168}$

d) $\frac{2}{21}$

(f) Number of onto (subjective) functions from A to B if n(A) = 6 and n(B) = 3 are **[1]**

a) 340

b) None of these

c) 2^6 - 2

d) 3^6 - 3

(g) If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$ then $\frac{dy}{dx}$ is equal to **[1]**

a) $\frac{4x^3}{1-x^4}$

b) $\frac{-4x^3}{1-x^4}$

c) $\frac{1}{4-x^4}$

d) $\frac{-4x}{1-x^4}$

(h) If f(x) = |3 – x| + (3 + x), where (x) denotes the least integer **[1]**

a) neither differentiable nor continuous at x = 3

b) continuous but not differentiable at x = 3

c) differentiable but not continuous at x = 3

d) continuous and differentiable at x = 3

(i) Which of the following is not correct in a given determinant of A, where A = $[a_{ij}]_{3\times3}$. **[1]**

a) Value of a determinant is obtained by multiplying elements of a row or column by corresponding cofactors

b) Order of minors and cofactors of elements of A is same

c) Order of minor is less than order of the det (A)

d) Minor of an element can never be equal to cofactor of the same element

(j) **Assertion (A):** If A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and C = $\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, then 3A - C = $\begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$. **[1]**

Reason (R): If the matrices A and B are of same order, say m $\times$ n, satisfy the commutative law, then A + B = B + A.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

(k) If f and g are real function defined by f (x) = x^2 + 7 and g (x) = 3x + 5, find the f (t) -f (-2) **[1]**

(l) Let A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$. Find A + B **[1]**

(m) Let g be real function defined by and g(x) = $(x + 4)^3$ .Find $\frac{1}{g}$. **[1]**

(n) Let E_1 and E_2 be two independent events such that P(E_1) = p_1 and P(E_2) = p_2. Describe in words of the events whose probability is: $p_1 + p_2 - 2p_1p_2$ **[1]**

(o) A bag contains 5 white, 7 red and 8 black balls. Four balls are drawn one by one with replacement, what is the probability that at least one is white? **[1]**

2. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to x, if $x \in (-\infty, -1)$ **[2]**

OR

Find the points on the curve $y = x^3$ where the slope of the tangent is equal to x-coordinate of the point.

3. Evaluate the Integral: $\int x\sqrt{x^2 - 1}dx$ **[2]**

4. Prove that function $f(x) = 3x^5 + 40x^3 + 240x$ is increasing on R. **[2]**

5. Evaluate: $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)}dx$ **[2]**

OR

Evaluate: $\int \frac{1}{1+x+x^2+x^3}dx$

6. Let S be the set of all real numbers and let R be a relation in S, defined by $\{R = \left\{(a, b) : a \leq b^3\right\}$. Show that R satisfies none of reflexivity, symmetry and transitivity. **[2]**

7. Prove the following. $\cos\left[\sin^{-1}\left(\frac{3}{5}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right] = \frac{6}{5\sqrt{13}}$ **[4]**

8. Evaluate: $\int \frac{x^2}{(x^4-x^2-12)}dx$. **[4]**

9. If $y = (\sin^{-1}x)^2$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$ **[4]**

OR

If $y = x^n$ {a cos (log x) + b sin (log x)}, prove that $x^2\frac{d^2y}{dx^2} + (1 - 2n)\frac{dy}{dx} + \left(1 + n^2\right)y = 0$

10. **Read the text carefully and answer the questions:** **[4]**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(a) Find the probability that both of them are selected.

(b) The probability that none of them is selected.

(c) Find the probability that only one of them is selected.

(d) Find the probability that atleast one of them is selected.

OR

Read the text carefully and answer the questions: **[4]**

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

(a) Calculate the probability that a randomly chosen seed will germinate.

(b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.

(c) A die is throw and a card is selected at random from a deck of 52 playing cards. Then find the probability of getting an even number on the die and a spade card.

(d) If A and B are any two events such that P(A) + P(B) - P(A and B) = P(A), then find P(A|B).

11. **Read the text carefully and answer the questions:** **[6]**

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association).

Let A be a 1×2 matrix and B be a 2×1 matrix, representing the investment and interest rate on each bond respectively.

Based on the above information, answer the following questions.

(a) If ₹ 15000 is invested in bond X, then what is the matrix representation of A and B?

(b) If ₹ 15,000 is invested in bond X, how can we determine the total amount of interest received on both bonds?

(c) How much is the investment in two bonds if the trust fund obtains an annual total interest of ₹3200?

12. a. Solve the differential equation for a particular solution: **[6]**

dy = (5x - 4y) dx, when y = 0 and x=0

b. Solve the differential equation ydx - $(x + 2y^2)$ dy = 0

OR

Solve the diff. eq $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

13. The sum of the surface areas of a cuboid with sides x, 2x and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal of three times the radius of sphere. Also, find the minimum value of the sum of their volumes. **[6]**

OR

Which of the following functions are decreasing on $\left(0, \frac{\pi}{2}\right)$?

i. cos x

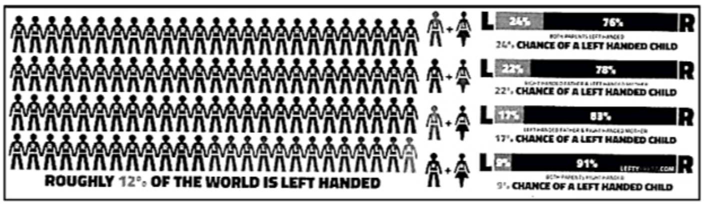
ii. cos 2x

iii. cos 3x

iv. tan x

14. **Read the text carefully and answer the questions:** **[6]**

Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows:

A. When both father and mother are left handed:

Chances of left handed child is 24%.

B. When father is right handed and mother is left handed:

Chances of left handed child is 22%.

C. When father is left handed and mother is right handed:

Chances of left handed child is 17%.

D. When both father and mother are right handed:

Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

(a) Find $P\left(\frac{L}{C}\right)$

(b) Find $P\left(\frac{\bar{L}}{A}\right)$

(c) Find $P\left(\frac{A}{L}\right)$

(d) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

SECTION B - 15 MARKS

15. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) The vectors $3\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} + \lambda\hat{j} - \hat{k}$ are coplanar if the value of λ is: **[1]**

a) -2 b) 0

c) 2 d) Any real number

(b) If a line makes an angle of 30°, 60°, 90° with the positive direction of x, y, z-axes, respectively, then find its direction cosines. **[1]**

(c) Find the value of $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$. **[1]**

(d) Determine whether the given plane 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z – 1 = 0 are parallel or perpendicular, and in case they are neither, find the angles between them. **[1]**

a) The planes are at 45° b) The planes are parallel

c) The planes are at 55° d) The planes are perpendicular

(e) Find the direction cosines of the line joining the points P(4, 3, -5) and Q(-2, 1, -8). **[1]**

16. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors, then prove that $|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$ **[2]**

OR

If $\vec{a} = (\hat{i} - 2\hat{j}), \vec{b} = (2\hat{i} - 3\hat{j})$ and $\vec{c} = (2\hat{i} + 3\hat{k})$, find $(\vec{a} + \vec{b} + \vec{c})$.

17. Write the value of k for which the planes x - 2 y + kz = 4 and 2x + 5y - z = 9 are perpendicular. **[4]**

OR

Find the angle between the lines whose direction cosines are given by the equations: 3l + m + 5n = 0 and 6mn - 2nl +

5lm = 0.

18. Using integration, find the area of the region bounded by the line y - 1 = x, the x -axis and the ordinates x = - 2 and x = 3. **[4]**

SECTION C - 15 MARKS

19. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) Which condition is true if AC is constant at all levels of output? **[1]**

a) MC $\neq$ AC   b) MC > AC

c) MC = AC   d) MC < AC

(b) In a LPP, the linear inequalities or restrictions on the variables are called **[1]**

a) Limits   b) Inequalities

c) Linear constraints   d) Constraints

(c) Two regression lines are represented by 2x + 3y - 10 = 0 and 4x + y - 5 = 0. Find the line of regression of y on x. **[1]**

(d) The total cost of producing x radio sets per day is ₹$\left(\frac{x^2}{4}+35x+25\right)$ and the price per set at which they may be sold is ₹$\left(50-\frac{x}{2}\right)$. Find the daily output to maximise the total profit. **[1]**

(e) A television manufacturer finds that the total cost for producing and marketing x television sets is $C(x) = 300x^2 + 4200x + 13500$. Each product is sold for ₹ 8400. Determine the break-even points. **[1]**

20. A T.V. company has 1000 subscribers who are each paying ₹ 100 per month. The company proposes to increase the monthly subscription and it is believed that for every increase of ₹ 1, five subscribers will discontinue the service. Find what increase will yield maximum revenue and what will this revenue be? **[2]**

OR

For the demand function p = $\frac{b}{a+x}$, show that the marginal revenue function is increasing for all b < 0, a > 0.

21. Find the line of regression of y on x from the following table: **[4]**

x	1	2	3	4	5
y	7	6	5	4	3

Hence, estimate the value of y when x = 6.

22. Draw the graph of the solution set of the system of inequations 2x + 3y $\geq$ 6, x + 4y $\leq$ 4, x $\geq$ 0 and y $\geq$ 0 **[4]**

OR

Maximize Z = x + 2y subject to the constraints

$x-y \geq 0, 2y \leq x+2, x, y \geq 0$

Solution

SECTION A - 65 MARKS

1. In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(i) **(a)** a symmetric matrix

Explanation: {

Symmetric matrix. Since, A' = A, therefore, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(ii) **(d)** $\frac{1}{2}\log(ab).\,log\left(\frac{b}{a}\right)$

Explanation: {

$= \int_a^b (log\ x)^1 (\frac{1}{x})dx$

(let log x = t then 1x dx = dt)

$= \left[\frac{(log\ x)^2}{2}\right]_a^b$

$= \frac{1}{2}$ [(log b)^2 − (log a)^2]

$= \frac{1}{2}$ (log b + log a)(log b − log a)

$= \frac{1}{2}\log(ab)\log\frac{b}{a}$

(iii) **(b)** $\frac{\pi}{6}$

Explanation: {

$\sin^{-1}\frac{1}{2} = \alpha$, say $\Rightarrow \sin\alpha = \frac{1}{2} = \sin\frac{\pi}{6}$

$\Rightarrow \alpha = \frac{\pi}{6} \in \left|-\frac{\pi}{2}, \frac{\pi}{2}\right|$

$\Rightarrow$ Principal value of $\sin^{-1}\frac{1}{2}$ is $\frac{\pi}{6}$.

(iv) **(c)** 2

Explanation: {

2

(v) **(b)** $\frac{3}{28}$

Explanation: {

Probability of drawing 2 green balls and one blue ball

$= P_G \cdot P_G \cdot P_B + P_B \cdot P_G \cdot P_G + P_G \cdot P_B \cdot P_G$

$= \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{2}{7} + \frac{2}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} + \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{2}{6}$

$= \frac{1}{28} + \frac{1}{28} + \frac{1}{28} = \frac{3}{28}$

(vi) **(b)** None of these

Explanation: {

Number of onto function

$= 3^6 - {}^3C_1 (3 - 1)^6 + {}^3C_2(3 - 2)^6 - {}^3C_3 (3 - 3)^6$

$= 36 - 3 \times 26 + 3 \times 1 = 3^6 - 3 \times 26 + 3$

$= 3 \times (35 - 26 + 1) = 3(243 - 64 + 1)$

$= 3 \times (244 - 64) = 3 \times 180 = 540$

(vii) **(d)** $\frac{-4x}{1-x^4}$

Explanation: {

We have, $y = \log\left(\frac{1-x^2}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\frac{1-x^2}{1+x^2}} \times \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{1+x^2}{1-x^2} \times \frac{\left[(1+x^2)(-2x) - (1-x^2)(+2x)\right]}{(1+x^2)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{(1+x^2)}{(1-x^2)} \times \frac{\left[-2x-2x^3-2x+2x^3\right]}{(1+x^2)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\frac{1-x^2}{1+x^2}} \cdot \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$

$= \frac{-2x\left[1+x^2+1-x^2\right]}{(1-x^2)\cdot(1+x^2)} = \frac{-4x}{1-x^4}$

$\Rightarrow \frac{dy}{dx} = \frac{1\times -4x}{(1-x^2)(1+x^2)}$

$\therefore \frac{dy}{dx} = \frac{-4x}{1-x^4}$

(viii) **(a)** neither differentiable nor continuous at x = 3

Explanation: {

Given that f(x) = |3 – x| + (3 + x), where (x) denotes the least integer greater than or equal to x.

$f(x) = \left\{\begin{array}{l} 3-x+3+3, 2<x<3 \\ x-3+3+4, 3<x<4 \end{array}\right\}$

$\Rightarrow \mathrm{f(x)} = \left\{\begin{array}{l} 9-\mathrm{x}, 2<\mathrm{x}<3 \\ \mathrm{x}+4, 3<\mathrm{x}<4 \end{array}\right\}$

Checking continuity at x =3,

Here, LHL at x = 3

$\lim_{x\to 3^-} 9-x=6$

RHL at x = 3

$\lim_{x\to 3^+} x+4=7$

$\because$ LHL $\neq$ RHL

$\therefore$ f(x) is neither continuous nor differentiable at x = 3.

(ix) **(d)** Minor of an element can never be equal to cofactor of the same element

Explanation: {

Minor of an element can never be equal to the cofactor of the same element.

$C_{ij} = (-1)^{i+j} M_{ij}$

So, for even values of i + j, $C_{ij} = M_{ij}$

(x) **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: {

3A - C = $3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 6-(-2) & 12-5 \\ 9-3 & 6-4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$

(xi) Here we have f (x) = x^2 + 7 and g (x) = 3x + 5

f(t) = t^2 + 7 and f(-2) = $(-2)^2$ + 7 = 4 + 7 = 11

$\therefore$ f(t) - f(-2) = t^2 + 7 - 11 = t^2 - 4

(xii) A + B = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

= $\begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix}$

= $\begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$

(xiii) We observe that g (x) = $(x + 4)^3$ = 0 for x = -4. Therefore, $\frac{1}{g} : R - \{-4\} \to R$ is given by

$\left(\frac{1}{g}\right)(x) = \frac{1}{g(x)} = \frac{1}{(x+4)^3}$

(xiv) Clearly,

$p_1 + p_2 - 2p_1p_2$

$= P(E_1) + P(E_2) - 2P(E_1) P(E_2)$

= $P(E_1) + P(E_2) - 2P(E_1 \cap E_2)$

= $P(E_1 \cup E_2) - 2P(E_1 \cap E_2)$

So, either E_1 or E_2 occurs but not both.

(xv) Let A_i be the event that ball drawn in the ith draw is white $1 \leq i \leq 4$.

Since the balls are drawn one by one with replacement. Therefore, A_1, A_2, A_3, A_4 are independent events such that

$P(A_i) = \frac{5}{20} = \frac{1}{4}$, i = 1, 2, 3, 4

Therefore, required probability is given by,

$P(A_1 \cup A_2 \cup A_3 \cup A_4)$

$= 1 - P\left(\bar{A}_1\right) P\left(\bar{A}_2\right) P\left(\bar{A}_3\right) P\left(\bar{A}_4\right)$ ...[$\because A_1, A_2, A_3, A_4$ are independent]

$= 1 - \left(\frac{3}{4}\right)^4$

2. Let $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Put, x = tan θ, we get

$y = \tan^{-1}(\tan 2\theta)$

If $-\infty < x < -1$, then

$x = \tan\theta \Rightarrow -\infty < \tan\theta < -1$

$\Rightarrow -\frac{\pi}{2} < \theta < -\frac{\pi}{4} \Rightarrow -\pi < 2\theta < -\frac{\pi}{2}$

$\therefore y = \tan^{-1}(\tan 2\theta)$

$\Rightarrow \quad y = \tan^{-1}\{\tan(\pi + 2\theta)\}$

$\Rightarrow \quad y = \tan^{-1}(\tan(\pi + 2\theta)) \quad \left[\because -\pi < 2\theta < -\frac{\pi}{2} \Rightarrow 0 < \pi + 2\theta < \frac{\pi}{2}\right]$

$\Rightarrow \quad y = \pi + 2\theta$

$\Rightarrow \quad y = \pi + 2\tan^{-1} x$

$\Rightarrow \quad \frac{dy}{dx} = 0 + \frac{2}{1+x^2} = \frac{2}{1+x^2}$

OR

Given: equation of curve, $y = x^3$

Let (x_1, y_1) be the required point.

x coordinate of the point is x_1.

Since, the point lies on the curve.

Hence, $y_1 = x_1^3$ ...(i)

Now, $y = x^3$

$\Rightarrow \frac{dy}{dx} = 3x^2$

Slope of tangent at (x, y) = $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 3x_1^2$

Given that

Slope of tangent at (x_1, y_1) = x co-ordinate of the point

$\Rightarrow 3x_1^2 = x_1$

$\Rightarrow x_1(3x_1 - 1) = 0$

$\Rightarrow x_1 = 0$ or $x_1 = \frac{1}{3}$

$\Rightarrow y_1 = 0$ or $y_1 = \frac{1}{27}$

So, the points are $(x_1, y_1) = (0, 0), \left(\frac{1}{3}, \frac{1}{27}\right)$

3. Let I = $\int x\sqrt{x^2 - 1}dx$

Since $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$

We have, $I = \int x\sqrt{x^2 - 1}dx$ .... (i)

Let $x^2 - 1 = t$

$\Rightarrow 2x = \frac{dt}{dx}$

$\Rightarrow xdx = \frac{dt}{2}$

Putting this value in equation (i), we get

$I = \int \frac{1}{2}\sqrt{t}dt \ \left[x = x^2 - 1\right]$

$\Rightarrow I = \frac{1}{2}\int t^{\frac{1}{2}}dx$

$\Rightarrow \mathrm{I} = \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

$\Rightarrow \mathrm{I} = \frac{1}{3}t^{\frac{3}{2}} + c$

$\Rightarrow \mathrm{I} = \frac{1}{3}\left(x^2 - 1\right)^{\frac{3}{2}} + c$

4. Given:

f(x) = $3x^5 + 40x^3 + 240x$

$\therefore$ f'(x)=15x4 + $120x^2$ + 240

= $15(x^4 + 8x^2 + 16)$

=$15(x^2 + 4)^2$

Now, $x \in_R$

$\Rightarrow \left(x^2 + 4\right)^2 > 0$

$\Rightarrow f'(x) > 0$

Hence, f(x) is an increasing function for all x∈R

5. Let I = $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)}dx$ ... (i)

Also let $\sqrt{x}$ + 1 = t then, we have

$d(\sqrt{x} + 1) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow \quad dx = 2\sqrt{x}dt$

Put $\sqrt{x}$ + 1 = t and $dx = 2\sqrt{x}dt$ in equation (i), we get

$I = \int \frac{1}{\sqrt{x}t} \times 2\sqrt{x}dt$

$= 2\int \frac{dt}{t}$

= 2 log |t| + c

= 2 log |$\sqrt{x}$ + 1| + c

$\therefore$ I = 2 log |$\sqrt{x}$ + 1| + c

OR

Let I = $\int \frac{dx}{1+x+x^2+x^3}$, then we have

I = $\int \frac{dx}{(x^2+1)(x+1)}$

Now,

Let $\frac{1}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$

$\Rightarrow$ 1 = (Ax + B)(x + 1) + C (x^2 + 1)

Equating similar terms, we get

A + C = 0, A + B = 0, B + C = 1

Solving, we get, A = -$\frac{1}{2}$, B = $\frac{1}{2}$, C = $\frac{1}{2}$

Thus,

I = - $\frac{1}{2}\int \frac{xdx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x+1}$

I = $-\frac{1}{4}\log\left|x^2 + 1\right| + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x + 1| + c.$

6. i. Non-reflexivity

Clearly, $\frac{1}{2}$ is a real number an $\frac{1}{2} \le \left(\frac{1}{2}\right)^3$ is not true.

$\therefore \left(\frac{1}{2}, \frac{1}{2}\right) \notin R$

Therefore, R is not reflexive.

ii. Non-symmetry

Take the real numbers $\frac{1}{2}$ and 1

Clearly, $\frac{1}{2} \le 1^3$ is true and therefore, $\left(\frac{1}{2}, 1\right) \in R$

But, $1 \le \left(\frac{1}{2}\right)^3$ is not true and so $\left(1, \frac{1}{2}\right) \notin R$

Therefore, R is not symmetric.

iii. Non-transitive

consider the real numbers 3, $\frac{3}{2}$ and $\frac{4}{3}$

Clearly, $3 \leq \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} \leq \left(\frac{4}{3}\right)^3$ but $3 \leq \left(\frac{4}{3}\right)^3$ is not true

Therefore,we have, $\left(3, \frac{3}{2}\right) \in R$ and $\left(\frac{3}{2}, \frac{4}{3}\right) \in R,$ but $\left(3, \frac{4}{3}\right) \notin R$

Hence, R is not transitive.

7. Here, we have to prove that $\cos\left[\sin^{-1}(\frac{3}{5}) + \cot^{-1}(\frac{3}{2})\right] = \frac{6}{5\sqrt{13}}$

Let us consider , $\sin^{-1}(\frac{3}{5}) = x$ and $\cot^{-1}(\frac{3}{2}) = y; \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

and $y \in (0, \pi)$

$\Rightarrow \quad \sin x = \frac{3}{5}$ and $\cot y = \frac{3}{2}$

$\Rightarrow \quad \cos x = \sqrt{1 - \sin^2 x}$ and

cosec y $= \sqrt{1 + \cot^2 y}$ [taking positive sign as $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $y \in (0, \pi)$]

$\Rightarrow \cos x = \sqrt{1 - \left(\frac{3}{5}\right)^2}$ and cosec y $= \sqrt{1 + \left(\frac{3}{2}\right)^2}$

$\Rightarrow \quad \cos x = \sqrt{1 - \frac{9}{25}}$ and cosec y $= \sqrt{1 + \frac{9}{4}}$

$\Rightarrow \quad \cos x = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

and cosec y $= \sqrt{\frac{4+9}{4}} = \sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{2}$

$\Rightarrow \quad \cos x = \frac{4}{5}$ and $\frac{1}{\sin y} = \frac{\sqrt{13}}{2}$

$\Rightarrow \quad \cos x = \frac{4}{5}$ and $\sin y = \frac{2}{\sqrt{13}}$

Also, cos y = sin y - cot y = $\frac{2}{\sqrt{13}} \times \frac{3}{2} = \frac{3}{\sqrt{13}}$

Now, cos(x + y) = cos x cos y - sin x sin y $= \frac{4}{5} \times \frac{3}{\sqrt{13}} - \frac{3}{5} \times \frac{2}{\sqrt{13}} = \frac{12}{5\sqrt{13}} - \frac{6}{5\sqrt{13}} = \frac{6}{5\sqrt{13}} =$ RHS

Hence proved.

8. Let, $I = \int \frac{x^2}{(x^4 - x^2 - 12)} dx$

Using partial fractions,

$\frac{x^2}{(x^4 - x^2 - 12)} = \frac{t}{t^2 - t - 12} = \frac{t}{(t-4)(t+3)} = \frac{A}{t-4} + \frac{B}{t+3}$ .... (1)

Where t = x^2

A(t + 3) + B(t - 4) = t

Now put t + 3 = 0

t = -3

A(0) + B(-7) = -3

$B = \frac{3}{7}$

Now put t - 4 = 0

t = 4

A(4 + 3) + B(0) = 4

$A = \frac{4}{7}$

From equation(1)

$\frac{t}{(t-4)(t+3)} = \frac{4}{7} \times \frac{1}{t-4} + \frac{3}{7} \times \frac{1}{t+3}$

$\frac{x^2}{(x^2-4)(x^2+3)} = \frac{4}{7} \times \frac{1}{x^2 - 2^2} + \frac{3}{7} \times \frac{1}{x^2 + (\sqrt{3})^2}$

$\int \frac{x^2}{(x^2-4)(x^2+3)} dx = \frac{4}{7} \int \frac{1}{x^2 - 2^2} dx + \frac{3}{7} \int \frac{1}{x^2 + (\sqrt{3})^2} dx$

$= \frac{4}{7} \times \frac{1}{2} \times \frac{1}{2} \log\left|\frac{x-2}{x+2}\right| + \frac{3}{7} \times \frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + c$

$= \frac{1}{7} \log\left|\frac{x-2}{x+2}\right| + \frac{\sqrt{3}}{7} \tan^{-1} \frac{x}{\sqrt{3}} + c$

9. We seek to show that

$\left(1 - x^2\right) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2$ = 0 where y = $(\sin^{-1} x)^2$

Using the result:

$\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

In conjunction with the chain rule, then differentiating y = $(\sin\text{-}1\ x)^2$ w.r.t. x we have:

$\frac{dy}{dx} = \frac{2sin^{-1}x}{\sqrt{1-x^2}}$

And differentiating a second time, in conjunction with the quotient rule, we have:

$$\frac{d^2y}{dx^2} = \frac{(\sqrt{1-x^2})\left(\frac{2}{\sqrt{1-x^2}}\right) - \left(\frac{\frac{1}{2}(-2x)}{\sqrt{1-x^2}}\right)(2sin^{-1}x)}{(\sqrt{1-x^2})^2}$$

$$= \frac{2+\frac{2xsin^{-1}x}{\sqrt{1-x^2}}}{1-x^2}$$

, considering the LHS of the given expression:

LHS = $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2$

= $\left(1-x^2\right)\left\{\frac{2+\frac{2xsin^{-1}x}{\sqrt{1-x^2}}}{1-x^2}\right\}$ - $x\left\{\frac{2sin^{-1}x}{\sqrt{1-x^2}}\right\}$ - 2

= 2 + $\frac{2xsin^{-1}x}{\sqrt{1-x^2}}$ - $\frac{2xsin^{-1}x}{\sqrt{1-x^2}}$ - 2

= 0

OR

y = x^n {a cos (log x) + b sin (log x)}

y = ax^n cos (log x) + bx^n sin (log x)

$\frac{dy}{dx}$ = an x^{n-1} cos (log x) - ax^{n-1} sin (log x) + bnx^{n-1} sin (log x) + bx^{n-1} cos (log x)

$\frac{dy}{dx}$ = x^{n-1} cos (log x) (na + b) + x^{n-1} sin (log x) (bn - a)

$\frac{d^2y}{dx^2} - \frac{d}{dx}$ (x^{n-1} cos (log x) (na + b) + x^{n-1} sin (log x) (bn - a))

$\frac{d^2y}{dx^2}$ = (na + b) [(n - 1) x^{n-2} cos (log x) - x^{n-2} sin (log x)] + (bn - a) (n - 1) x^{n-2} sin (log x) + x^{n-2} cos (log x)]

$\frac{d^2y}{dx^2}$ = (na + b) x^{n-2} [(n - 1) cos (log x) - sin (log x)] + (bn - a) x^{n-2} [(n - 1) sin (log x) + cos (log x)]

$x^2\frac{dy}{dx^2} + (1-2n)\frac{dy}{dx} + \left(1+n^2\right)y$

= (na + b) x^n [(n - 1) cos (log x) - sin (log x)] + (bn - a) x^n [(n - 1) sin (log x) + cos (log x)]

+ (1 - 2n) x^{n-1} cos (log x) (na + b) + (1 - 2n) x^{n-1} sin (log x) (bn - a)

+ a(1 + n^2) x^n cos (log x) + b (1 + n^2) x^n sin (log x)

= 0

Hence $x^2\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + \left(1+n^2\right)y = 0$

10. **Read the text carefully and answer the questions:**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

(i) P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(Both are selected) = $P(A\cap B) = P(A)\cdot P(B)$ = $\frac{1}{3}.\frac{1}{2}$

P(Both are selected) = $\frac{1}{6}$

(ii) P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(none of them selected) = $P(A' \cap B') = P(A') \cdot P(B')$ = $\frac{2}{3} . \frac{1}{2}$

P(Both are selected) = $\frac{1}{3}$

(iii)P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(none of them selected) = $P(A') \cdot P(B) + P(A) \cdot P(B')$ = $\frac{2}{3} . \frac{1}{2} + \frac{1}{3} . \frac{1}{2}$

P(Both are selected) = $\frac{3}{6} = \frac{1}{2}$

(iv)P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(atleast one of them selected) = 1 - P(none selected) = $1 - \frac{1}{3}$

P(atleast one of them selected) = $\frac{2}{3}$

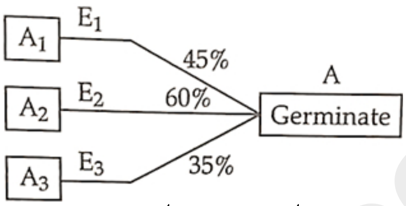
OR

Read the text carefully and answer the questions:

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

(i)



Here, P(E_1) = $\frac{4}{10}$, P(E_2) = $\frac{4}{10}$, P(E_3) = $\frac{2}{10}$

$P\left(\frac{A}{E_1}\right) = \frac{45}{100}, P\left(\frac{A}{E_2}\right) = \frac{60}{100}, P\left(\frac{A}{E_3}\right) = \frac{35}{100}$

$\therefore$ P(A) = P(E_1) · P$\left(\frac{A}{E_1}\right)$ + P(E_2) · $P\left(\frac{A}{E_2}\right)$ + P(E_3) · $P\left(\frac{A}{E_3}\right)$

$= \frac{4}{10} \times \frac{45}{100} + \frac{4}{10} \times \frac{60}{100} + \frac{2}{10} \times \frac{35}{100}$

$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{100}$

$= \frac{490}{1000}$ = 4.9

(ii) Required probability = $P\left(\frac{E_2}{A}\right)$

$= \frac{P(E_2) \cdot P\left(\frac{A}{E_2}\right)}{P(A)}$

$= \frac{\frac{4}{10} \times \frac{60}{100}}{\frac{490}{1000}}$

$= \frac{240}{490} = \frac{24}{49}$

(iii)Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$\therefore$ P(E_1) = $\frac{3}{6}$

$= \frac{1}{2}$

and P(E_2) = $\frac{13}{52} = \frac{1}{4}$

Then, P($E_1 \cap E_2$) = P(E_1) · P(E_2)

$= \frac{1}{2}, \frac{1}{4} = \frac{1}{8}$

(iv) P(A) + P(B) - P(A and B) = P(A)

$\Rightarrow$ P(A) + P(B) - P(A $\cap$ B) = P(A)

$\Rightarrow$ P(B) - P(A $\cap$ B) = 0

$\Rightarrow$ P(A $\cap$ B) = P(B)

$\therefore$ P(A|B) = $\frac{P(A\cap B)}{P(B)}$

= $\frac{P(B)}{P(B)}$

= 1

11. **Read the text carefully and answer the questions:**

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association).

Let A be a 1 $\times$ 2 matrix and B be a 2 $\times$ 1 matrix, representing the investment and interest rate on each bond respectively.

Based on the above information, answer the following questions.

(i) If ₹ 15000 is invested in bond X, then the amount invested in bond Y = ₹ (35000 - 15000) = ₹ 20000.

A = Investment $\begin{bmatrix} X & Y \\ 15000 & 20000 \end{bmatrix}$

and B = $\begin{matrix} X \\ Y \end{matrix} \overset{Interest\ rate}{\begin{bmatrix} 10\% \\ 8\% \end{bmatrix}} = \begin{matrix} X \\ Y \end{matrix} \overset{Interest\ rate}{\begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}}$

(ii) The amount of interest received on each bond is given by

AB = $\begin{bmatrix} 15000 & 20000 \end{bmatrix} \times \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$

= [15000 $\times$ 0.1 + 20000 $\times$ 0.08] = [1500 + 1600] = 3100

(iii) Let ₹ x be invested in bond X and then ₹ (35000 - x) will be invested in bond Y.

Now, total amount of interest is given by

$\begin{bmatrix} x & 35000 - x \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$ = [0.1x + (35000 - x)0.08]

But, it is given that total amount of interest = ₹ 3200

$\therefore$ 0.1x + 2800 - 0.08x = 3200

$\Rightarrow$ 0.02x = 400 $\Rightarrow$ x = 20000

Thus, ₹ 20000 invested in bond X and ₹ 35000 - ₹ 20000 = ₹ 15000 invested in bond Y.

12. a. Given, dy = (5x - 4y) dx

$\frac{dy}{dx}$ + 4y = 5x, is a linear differential equation of the form $\frac{dy}{dx} + p_y = Q$

Here, P = 4, Q = 5x

Now, I.F. = $e^{\int 4dx} = e^{4x}$

Solution of given equation is given by

$y \cdot (I.F.) = \int Q \cdot (I.F.) dx$

$y \cdot e^{4x} = \int 5x \cdot e^{4x} dx$

$= 5x \cdot \frac{e^{4x}}{4} - \frac{5}{4} \int e^{4x} dx + c$ [using integration by parts]

$= \frac{5}{4} x \cdot e^{4x} - \frac{5}{16} e^{4x} + c$

Given, when y = 0, x = 0, $\therefore\ c = \frac{5}{16}$

$\therefore\ y = \frac{5}{4} x - \frac{5}{16} + \frac{5}{16} e^{-4x}$

b. Given, y dx - (x + 2y^2)dy = 0

$\Rightarrow$ y dx = (x + 2 y^2)dy

$\Rightarrow \frac{dx}{dy} = \frac{x+2y^2}{y}$

$\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 2y \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y$

It is linear differential equation of the form, $\frac{dx}{dy} + Px = Q$

Here, $P=-\frac{1}{y}$ and Q = 2y

$\therefore$ I.F. $=e^{\int Pdy}=e^{-\int \frac{1}{y}dy}$

$\Rightarrow$ I.F. $=e^{-\log|y|}=e^{\log\frac{1}{|y|}}=\frac{1}{y}$

The solution is

$x\times l.F.=\int(Q\times I.F.)dy+C$

$\Rightarrow \frac{x}{y}=\int\frac{2y}{y}dy+C$

$\Rightarrow \frac{x}{y}=2\int dy+C$

$\Rightarrow \frac{x}{y}=2y+C$

$\Rightarrow$ x = 2y^2 + yC

OR

$\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$

$\frac{dy}{dx}=-\frac{y^2+y+1}{x^2+x+1}$

$\int\frac{dy}{y^2+y+1}=-\int\frac{dx}{x^2+x+1}$

$\int\frac{dy}{y^2+y+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}=-\int\frac{dx}{x^2+x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}$

$\int\frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=-\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

$\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)=-\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+c$

$\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)=\frac{\sqrt{3}}{2}c$

$\tan^{-1}\left[\frac{\frac{2y+1}{\sqrt{3}}+\frac{2x+1}{\sqrt{3}}}{1+\left(\frac{2y+1}{\sqrt{3}}\right)\left(\frac{2x+1}{\sqrt{3}}\right)}\right]=\frac{\sqrt{3}}{2}c$

let $\frac{\sqrt{3}}{2}c=A_1$

$\frac{\frac{2y+1}{\sqrt{3}}+\frac{2x+1}{\sqrt{3}}}{1-\left(\frac{2y+1}{\sqrt{3}}\right)\left(\frac{2x+1}{\sqrt{3}}\right)}=\tan A_1$

$\frac{2\sqrt{3}(x+y+1)}{3-(4xy+2x+2y+1)}=\tan A_1$

$x+y+1=A(1-x-y-2xy)$

$\left[\because \frac{1}{\sqrt{3}}\tan A_1=A\right.$

13. Let r be the radius of the sphere and dimensions of cuboid are x, 2x and $\frac{x}{3}$.

$\therefore 4\pi r^2+2\left[\frac{x}{3}\times x+x\times 2x+2x\times\frac{x}{3}\right]$ =k (constant) [given]

$\Rightarrow 4\pi r^2+6x^2=k$

$\Rightarrow \quad r^2=\frac{k-6x^2}{4\pi}\Rightarrow r=\sqrt{\frac{k-6x^2}{4\pi}}$ ......(i)

Sum of the volumes, $V=\frac{4}{3}\pi r^3+\frac{x}{3}\times x\times 2x$

$=\frac{4\pi r^3}{3}+\frac{2}{3}x^3$ ....(ii)

$\Rightarrow \quad V=\frac{4}{3}\pi\left(\frac{k-6x^2}{4\pi}\right)^{\frac{3}{2}}+\frac{2}{3}x^3$

On differentiating both sides w.r.t. x, we get

$\frac{dV}{dx}=\frac{4}{3}\pi\times\frac{3}{2}\left(\frac{k-6x^2}{4\pi}\right)^{\frac{1}{2}}\left(\frac{-12x}{4\pi}\right)+\frac{2}{3}\times 3x^2$

$=2\pi\sqrt{\frac{k-6x^2}{4\pi}}\left(\frac{-3x}{\pi}\right)+2x^2$

$=(-6x)\sqrt{\frac{k-6x^2}{4\pi}}+2x^2$

For maxima or minima, put $\frac{dV}{dx}=0$

$\Rightarrow \quad (-6x)\sqrt{\frac{k-6x^2}{4\pi}}+2x^2=0$

$\Rightarrow \quad 2x^2=6x\sqrt{\frac{k-6x^2}{4\pi}}$

$\Rightarrow \quad x = 3\sqrt{\frac{k-6x^2}{4\pi}}$

⇒ x=3r

[using Eq. (i)]

Again, on differentiating $\frac{dV}{dx}$ w.r.t. x, we get

$\frac{d^2V}{dx^2} = -6\frac{d}{dx}\left(x\sqrt{\frac{k-6x^2}{4\pi}}\right) + 4x$

$= -6\left(\sqrt{\frac{k-6x^2}{4\pi}} + x \cdot \frac{1}{2} + \frac{1}{\sqrt{\frac{k-6x^2}{4}}}\left(\frac{-12x}{4\pi}\right)\right) + 4x$

$= -6\left(r - \frac{3x^2}{2\pi r}\right) + 4x$

$= -6r + \frac{9x^2}{\pi r} + 4x$

Now, $\left(\frac{d^2r}{dx^2}\right)_{x=3r} = -6r + \frac{9\times 9r^2}{\pi} + 12r = 6r + \frac{18r}{\pi} > 0$

Hence, V is minimum when x is equal to three times the radius of the sphere.

Hence proved.

Now, on putting $r = \frac{x}{3}$ in Eq. (ii), we get

$V_{\text{min}} = \frac{4\pi}{3}\left(\frac{x}{3}\right)^3 + \frac{2}{3}x^3 = \frac{4\pi}{81}x^3 + \frac{2}{3}x^3$

$= \frac{2}{3}x^2\left(\frac{2\pi}{27} + 1\right) = \frac{2}{3}x^3\left(\frac{44}{189} + 1\right)$

$= \frac{2}{3}x^3\left(\frac{233}{189}\right) = \frac{466}{567}x^3$

OR

i. Let $f_1(x)$ = cos x

   $\therefore f_1'(x)$ = -sin x

   In interval $\left(0, \frac{\pi}{2}\right)$, $f_1'(x)$ = -sin x < 0.

   Therefore, $f_1(x)$ = cos x is strictly decreasing in interval $\left(0, \frac{\pi}{2}\right)$.

ii. Let $f_2(x)$ = cos 2x

   $\therefore f_2'(x)$ = -2 sin 2x

   Now, $0 < x < \frac{\pi}{2}$

   $\Rightarrow 0 < 2x < \pi$

   ⇒ sin 2x > 0

   ⇒ -2 sin 2x < 0

   $\therefore f_2'(x)$ = -2 sin 2x < 0 on $\left(0, \frac{\pi}{2}\right)$

   Therefore, $f_2(x)$ = cos 2x is strictly decreasing in interval $\left(0, \frac{\pi}{2}\right)$.

iii. Let $f_3(x)$ = cos 3x

   $\therefore f_3'(x)$ = -3 sin 3x

   Now, f_3' = 0

   ⇒ sin 3x = 0

   ⇒ 3x = π, as $x \in \left(0, \frac{\pi}{2}\right)$

   $\Rightarrow x = \frac{\pi}{3}$

   The point $x = \frac{\pi}{3}$ divides the interval $\left(0, \frac{\pi}{2}\right)$ into two distinct intervals.

   i.e. $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

   Now, in interval, $\left(0, \frac{\pi}{3}\right)$

   $f_3'(x) = -3\sin 3x < 0$ as $\left(0 < x < \frac{\pi}{2} \Rightarrow 0 < 3x < \pi\right)$

   Therefore, f_3 is strictly decreasing in interval $\left(0, \frac{\pi}{3}\right)$

   Now, in interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

   $f_3'(x) = -3\sin 3x >$ as $\frac{\pi}{3} < \text{x} < \frac{\pi}{2} \Rightarrow \pi < 3\text{x} < \frac{3\pi}{2}$

   Therefore, f_3 is strictly increasing in interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

iv. Let f_4 = tan x

   $\therefore f_3'(\text{x}) = \sec^2 \text{x}$

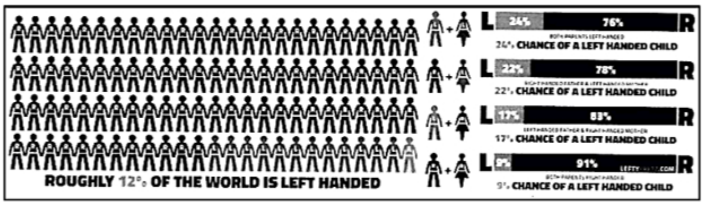
In interval $\left(0, \frac{\pi}{2}\right)$

$f_3'(x) = \sec^2 x > 0$

Therefore, f_4 is strictly increasing in interval $\left(0, \frac{\pi}{2}\right)$

14. **Read the text carefully and answer the questions:**

Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows:

A. When both father and mother are left handed:
Chances of left handed child is 24%.

B. When father is right handed and mother is left handed:
Chances of left handed child is 22%.

C. When father is left handed and mother is right handed:
Chances of left handed child is 17%.

D. When both father and mother are right handed:
Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

(i) $P\left(\frac{L}{C}\right) = \frac{17}{100}$

(ii) $P\left(\frac{\bar{L}}{A}\right) = 1 - P\left(\frac{L}{A}\right) = 1 - \frac{24}{100} = \frac{76}{100}$ or $\frac{19}{25}$

(iii) $P\left(\frac{A}{L}\right) = \frac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100}+\frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}+\frac{1}{4}\times\frac{9}{100}} = \frac{24}{72} = \frac{1}{3}$

(iv) Probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed.

$= P\left(\frac{L}{B\cup C}\right) = \frac{22}{100} + \frac{17}{100} = \frac{39}{100}$

SECTION B - 15 MARKS

15. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) **(a)** -2

Explanation: {

If three vectors are coplanar $\begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & \lambda & -1 \end{vmatrix} = 0$

$3(1 \times (-1) - 3\lambda) - (-1)(-2 - 3) + 2(2\lambda - 1) = 0$

$-3 - 9\lambda - 2 - 3 + 4\lambda - 2 = 0$

$-5\lambda = 10$

$\lambda = \frac{10}{-5}$

$\lambda = -2$

(ii) The direction cosines of a line which makes an angle of α, β, γ with the axes, are $\cos\alpha, \cos\beta, \cos\gamma$

Therefore,direction cosines of the line are cos 30°, cos 60°, cos 90° i.e., $\pm\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right)$

(iii) $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j} = \hat{k} \cdot \hat{k} + \hat{i} \cdot \hat{j}$

= 1 + 0 = 1

(iv) **(b)** The planes are parallel

Explanation: {

We have, 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z = 0. Here ,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{2}{3}$

Therefore , the given planes are parallel.

(v) DRs are (6, 2, 3) $\therefore$ DC’s are $\left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$

16. Given that a, b, c are mutually prependicular vectors,

So,

$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$ ....(1)

Also, a,b and c are unit vectors, so

$|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

$|\vec{a}+\vec{b}+\vec{c}|^2=(\vec{a}+\vec{b}+\vec{c})^2$

$=(\vec{a})^2+(\vec{b})^2+(\vec{c})^2+2\vec{a}.\vec{b}+2\vec{b}.\vec{c}+2\vec{c}.\vec{a}$

$=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(0)+2(0)+2(0)$[from (1)]

$=(1)^2+(1)^2+(1)^2+0$

$|\vec{a}+\vec{b}+\vec{c}|^2=1+1+1$

$|\vec{a}+\vec{b}+\vec{c}|^2=3$

$|\vec{a}+\vec{b}+\vec{c}|=\sqrt{3}$

OR

Given,$\vec{a}=\hat{i}-2\hat{j}$

$\vec{b}=2\hat{i}-3\hat{j}$

$\vec{c}=2\hat{i}+3\hat{k}$

$\therefore=\vec{a}+\vec{b}+\vec{c}$

$=(\hat{i}-2\hat{j})+(2\hat{i}-3\hat{j})+(2\hat{i}+3\hat{k})$

$=5\hat{i}-5\hat{j}+3\hat{k}$

17. We know that the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$

The given planes are x - 2y + kz = 4 and 2x + z = 9

$\Rightarrow a_1=1;b_1=-2;c_1=k;a_2=2;b_2=5;c_2=-1$

It is given that the given planes are perpendicular.

$\Rightarrow a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow$ (1) (2) + (-2) (5) + (k) (-1) = 0

$\Rightarrow$ 2 - 10 - k = 0

$\Rightarrow$ -8 - k = 0

$\Rightarrow$ k = -8

Hence the value of k = -8

OR

Given equations are;

3l + m + 5n = 0 and

6mn - 2nl + 5lm = 0

Eliminating m from the above two equations, we get

$\Rightarrow 2n^2+3ln+l^2=0$

$\Rightarrow$ (n + l)(2n + l) = 0

$\Rightarrow$ either n = -l or l = -2n

Now if l = -n, then m = -2n

and if l = -2n, then m = n.

Thus the direction ratios of two lines are proportional to -n, -2n, n and -2n, n, n,

i.e. 1, 2, -1 and -2, 1, 1.

So, vectors parallel to these lines are

$\vec{a}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=-2\hat{i}+\hat{j}+\hat{k}$, respectively.

If θ is the angle between the lines, then

$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

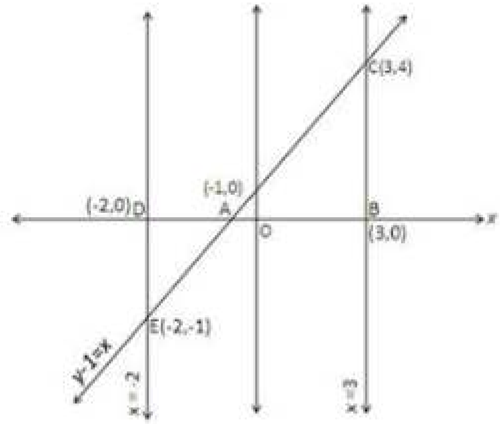
$=\frac{(\hat{i}+2\hat{j}-\hat{k})\cdot(-2\hat{i}+\hat{j}+\hat{k})}{\sqrt{1^2+2^2+(-1)^2}\sqrt{(-2)^2+1^2+1^2}}=-\frac{1}{6}$

Hence $\theta=\cos^{-1}\left(-\frac{1}{6}\right)$

18. p>To find area of region bounded by x-axis the ordinates x = -2 and x = 3 and y -1 = x ...(i)

Equation (i) is a line that meets at axes at (0, 1) and (-1, 0)

A rough sketch of the given information is as under:-



Bounded region provides the required area.

Now Required area = Area of Region ABCA + Area of Region ADEA

A = $\int_{-1}^{3} ydx + \left|\int_{-2}^{-1} ydx\right|$

$= \int_{-1}^{3}$ (x + 1)dx + |[-2^{-1} (x + 1)dx|

$= \left(\frac{x^2}{2} + x\right)_{-1}^{3} + \left|\frac{x^2}{2} + x\right|_{-2}^{-1}$

= [($\frac{9}{2}$ + 3) - ($\frac{1}{2}$ - 1)] + ($\frac{1}{2}$ - 1) - (2 - 2)]

$= \left[\frac{15}{2} + \frac{1}{2}\right] + \left|-\frac{1}{2}\right|$

= 8 + $\frac{1}{2}$

A = $\frac{17}{2}$ sq. units

SECTION C - 15 MARKS

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) **(c)** MC = AC

Explanation: {

MC = AC

(ii) **(c)** Linear constraints

Explanation: {

In a LPP, the linear inequalities or restrictions on the variables are called Linear constraints.

(iii) Let the line of regression of y on x be

2x + 3y - 10 = 0

$\Rightarrow$ 3y = -2x + 10

$\Rightarrow$ y = -$\frac{2}{3}$x - $\frac{10}{3}$

$\therefore b_{yx} = -\frac{2}{3}$

Let the line of regression of x on y be

4x + y - 5 = 0

$\Rightarrow$ 4x = -y + 5

$\Rightarrow$ x = -$\frac{1}{4}$y + $\frac{5}{4}$

$\therefore$ bxy = -$\frac{1}{4}$

Here, $b_{yx} \times b_{xy} = \left(-\frac{2}{3}\right)\left(-\frac{1}{4}\right) = \frac{1}{6} < 1$

Which is true. Hence, our assumption is correct and line of regression of y on x is 2x + 3y - 10 = 0

(iv) P = Revenue - Cost = ₹$\left\{\left(50 - \frac{x}{2}\right)x - \left(\frac{x^2}{4} + 35x + 25\right)\right\}$ = ₹ $\left(-\frac{3}{4}x^2 + 15x - 25\right)$

P = $\left(-\frac{3}{4}x^2 + 15x - 25\right)$

differentiating the above equation, we get,

$\frac{dP}{dx} = -\frac{3}{2}x + 15$

equate $\frac{dP}{dx} = 0$

$\Rightarrow -\frac{3}{2}x + 15 = 0$

$\therefore$ x = 10

again differentiating the above equation, we get,

$\frac{d^2P}{dx^2} = -\frac{3}{2} < 0$ maxima

Therefore profit is maximum when daily output is 10 items.

(v) Let R(x) be the revenue received for selling x television sets. Then,

R(x) = px

$\Rightarrow$ R (x) = 8400x

We have, C(x) = $300x^2$ + 4200x + 13500. At the break even points, we have

R(x) = C(x)

$\Rightarrow$ 8400x = $300x^2$ + 4200x + 13500

$\Rightarrow$ $300x^2$ - 4200x + 13500 = 0

$\Rightarrow$ x^2 - 14x + 45 = 0

$\Rightarrow$ x = 5, 9

Hence, the sell of either 5 or 9 television sets will give break-even points.

20. Let ₹ x be the increase in the monthly subscription. Then, New rate = 100 + x.

Since, for each increase of ₹ 1, 5 subscribers will discontinue the service.

$\therefore$ Total number of subscribers = 1000 - 5x

Let R be the total revenue of the company. Then,

R = Rate $\times$ Number of subscribers

$\Rightarrow$ R = (100 + x)(1000 - 5x) = 100000 + 500x - $5x^2$ ... (i)

$\Rightarrow \frac{dR}{dx}$ = 500 - 10x and, $\frac{d^2R}{dx^2}$ = -10

For R to be maximum, we must have

$\frac{dR}{dx} = 0 \Rightarrow$ 500 - 10x = 0

$\Rightarrow$ x = 50

Clearly, $\frac{d^2R}{dx^2}$ = -10 < 0 for all x. So, R is maximum when x = 50.

Putting x = 50 in (i) we get: R = 10000 + 500 $\times$ 50 - 5 $\times$ $(50)^2$ = 112500

Thus, R is maximum when monthly subscription is increased by ₹ 50 and the maximum revenue is ₹ 112500.

OR

We have, p = $\frac{b}{a+x}$, Let R be the revenue function. Then,

R = px $\Rightarrow$ R = $\frac{x}{a+x} \Rightarrow \frac{dR}{dx} = \frac{(a+x)b-bx}{(a+x)^2}$

$\Rightarrow$ MR = $\frac{ab}{(a+x)^2}$

$\Rightarrow \frac{d}{dx}(MR) = -\frac{2ab}{(a+x)^3}$

Clearly, $\frac{d}{dx}(MR) > 0$ ...[$\because$ b < 0 and a > 0 $\therefore$ ab < 0 $\Rightarrow \frac{-2ab}{(a+x)^2} > 0$]

$\therefore$ MR is increasing for all b < 0, a > 0

21.

x	y	xy	x^2
1	7	7	1
2	6	12	4
3	5	15	9
4	4	16	16
5	3	15	25
Σx = 15	Σy = 25	Σxy = 65	Σx^2 = 55

$\bar{x} = \frac{\sum x}{n} = \frac{15}{5} = 3$

and $\bar{y} = \frac{\sum y}{n} = \frac{25}{5} = 5$

$b_{yx} = \frac{\sum xy - \frac{\sum x \cdot \Sigma y}{n}}{\sum x^2 - \frac{(\Sigma x)^2}{n}}$

$= \frac{65 - \frac{15.25}{5}}{55 - \frac{(15)^2}{5}}$

$= \frac{65-75}{55-45} = \frac{-10}{10}$ = -1

Regression line y on x is given by

(y - $\bar{y}$) = b_{yx} (x - $\bar{x}$)

∴ y - 5 = -1(x - 3)

⇒ y - 5 = -x + 3

⇒ x + y = 8 ...(i)

When x = 6,

From (i), we get

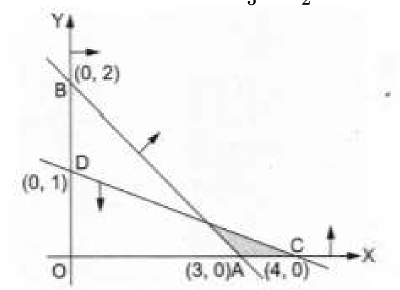
y = 8 - 6

⇒ y = 2

22. Here, given equations

$2x + 3y = 6, x + 4y = 4$, $x = 0$ *and* $y = 0$

Now, $2x + 3y = 6 \Rightarrow \frac{x}{3} + \frac{y}{2} = 1$



Now the line meets the axes at A(3,0) and B(0,2). Join these points and draw a thick line. Clearly, the portion not containing (0, 0) represents the solution set of the inequation 2x + 3y > 6

Again, $x + 4y = 4 \Rightarrow \frac{x}{4} + \frac{y}{1} = 1$

This line meets the axes at C(4, 0) and D(0, 1).

Join these points and draw a thick line. Clearly, the portion containing (0, 0) represents the solution set of the inequation x + 4y < 4

Clearly, x > 0 is represented by the y-axis and the portion on its right-hand side

Also, y > 0 is represented by the x-axis and the portion above the x-axis.

Therefore, the shaded region represents the solution set of the given inequations.

OR

According to the question,

Maximize $Z = x + 2y$

Subject to the constraints

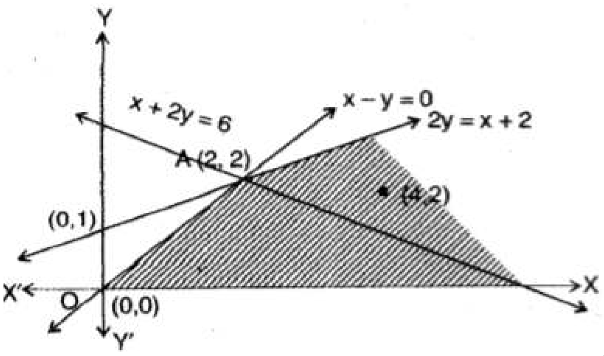
$x - y \geq 0$

$2y \leq x + 2$

such that $x \geq 0, y \geq 0$

We draw line $x - y = 0$ and $2y = x + 2$ and shaded the feasible region with respect to constraints sign.

We observe that the feasible region is unbounded and corner points are $O(0,0), A(2,2)$ only. Evaluating Z at all corner points,



Corner Points	Z = x + 2y
$(0, 0)$	$Z = 0\ (Smallest)$
$(2, 2)$	$Z = 6\ (largest)$

We are to determine the maximum value. As the feasible region is unbounded we can not say whether the largest value Z = 6 is maximum or not.

We draw the half plane $x + 2y > 6$ and notice that it has common points with feasible region. There does not exist any maximum value. For testing we let $x = 4, y = 2$, this point lie in the feasible region and at $(4, 2)$ the value of $Z = 8$ i.e., greater than '6'.