ISC 2025 EXAMINATION

Sample Question Paper - 4

Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

This Question Paper consists of three sections A, B and C.

Candidates are required to attempt all questions from ***Section A*** *and all questions*

EITHER *from* ***Section B OR Section C.***

Section A*: Internal choice has been provided in* ***two questions of two marks each, two questions of four marks each and two questions of six marks each.***

Section B: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

Section C: *Internal choice has been provided in* ***one question of two marks and one question of four marks.***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

1. **In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.** **[15]**

(a) If $\begin{bmatrix} 1 & 2 \\ -2 & -b \end{bmatrix} + \begin{bmatrix} a & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix}$, then $a^2 + b^2$ is equal to **[1]**

a) 12 b) 10

c) 20 d) 22

(b) The primitive of the function $f(x) = \left(1 - \frac{1}{x^2}\right) a^{x+\frac{1}{x}}, a > 0$ is **[1]**

a) $\frac{a^{x+\frac{1}{x}}}{x} \log_e a$ b) $\log_e a \cdot a^{x+\frac{1}{x}}$

c) $\frac{a^{x+\frac{1}{x}}}{\log_e a}$ d) $x \frac{a^{\frac{1}{x}}}{\log_e a}$

(c) Which of the following is the principal value branch of $\text{cosec}^{-1}x$? **[1]**

a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

c) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

(d) The solution of $\frac{dy}{dx} = |x|$ is **[1]**

a) $y = \frac{x^2}{2} + C$ b) $y = \frac{x|x|}{2} + C$

c) $y = \frac{|x|}{2} + C$ d) $y = \frac{x^3}{2} + C$

(e) If A and B are two independent events such that P(A) = 0.3, P(A ∪ B) = 0.5, then P(A / B) - P(B / A) **[1]**

=

a) $\frac{2}{7}$

b) $\frac{1}{7}$

c) $\frac{1}{70}$

d) $\frac{3}{35}$

(f) If f(x) = $\frac{x}{x-1}$ then $\frac{f(a)}{f(a+1)}$ = **[1]**

a) $f\left(-\frac{a}{a-1}\right)$

b) f(a^2)

c) f(-a)

d) $f\left(\frac{1}{a}\right)$

(g) $\frac{d}{dx}$(e^x + e^{x^2} + ... + e^{x^5}) is equal to **[1]**

a) $e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$

b) $e^x - 2e^x - 3e^x - 4e^x - 5e^x$

c) $e^x + 2e^x + 3e^x + 4e^x + 5e^x$

d) $e^x + 2xe^x + 3xe^{x^2} + 4e^{x^3} + 5xe^{x^4}$

(h) If e^{x+y} = xy then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{(x-xy)}{(xy-y)}$

b) $\frac{1}{2}\frac{(x-xy)}{(xy-y)}$

c) $\frac{y(1-x)}{x(y-1)}$

d) $\frac{x(1-y)}{y(x-1)}$

(i) If A is an invertible matrix of any order, then which of the following options is NOT true? **[1]**

a) $(A^2)^{-1} = (A^{-1})^2$

b) $|A^{-1}| = |A|^{-1}$

c) $(A^T)^{-1} = (A^{-1})^T$

d) $|A| \neq 0$

(j) Let A and B be two symmetric matrices of order 3. **[1]**

Assertion (A): A(BA) and (AB) A are symmetric matrices.

Reason (R): AB is symmetric matrix, if matrix multiplication of A with B is commutative.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

(k) Let S = {1, 2, 3} Determine whether the function f: S $\rightarrow$ S defined as below have inverse. Find f^{-1}, if it exists. **[1]**

f = { (1, 2) (2, 1) (3, 1) }

(l) For A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ write A^{-1}. **[1]**

(m) Find the domain of the function $f(x) = \frac{x^2+3x+5}{x^2+x-6}$. **[1]**

(n) A coin is tossed three times. Let the events A and B be defined as follows: A = first toss is head, B = second toss is head. Check the independence of A and B. **[1]**

(o) The probability that a student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university none will graduate. **[1]**

2. Differentiate $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, if $x > \frac{1}{\sqrt{3}}$ **[2]**

OR

A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

3. Evaluate: $\int \frac{x}{(x^2+4)\sqrt{x^2+9}}dx$ **[2]**

4. Find the intervals in which $f(x) = \frac{x}{\log x}$ is increasing or decreasing. **[2]**

5. Evaluate the Integral: $\int \frac{1}{x\sqrt{x^4-1}}dx$ **[2]**

OR

Evaluate: $\int \frac{\tan x \sec^2 x}{(1-\tan^2 x)}dx.$

6. Check whether the relation R in **R** defined by R = {(a, b) : a $\leq$ b^3} is reflexive, symmetric or transitive. **[2]**

7. Find the value of $\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$ **[4]**

8. Evaluate $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x+\cos^n x}dx$ **[4]**

9. Let f(x + y) = f(x) + f(y) for all x, y $\in$ R. If f(x) is continuous at x = 0, show that f(x) is continuous at all x. **[4]**

OR

If $x = a.sin(2t).(1 + cos2t)$ and $y = b.cos2t.(1-cos2t)$, show that $\left(\frac{dy}{dx}\right)_{at\ t=\frac{\pi}{4}} = \frac{b}{a}$

10. **Read the text carefully and answer the questions:** **[4]**

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

(c) Find the conditional probability of obtaining the sum 10, given that the black die resulted in even number.

(d) Find the conditional probability of obtaining the doublet, given that the red die resulted in a number more than 4.

OR

Read the text carefully and answer the questions: **[4]**

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(a) What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work.

(b) An application selected at random is found to be under the below 18 category. Find the probability that it is under the folk genre. Show your work.

(c) If P(A) = 0.4, P(B) = 0.8 and P(B|A) = 0.6, then P(A $\cup$ B) is equal to

(d) If A and B are two independent events with
P(A) = $\frac{3}{5}$ and P(B) = $\frac{4}{9}$, then find P(A' $\cap$ B').

11. **Read the text carefully and answer the questions:** **[6]**

Three shopkeepers A, B and C go to a store to buy stationery. A purchase 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs ₹40, a pen costs ₹12 and a pencil costs ₹3.

(a) How are the number of items purchased by shopkeepers A, B, and C represented in matrix form?

(b) If X represents a matrix, and Y represents the matrix formed by the cost of each item, what does the product X Y equal?

(c) If $A^2 = A$, then what is the value of $(A + I)^3 - 7A$?

12. Solve $(x^2 - y^2)dx + 2xy\,dy = 0$ **[6]**

OR

Show that the differential equation of $\left(x^2 + xy\right) dy = \left(x^2 + y^2\right) dx$ is homogeneous and solve it.

13. Show that the curves intersect orthogonally at the indicated points $y^2 = 8x$ and $2x^2 + y^2 = 10$ at $(1, 2\sqrt{2})$. **[6]**

OR

Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

14. **Read the text carefully and answer the questions:** **[6]**

In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.

(a) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is NOT processed by Govind.

(b) Find the probability that Priyanka processed the form and committed an error.

(c) Find the total probability of committing an error in processing the form.

(d) Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form. The value of $\sum_{i=1}^{3} P\left(E_i \mid A\right)$?

SECTION B - 15 MARKS

15. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** **[5]**

(a) If $\vec{a} \cdot \vec{b} = \frac{1}{2}|\vec{a}||\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$is **[1]**

a) 60^o

b) 0^o

c) 90^o

d) 30^o

(b) Find the angle between the pair of lines [1]

$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$

$\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

(c) Write the length (magnitude) of a vector whose projections on the coordinate axes are 12, 3 and 4 units. [1]

(d) If a line makes an angle of 30^o with the positive direction of x-axis, 120^o with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is: [1]

a) 0^o b) 60^o

c) 90^o d) 120^o

(e) Write the sum of intercepts cut off by the plane $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) - 5 = 0$ on the three axes. [1]

16. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, show that $(\vec{a} + \vec{b})$ is perpendicular $(\vec{a} - \vec{b})$. [2]

OR

Prove, using vectors, Medians of a triangle are concurrent.

17. Find the angle between the planes whose vector equations are $\vec{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 5$ and $\vec{r}.\left(3\hat{i} - 3\hat{j} + 5\hat{k}\right) = 3$. [4]

OR

Find the image of the point (3, -2, 1) in the plane 3x - y + 4z = 2.

18. Make a sketch of the region $\left\{(x,y) : 0 \le y \le x^2 + 3; 0 \le y \le 2x + 3; 0 \le x \le 3\right\}$ and find its area using integration. [4]

SECTION C - 15 MARKS

19. **In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.** [5]

(a) The revenue of a monopolist is given by $R(x) = 12x^2 + 300 - x$. Then the average revenue function AR(x) at x = 10 will be: [1]

a) 1500 b) 1210

c) 12310 d) 1229

(b) A linear programming problem deals with the optimization of a/an: [1]

a) logarithmic function b) exponential function

c) linear function d) quadratic function

(c) If $\bar{x}$ = 18, $\bar{y}$ = 100, σ_x = 14, σ_y = 20 and correlation coefficient r_{xy} = 0.8, find the regression equation of y on x. [1]

(d) A company has fixed costs of ₹ 26,000. The cost of producing one item is ₹ 30. If this item sells for ₹ 43, find the break-even point. [1]

(e) The demand function of a monopolist is given by $p = 1500 - 2x - x^2$. Find the marginal revenue for any level of output x. Also, find the marginal revenue (MR) when x = 10. [1]

20. A tour operator charges ₹ 136 per passenger for 100 passengers with a refund of ₹ 4 for each 10 passengers in excess of 100. Determine the number of passengers that will maximize the amount of money the tour operator receives. [2]

OR

Suppose the cost to produce some commodity is a linear function of output. Find cost as a function of output, if costs are ₹ 4000 for 250 units and ₹ 5000 for 350 units.

21. From the equation of the two regression lines, 4x + 3y + 7 = 0 and 3x + 4y + 8 = 0, find: **[4]**

i. Mean of x and y
ii. Regression coefficient
iii. Coefficient of correlation

22. A man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs 3 per km. He has at most Rs 120 to spend on petrol and one hour time. He wishes to find the maximum distance that he can travel. **[4]**

Express this problem as a linear programming problem.

OR

Maximise the function Z = 11x + 7y, subject to the constraints: $x \leqslant 3, y \leqslant 2, x \geqslant 0, y \geqslant 0$.

Solution

SECTION A - 65 MARKS

1. In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(i) **(c)** 20

Explanation: {

We have, $\begin{bmatrix} 1 & 2 \\ -2 & -b \end{bmatrix} + \begin{bmatrix} a & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+1 & 6 \\ 1 & 2-b \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow$ a + 1 = 5, 2 - b = 0

$\Rightarrow$ a = 4, b = 2

$\therefore a^2 + b^2 = 4^2 + 2^2 = 16 + 4 = 20$

(ii) **(c)** $\frac{a^{x+\frac{1}{x}}}{\log_e a}$

Explanation: {

$f(x) = \left(1 - \frac{1}{x^2}\right) a^{x+\frac{1}{x}}$

$\Rightarrow \int f(x) dx = \int \left(1 - \frac{1}{x^2}\right) a^{x+\frac{1}{x}} dx$

Put $x + \frac{1}{x} = t$

$\Rightarrow \left(1 - \frac{1}{x^2}\right) \mathrm{dx} = \mathrm{dt}$

$I = \int a^t dt$

$I = \frac{a^t}{\log_e a} + c$

$I = \frac{a^{x+\frac{1}{x}}}{\log_e a} + c$

(iii) **(b)** $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Explanation: {

We know that the principal value branch of $\operatorname{cosec}^{-1}$ x is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

(iv) **(b)** y = $\frac{x|x|}{2}$ + C

Explanation: {

y = $\frac{x|x|}{2}$ + C

(v) **(c)** $\frac{1}{70}$

Explanation: {

P(A) = 0.3 , P(A∪B) = 0.5 (Given)

Since, A and B are two independent events,

P(A∩B) = P(A) P(B)

P(A∩B) = 0.3 × P(B) ...(i)

Also, according to the addition theorem of probability,

P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

0.5 = 0.3 + P(B) - 0.3 P(B) From (Given) & (i)

0.7 P(B) = 0.2

P(B) = $\frac{0.2}{0.7}$ = $\frac{2}{7}$ ...(ii)

Putting value of P(B) in equation (i) we get,

P(A ∩ B) = 0.3 $\times \frac{2}{7} = \frac{3}{10} \times \frac{2}{7}$

P(A ∩ B) = $\frac{6}{70}$ ...(iii)

Now,

$P\left(\frac{A}{B}\right) - P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(B)} - \frac{P(A \cap B)}{P(A)}$

$= \frac{\frac{6}{70}}{\frac{2}{7}} - \frac{\frac{6}{70}}{\frac{3}{10}}$ From (iii) & (ii) and (Given)

$= \frac{6}{70} \times \frac{7}{2} - \frac{6}{70} \times \frac{10}{3}$

$= \frac{3}{10} - \frac{2}{7}$

$= \frac{1}{70}$

(vi) **(b)** $f(a^2)$

Explanation: {

$\frac{f(a)}{f(a+1)} = \frac{\frac{a}{a-1}}{\frac{a+1}{a+1-1}} = \frac{\frac{a}{a-1}}{\frac{a+1}{a}}$

$= \frac{a}{a-1} \times \frac{a}{a+1} = \frac{a^2}{a^2-1}$

$f(a^2) = \frac{a^2}{a^2-1}$

$\therefore \frac{f(a)}{f(a+1)} = f(a^2)$

(vii) **(a)** $e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$

Explanation: {

Let $y = e^x + e^{x^2} + ... + e^{x^5}$

On differentiating both sides w.r.t. x, we get

$\frac{d}{dx}(y) = \frac{d}{dx}\left\{e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}\right\}$

$= \frac{d}{dx}(e^x) + \frac{d}{dx}\left(e^{x^2}\right) + \frac{d}{dx}\left(e^{x^3}\right) + \frac{d}{dx}\left(e^{x^4}\right) + \frac{d}{dx}\left(e^{x^5}\right)$

$= e^x + e^{x^2}\frac{d}{dx}x^2 + e^{x^3}\frac{d}{dx}x^3 + e^{x^4}\frac{d}{dx}\left(x^4\right) + e^{x^5}\frac{d}{dx}\left(x^5\right)$[using chain rule]

$= e^x + e^{x^2}(2x) + e^{x^3}\left(3x^2\right) + e^{x^4}\left(4x^3\right) + e^{x^5}\left(5x^4\right)$

$= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$

(viii) **(c)** $\frac{y(1-x)}{x(y-1)}$

Explanation: {

Given that $xy = e^{x + y}$

Taking log both sides, we get

$\log_e xy = x + y$ (Since $\log_a b^c = c\log_a b$)

Since $\log_a bc = \log_a b + \log_a c$, we get

$\log_e x + \log_e y = x + y$

Differentiating with respect to x, we obtain

$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$

Or

$\frac{dy}{dx}\left(\frac{y-1}{y}\right) = \frac{1-x}{x}$

Therefore, $\frac{dy}{dx} = \frac{y(1-x)}{x(y-1)}$

(ix) **(b)** $|A^{-1}| = |A|^{-1}$

Explanation: {

Since the determinant value of matrix and its reciprocal is same as the determinant value of an invertible matrix

(x) **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: {

Assertion: Since, A and B are symmetric matrices.

$\therefore A^T = A$ and $B^T = B$.

Now, to check A (BA) is symmetric.

Consider $[A(BA)]^T = (BA)^T \cdot A^T = (A^TB^T)A^T$

$= (AB)A = A(BA)$

So, $[A(BA)]^T = A(BA)$

$\Rightarrow$ A(BA) is symmetric.

Similarly, (AB) A is symmetric.

So, Assertion is true.

Reason: Now, (AB)' = B'A'

= BA

This will be symmetric, if A and B is commutative i.e. AB = BA.

Hence, both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(xi) f(2) = 1, f(3) =1 , as two elements of domain have same image in co-domain.

f is not one – one so that f is not invertible

Hence no inverse.

(xii) |A| = 1

$A^{-1} = \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$

(xiii) Here $f(x) = \frac{x^2+3x+5}{x^2+x-6} = \frac{x^2+3x+5}{(x+3)(x-2)}$

The function f(x) is defined for all values of x except

x + 3 = 0, x - 2 = 0 i.e. x = -3 and x = 2

Thus domain of f(x) = R - {-3, 2}

(xiv) The sample space is given by,

S = {(H H H),(H H T),(H T H),(H T T),(T H H),(T H T),(T T H),(T T T)}

$P(A) = \frac{4}{8} = \frac{1}{2}$

$P(B) = \frac{4}{8} = \frac{1}{2}$

$P(A \cap B) = \frac{2}{8} = \frac{1}{4} = P(A)P(B)$

Thus, A and B are independent events .

(xv) Let X be a random variable denoting number of students that graduate from among 3 students.

Let p = probability that a student entering a university will graduate.

Here, n = 3, p = 0.4 and q = 0.6

Therefore, the distribution is given by

$P(X = r) = {}^3C_r (0.4)^r (0.6)^{3-r}$, r = 0, 1, 2, 3

P(X = 0) = q^3 = 0.216

2. Let $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

Put, x = tan θ, we get

$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right) = \tan^{-1}(\tan 3\theta)$

If $x > \frac{1}{\sqrt{3}}$, then

$x = \tan\theta \Rightarrow \tan\theta > \frac{1}{\sqrt{3}} \Rightarrow \frac{\pi}{6} < \theta < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < 3\theta < \frac{3\pi}{2}$

$\therefore \quad y = \tan^{-1}(\tan 3\theta)$

$\Rightarrow \quad y = \tan^{-1}(-\tan(\pi - 3\theta)\}$

$\Rightarrow \quad y = \tan^{-1}\{\tan(3\theta - \pi)\}$

$\Rightarrow \quad y = 3\theta - \pi \quad \left[\because \frac{\pi}{2} < 3\theta < \frac{3\pi}{2} \Rightarrow -\frac{\pi}{2} < 3\theta - \pi < \frac{\pi}{2}\right]$

$\Rightarrow \quad y = 3\tan^{-1}x - \pi \quad [\because x = \tan\theta \Rightarrow \theta = \tan^{-1}x]$

$\Rightarrow \quad \frac{dy}{dx} = \frac{3}{1+x^2} - 0 = \frac{3}{1+x^2}$

OR

Let x cm be the radius and y be the enclosed area of the circular wave at any time t.

Rate of increase of radius of circular wave = 5 cm/sec

$\Rightarrow \frac{dx}{dt}$ is positive and = 5 cm/sec

$\Rightarrow \frac{dx}{dt} = 5cm/\sec$ ...(i)

$y = \pi x^2$

$\therefore$ Rate of change of area $= \frac{dy}{dt} = \pi \frac{d}{dt} x^2$

$= \pi.2x \frac{dx}{dt} = 2\pi x\,(5)$ (from (i)

$= 10\pi x cm^2/\sec$

Putting $x = 8cm$ (given),

$\frac{dy}{dt} = 10\pi\,(8) = 80\pi cm^2/\sec$

Since $\frac{dy}{dt}$ is positive, therefore area of circular wave is increasing at the rate of $80\pi cm^2/\sec$.

3. Let I = $\int \frac{x}{(x^2+4)\sqrt{x^2+9}} dx$

Assume $x^2 + 9 = u^2$ then xdx = udu

$\therefore \quad I = \int \frac{\text{udu}}{(\text{u}^2-5)\text{u}} = \int \frac{du}{(u^2-5)}$

Using identity $\int \frac{dz}{(z)^2-1} = \frac{1}{2}\log\left|\frac{z-1}{z+1}\right| + c$

$I = \frac{1}{2\sqrt{5}}\log \left|\frac{u-\sqrt{5}}{u+\sqrt{5}}\right| + c$

Substituting, u = $\sqrt{9+x^2}$, we get

$I = \frac{1}{2\sqrt{5}}\log \left|\frac{\sqrt{9+x^2}-\sqrt{5}}{\sqrt{9+x^2}+\sqrt{5}}\right| + c$

4. Note that the domain of f(x) is the set of all positive real numbers other than unity ie; $(0,1) \cup (1,\infty)$

Now $f(x) = \frac{x}{\log x}$

$\Rightarrow \quad f'(x) = \frac{\log x - 1}{(\log x)^2}$

For f(x) to be increasing function, we must have

$f'(x) > 0$

$\Rightarrow \quad \frac{\log x-1}{(\log x)^2} > 0$

$\Rightarrow \quad \log x - 1 > 0$

$\log x > 1$

$x > e^1$

So, f(x) is increasing on (e, ∞)

For f(x) to be decreasing we must have

$\Rightarrow \quad \frac{\log x-1}{(\log x)^2} < 0$

$\Rightarrow \quad \log x - 1 < 0$

$\Rightarrow \quad \log x < 1$

$\Rightarrow \quad x < e^1$

So f(x) is decreasing on $(0, e)$

5. Let I = $\int \frac{1}{x\sqrt{x^4-1}} dx$

Since $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c$

We have, $I = \int \frac{1}{x\sqrt{x^4-1}} dx$ .... (i)

Multiplying numerator and denominator with x

$I = \int \frac{x}{x^2\sqrt{(x^2)^2-1}} dx$

Let $x^2=t$

$\Rightarrow 2x = \frac{dt}{dx}$

$\Rightarrow$ xdx = $\frac{dt}{2}$

Putting this value in equation (i), we get

$I = \frac{1}{2}\int \frac{dt}{t\sqrt{t^2-1}} \left[x^2 = t\right]$

$\Rightarrow \text{I} = \frac{1}{2}\sec^{-1}\text{t} + \text{c}$

$\Rightarrow \text{I} = \frac{1}{2}\sec^{-1}\left(\text{x}^2\right) + \text{c}$

OR

Take tan x = a

Hence, $\sec^2 x \, dx = da$

$\therefore \int \frac{\tan x \sec^2 x}{1-\tan^2 x} dx$

$= \int \frac{\text{ada}}{1-a^2}$

Now, taking $1-a^2 = k$, -2a da = dk i.e. a da = -dk/2

$\therefore \int \frac{\text{ada}}{1-a^2}$

$= \int \frac{-\text{dk}}{2\text{k}}$

$= -\frac{1}{2}\ln|\text{k}| + \text{c}$

Replacing the value of k,

$-\frac{1}{2}\ln|\text{k}| + \text{c}$

$= -\frac{1}{2}\ln\left|1 - a^2\right| + c$

Replacing the value of a,

$-\frac{1}{2}\ln\left|1 - a^2\right| + c$

$= -\frac{1}{2}\ln\left|1 - \tan^2 x\right| + c$

Where c is the integrating constant

6. i. For (a, a), $a < a^3$ which is false. ∴ R is not reflexive.

ii. For (a, b), $a < b^3$ and (b, a), $b > a^3$ which is false. ∴R is not symmetric.

iii. For $a < b^2$ $b < c^3$. Now $b < c^3$ implies $b^3 < c^9$

Thus, we get $a < c^9$, therefore (a,c) does not belong to $\mathbb{R}$ and hence R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

7. Let $\tan^{-1}\frac{2}{3} = x$ and $\tan^{-1}\sqrt{3} = y$

so that $\tan x = \frac{2}{3}$ and $\tan y = \sqrt{3}$

Therefore,

$\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$

= sin(2x) + cos y

$= \frac{2\tan x}{1+\tan^2 x} + \frac{1}{\sqrt{1+\tan^2 y}}$

$= \frac{2.\frac{2}{3}}{1+\frac{4}{9}} + \frac{1}{\sqrt{1+(\sqrt{3})^2}}$

$= \frac{12}{13} + \frac{1}{2} = \frac{37}{26}$

8. $\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x+\cos^n x}dx$

Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x+\cos^n x}dx$ ..(i)

So, using the property of defnite integrals we have

$I = \int_0^{\frac{\pi}{2}} \frac{\sin^n\left(\frac{\pi}{2}-x\right)}{\sin^n\left(\frac{\pi}{2}-x\right)+\cos^n\left(\frac{\pi}{2}-x\right)}dx \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$= \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x+\cos^n x}$ ..(ii)

Adding (i) & (ii)

$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x+\cos^n x}dx + \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x+\cos^n x}$

$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n + \cos^n x}{\sin^n + \cos^n x}dx$

$2I = \int_0^{\frac{\pi}{2}} dx$

$2I = [x]_0^{\frac{\pi}{2}}$

$2I = \left[\frac{\pi}{2} - 0\right]$

$I = \frac{\pi}{4}$

9. Since f(x) is continuous at x = 0.

Therefore,

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = f(0)$

$\Rightarrow \quad \lim\limits_{h\to 0} f(0-h) = \lim\limits_{h\to 0} f(0+h) = f(0)$

$\Rightarrow \quad \lim\limits_{h\to 0} f(0+(-h)) = \lim\limits_{h\to 0} f(0+h) = f(0)$

$\Rightarrow \quad \lim\limits_{h\to 0}[f(0)+f(-h)] = \lim\limits_{h\to 0}[f(0)+f(h)] = f(0)$ ......[Using: f (x + y) = f(x) + f(y)]

$\Rightarrow \quad f(0) + \lim\limits_{h\to 0} f(-h) = f(0) + \lim\limits_{h\to 0} f(h) = f(0)$

$\Rightarrow \quad \lim\limits_{h\to 0} f(-h) = \lim\limits_{h\to 0} f(h) = 0$ ....(i)

Let 'a' be any real number.

Then,

$\lim\limits_{x\to a^-} f(x) = \lim\limits_{h\to 0} f(a-h) = \lim\limits_{h\to 0} f(a+(-h))$

$\Rightarrow \lim_{x \to a^-} f(x) = \lim_{h \to 0}[f(a) + f(-h)]$ .....[∵ f (x + y) = f(x) + f(y)]

$\Rightarrow \lim_{x \to a^-} f(x) = f(a) + \lim_{h \to 0} f(-h)$

$\Rightarrow \lim_{x \to a^-} f(x) = f(a) + 0$ [Using (i)]

$\Rightarrow \lim_{x \to a^-} f(x) = f(a)$

and, $\lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a + h)$

$\Rightarrow \lim_{x \to a^+} f(x) = \lim_{h \to 0}[f(a) + f(h)]$ ... [∵ f (x + y) = f(x) + f(y)]

$\Rightarrow \lim_{x \to a^+} f(x) = f(a) + \lim_{h \to 0} f(h)$

$\Rightarrow \lim_{x \to a^+} f(x) = f(a) + 0 = f(a)$ ...[Using (i)]

Thus, we have

$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

∴ f(x) is continuous at x = a

Since 'a' is an arbitrary real number.

So, f(x) is continuous at all x ∈ R

OR

Here,

$x = a. sin(2t).(1 + cos2t)$ *and* $y = b. cos(2t).(1 - cos2t)$

$\therefore \frac{dx}{dt} = a\left[\sin 2t \cdot \frac{d}{dt}(1 + \cos 2t) + (1 + \cos 2t) \cdot \frac{d}{dt}\sin 2t\right]$

$= a\left[\sin 2t.(-\sin 2t) \cdot \frac{d}{dt}2t + (1 + \cos 2t) \cdot \cos 2t \cdot \frac{d}{dt}2t\right]$

$= -2a. sin^2(2t) + 2a. cos(2t)(1 + cos2t)$

$\Rightarrow \frac{dx}{dt} = -2\alpha\left[\sin^2 2t - \cos 2t(1 + \cos 2t)\right]$ .... (i)

and $\frac{dy}{dt} = b\left[\cos 2t \cdot \frac{d}{dt}(1 - \cos 2t) + (1 - \cos 2t) \cdot \frac{d}{dt}\cos 2t\right]$

$= b\left[\cos 2t.(\sin 2t)\frac{d}{dt}2t + (1 - \cos 2t)(-\sin 2t) \cdot \frac{d}{dt}2t\right]$

$= b[2\sin 2t \cdot \cos 2t + 2(1 - \cos 2t)(-\sin 2t)]$

$= b[2\sin 2t \cdot \cos 2t + 2(1 - \cos 2t)(-\sin 2t)]$ ..... (ii)

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2b[-\sin 2t \cdot \cos 2t + (1 - \cos 2t)\sin 2t]}{-2a[\sin^2 2t - \cos 2t(1 + \cos 2t)]}$

$= \frac{b}{a} \cdot \frac{(0+1)}{(1-0)} \quad \ldots \left[\because \sin\frac{\pi}{2} = 1 \text{ and } \cos\frac{\pi}{2} = 0\right]$

$= \frac{b}{a}$

Hence proved.

10. **Read the text carefully and answer the questions:**

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(i) Let A represents obtaining a sum greater than 9 and B represents black die resulted in a 5.

n(S) = 36

n(A) = {(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)} = 6

n(B) = {(5, 1), (5, 2), (5, 3), (5, 4), (5, 5)} = 6

n(A ∩ B) = {(5, 5), (5, 6)} = 2

P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}$

(ii) Let A represents obtaining a sum 8 and B represents red die resulted in number less than 4.

n(S) = 36

n(A) = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2) = 5

n(B) = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)} = 18

n(A ∩ B) = {(5, 3), (6, 2)} = 2

P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9}$

(iii) Let A represents obtaining a sum 10 and B represents black die resulted in even number.

n(S) = 36

n(A) = {(4, 6), (6, 4), (5, 5)} = 3

n(B) = {(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} = 18

n(A ∩ B) = {(4, 6), (6, 4)} = 2

P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9}$

(iv) Let A represents getting doublet and B represents red die resulted in number greater than 4.

n(S) = 36

n(A) = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} = 6

n(B) = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6), (5, 5), (5, 6), (6, 5), (6, 6)} = 12

n(A ∩ B) = {(4, 4), (5, 5), (6, 6)} = 3

P(A/B) = $\frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}} = \frac{\frac{3}{36}}{\frac{12}{36}} = \frac{1}{4}$

OR

Read the text carefully and answer the questions:

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

i. 960 of the total applications were the folk genre.
ii. 192 of the folk applications were for the below 18 category.
iii. 104 of the classical applications were for the 18 and above category.

(i) According to given information, we construct the following table.

Given, total applications = 2000

	Folk Genre	**Classical Genre**
	960 (given)	2000 - 960 = 1040
Below 18	192 (given)	1040 - 104 = 936
18 or Above 18	960 - 192 = 768	104 (given)

Let E_1 = Event that application for folk genre

E_2 = Event that application for classical genre

A = Event that application for below 18

B = Event that application for 18 or above 18

∴ P(E_2) = $\frac{1040}{2000}$

and P(B ∩ E_2) = $\frac{104}{2000}$

Required Probability = $\frac{P(B \cap E_2)}{P(E_2)}$

= $\frac{\frac{104}{2000}}{\frac{1040}{200}} = \frac{1}{10}$

(ii) Required probability = $P\left(\frac{\text{folk}}{\text{below 18}}\right)$

= $P\left(\frac{E_1}{A}\right)$

= $\frac{P(E_1 \cap A)}{P(A)}$

Now, P(E_1 ∩ A) = $\frac{192}{2000}$

and P(A) = $\frac{192+936}{2000} = \frac{1128}{2000}$

$\therefore$ Required probability = $\frac{\frac{192}{2000}}{\frac{1128}{2000}} = \frac{192}{1128} = \frac{8}{47}$

(iii)Here,

P(A) = 0.4, P(B) = 0.8 and P(B|A) = 0.6

$\because$ P(B|A) = $\frac{P(B \cap A)}{P(A)}$

$\Rightarrow$ P(B $\cap$ A) = P(B|A).P(A)

= 0.6 $\times$ 0.4 = 0.24

$\because$ P(A $\cup$ B) = P(A) + P(B) - P(A $\cap$ B)

= 0.4 + 0.8 - 0.24

= 1.2 - 0.24 = 0.96

(iv)Since, A and B are independent events, A' and B' are also independent. Therefore,

P(A' $\cap$ B') = P(A') $\cdot$ P(B')

= (1 - P(A)(1 - P(B))

= $\left(1-\frac{3}{5}\right)\left(1-\frac{4}{9}\right)$

= $\frac{2}{5} \cdot \frac{5}{9}$

= $\frac{2}{9}$

11. **Read the text carefully and answer the questions:**

Three shopkeepers A, B and C go to a store to buy stationery. A purchase 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs ₹40, a pen costs ₹12 and a pencil costs ₹3.

(i) Number of items purchased by shopkeepers A, B and C can be written in matrix form as

$$X = \begin{matrix} & \text{Notebooks} & \text{pens} & \text{pencils} & \\ & \left[\begin{matrix}144 \\ 120 \\ 132\end{matrix}\right. & \begin{matrix}60 \\ 72 \\ 156\end{matrix} & \left.\begin{matrix}72 \\ 84 \\ 96\end{matrix}\right] & \begin{matrix}A \\ B \\ C\end{matrix}\end{matrix}$$

(ii)

Since, Y = $\begin{bmatrix}40\\12\\3\end{bmatrix}\begin{matrix}Note\ book\\Pen\\Pencil\end{matrix}$

$\therefore$ XY = $\begin{bmatrix}144 & 60 & 72\\120 & 72 & 84\\132 & 156 & 96\end{bmatrix}\begin{bmatrix}40\\12\\3\end{bmatrix}$

= $\begin{bmatrix}5760+720+216\\4800+864+252\\5280+1872+288\end{bmatrix} = \begin{bmatrix}6696\\5916\\7440\end{bmatrix}$

(iii)$(A + I)^2 = A^2 + 2A + I = 3A + I$

$\Rightarrow (A + I)^3 = (3A + I)(A + I)$

$= 3A^2 + 4A + I = 7A + I$

$\therefore (A + I)^3 - 7A = I$

12. $(x^2 - y^2)dx + 2xy\,dy = 0$

$\Rightarrow (x^2 - y^2)dx = -2xy\,dy$

$\Rightarrow \frac{dy}{dx} = \frac{y^2-x^2}{2xy}$

$\Rightarrow \frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2-1}{2\left(\frac{y}{x}\right)}$ (1)

Put y = vx, then, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Put $\frac{dy}{dx}$ in eq (i),we get,

$v + x\frac{dv}{dx} = \frac{v^2-1}{2v}$
$\Rightarrow x\frac{dv}{dx} = \frac{v^2-1}{2v} - v$
$\Rightarrow x\frac{dv}{dx} = \frac{v^2-1-2v^2}{2v}$
$\Rightarrow x\frac{dv}{dx} = \frac{-1-v^2}{2v}$
$\Rightarrow \int \frac{2vdv}{v^2+1} = \int \frac{-dx}{x}$
$\Rightarrow \log(v^2+1) = -\log x + c$
$\Rightarrow \log((v^2+1).x) = c$
$\Rightarrow (v^2+1).x = e^c$
$\Rightarrow \left(\frac{y^2}{x^2}+1\right).x = e^c \ \left[v = \frac{y}{x}\right]$
$\Rightarrow \frac{x^2+y^2}{x} = A \ [\because e^c = A]$
$\Rightarrow x^2+y^2 = Ax$

OR

We have $\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$
Let $f(x,y) = \frac{x^2+y^2}{x^2+xy}$
Here, putting x = kx and y = ky
$f(kx,ky) = \frac{(kx)^2+(ky)^2}{(kx)^2+kx.ky}$ = k^0.f(x,y)
Therefore, the given differential equation is homogeneous.
$(x^2 + xy)dy = (x^2 + y^2)dx$
$\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$
To solve it we make the substitution.
y = vx
Differentiating above eq. with respect to x, we get
$\frac{dy}{dx} = v + x\frac{dv}{dx}$
$v + x\frac{dv}{dx} = \frac{x^2+(vx)^2}{x^2+x\cdot vx}$
$v + x\frac{dv}{dx} = \frac{x^2(1+v^2)}{x^2(1+v)}$
$v + x\frac{dv}{dx} = \frac{1+v^2}{1+v}$
$\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}} = \frac{1+\mathrm{v}^2}{1+\mathrm{v}} - \mathrm{v} = \frac{1+\mathrm{v}^2-\mathrm{v}-\mathrm{v}^2}{1+\mathrm{v}}$
$x\frac{dv}{dx} = \frac{1-v}{1+v}$
$\frac{1+v}{1-v}dv = \frac{1}{x}dx$
Integrating on both side,
$\int \frac{1+v}{1-v}dv = \int \frac{1}{x}dx$
$\int \left(-1+\frac{2}{1-v}\right)dv = \int \frac{1}{x}dx$
- v - 2log|1 - v| = log|x| + log c
$-\frac{y}{x} - 2\log\left|1-\frac{y}{x}\right| = \log|x| + \log C$
$-\frac{y}{x} = 2\log\left|1-\frac{y}{x}\right| + \log|x| + \log C$
$-\frac{y}{x} = \log\frac{(x-y)^2}{x^2} + \log|x| + \log C$
$-\frac{y}{x} = \log\frac{(x-y)^2}{x^2}\cdot Cx$
$-\frac{y}{x} = \log\frac{(x-y)^2}{x}c$
$-\frac{y}{x} = \log\frac{(x-y)^2}{x}c$
$\frac{\mathrm{C(x-y)^2}}{\mathrm{x}} = \mathrm{e}^{-\mathrm{y/x}}$
$\mathrm{C(x-y)^2} = \mathrm{xe}^{-\mathrm{y/x}}$
$(x-y)^2 = kxe^{-y/x}$
Which is the required solution of the given differential equation.

13. Given:

Curves $y^2 = 8x$ .... (i)

& $2x^2 + y^2 = 10$ ... (ii)

The point of intersection of two curves are (0, 0) & $(1, 2\sqrt{2})$

Now, Differentiating curves (i) & (ii) w.r.t. x, we get

$\Rightarrow y^2 = 8x$

$\Rightarrow 2y \cdot \frac{dy}{dx} = 8$

$\Rightarrow \frac{dy}{dx} = \frac{8}{2y}$

$\Rightarrow \frac{dy}{dx} = \frac{4}{y}$ ... (iii)

$\Rightarrow 2x^2 + y^2 = 10$

Differentiating above w.r.t. x,

$\Rightarrow 4x + 2y \cdot \frac{dy}{dx} = 0$

$\Rightarrow 2x + y \cdot \frac{dy}{dx} = 0$

$\Rightarrow y \cdot \frac{dy}{dx} = -2x$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{y}$ ... (iv)

Substituting $(1, 2\sqrt{2})$ for m_1 & m_2, we get,

$m_1 = \frac{4}{y}$

$\Rightarrow \frac{4}{2\sqrt{2}}$

$m_1 = \sqrt{2}$ ... (v)

$m_2 = \frac{-2x}{y}$

$\Rightarrow \frac{-2\times 1}{2\sqrt{2}}$

$m_2 = -\frac{-1}{\sqrt{2}}$ ... (vi)

When $m_1 = \sqrt{2}$ & $m_2 = \frac{-1}{\sqrt{2}}$

Two curves intersect orthogonally if $m_1 m_2 = -1$, where m_1 and m_2 the slopes of the two curves.

$\Rightarrow \sqrt{2} \times \frac{-1}{\sqrt{2}} = -1$

$\therefore$ Two curves $y^2 = 8x$ & $2x^2 + y^2 = 10$ intersect orthogonally.

OR

Let ABC be right-circular cone having radius 'r' and height 'h'. If V and S are its volume and surface area (curved) respectively, then

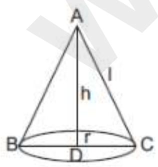
$S = \pi r l$

$S = \pi r\sqrt{h^2 + r^2}$ ..... (i)

Putting the value of h in (i), we get

$S = \pi r\sqrt{\frac{9V^2}{\pi^2 r^4} + r^2}$

$\Rightarrow \quad S^2 = \pi^2 r^2 \left(\frac{9V^2 + \pi^2 r^6}{\pi^2 r^4}\right)$



$\left[\begin{array}{l} \because V = \frac{1}{3}\pi r^2 h \\ \quad h = \frac{3V}{\pi r^2} \end{array}\right.$

[Maxima or Minima is same for S or S^2]

$\Rightarrow \quad S^2 = \frac{9V^2}{r^2} + \pi^2 r^4$

$\Rightarrow \quad (S^2)' = \frac{-18V^2}{r^3} + 4\pi^2 r^3$ ..... (ii) [Differentiating w.r.t. 'r']

Now, $(S^2)' = 0$

$\Rightarrow \quad -18\frac{V^2}{r^3} + 4\pi^2 r^3 = 0$

$\Rightarrow \quad 4\pi^2 r^6 = 18V^2$

$\Rightarrow \quad 4\pi^2 r^6 = 18 \times \frac{1}{9}\pi^2 r^4 h^2$ [Putting value of V]

$\Rightarrow \quad 2r^2 = h^2 \quad \Rightarrow \quad r = \frac{h}{\sqrt{2}}$

Differentiating (ii) w.r.t. 'r', again

$(S^2)'' = \frac{54V^2}{r^4} + 12\pi^2 r^2$

$\Rightarrow \quad (S^2)''\Big]_{r=\frac{h}{\sqrt{2}}} > 0$ (for any value of r)

Hence, S^2 i.e., is minimum for $r = \frac{h}{\sqrt{2}}$ or $h = \sqrt{2}r$.

i.e., For least curved surface, altitude is equal to $\sqrt{2}$ times the radius of the base.

14. **Read the text carefully and answer the questions:**

In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.

(i) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06, P(\frac{A}{E_2}) = 0.04, P(\frac{A}{E_3}) = 0.03$

Using Bayes' theorem, we have

$$P(\frac{E_1}{A}) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)+P(E_3)\cdot P\left(\frac{A}{E_3}\right)}$$

$$= \frac{0.5\times0.06}{0.5\times0.06+0.2\times0.04+0.3\times0.03} = \frac{30}{47}$$

$\therefore$ Required probability = $P\left(\frac{\bar{E}_1}{A}\right)$

$= 1 - P\left(\frac{E_1}{A}\right) = 1 - \frac{30}{47} = \frac{17}{47}$

(ii) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06, P(\frac{A}{E_2}) = 0.04, P(\frac{A}{E_3}) = 0.03$

$P(A \cap E_2) = P\left(\frac{A}{E_2}\right).P(E_2)$

$\Rightarrow 0.04 \times 0.2 = 0.008$

(iii) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06, P(\frac{A}{E_2}) = 0.04, P(\frac{A}{E_3}) = 0.03$

$P(A) = P\left(\frac{A}{E_1}\right).P(E_1) + P\left(\frac{A}{E_2}\right).P(E_2) + P\left(\frac{A}{E_3}\right).P(E_3)$

$= 0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03 = 0.047$

(iv) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5, P(E_2) = 0.2, P(E_3) = 0.3$

$P(\frac{A}{E_1})$ = 0.06, $P(\frac{A}{E_2})$ = 0.04, $P(\frac{A}{E_3})$ = 0.03

$\sum_{i=1}^{3} P\left(\frac{E_i}{A}\right) = P\left(\frac{E_1}{A}\right) + P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right)$

= 1 [∵ Sum of posterior probabilities is 1]

SECTION B - 15 MARKS

15. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) **(a)** 60^o

Explanation: {

$\vec{a} \cdot \vec{b} = \frac{1}{2}|\vec{a}||\vec{b}|$

$|\vec{a}|\ |\vec{b}|\cos\theta = \frac{1}{2}|\vec{a}||\vec{b}|$

$\cos\theta = \frac{1}{2}$

$\theta = 60^o$

(ii) The direction ratios of the first line are (3, 5, 4) and the direction ratios of the second line are (1, 1, 2).

If θ is the angle between them, then

$\cos\theta = \left|\frac{3.1+5.1+4.2}{\sqrt{3^2+5^2+4^2}\sqrt{1^2+1^2+2^2}}\right| = \frac{16}{\sqrt{50}\sqrt{6}} = \frac{16}{5\sqrt{2}\sqrt{6}} = \frac{8\sqrt{3}}{15}$

Hence, the required angle is $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

(iii) Given: Projection of the vector on the coordinate axes are 12, 3, 4 units.

Therefore, Length of the vector

$= \sqrt{12^2 + 3^2 + 4^2}$

$= \sqrt{169}$ = 13

(iv) **(c)** 90^o

Explanation: {

To find the angle with the z-axis, we use the relation:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

where $\alpha = 30^\circ$ and $\beta = 120^\circ$. Calculating, we get:

$\cos^2 30^\circ = \frac{3}{4}, \cos^2 120^\circ = \frac{1}{4}$

Thus,

$\frac{3}{4} + \frac{1}{4} + \cos^2\gamma = 1 \Longrightarrow \cos^2\gamma = 0 \Longrightarrow \gamma = 90^\circ.$

So, the angle with the z-axis is 90^o.

(v) We are given vector equation of plane which we will simplify to find the required intercepts and their sum.

$Now, \vec{r}.(2\hat{i} + \hat{j} - \hat{k}) = 5$

Put $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k},$ we get

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} + \hat{j} - \hat{k}) = 5$

$\Rightarrow \quad 2x + y - z = 5 \Rightarrow \frac{2x}{5} + \frac{y}{5} + \frac{z}{-5} = 1$

On comparing it with standard equation of plane in intercept form

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, we get

$a = \frac{5}{2}, b = 5$ and c= -5

Now, required sum of intercepts cut off by the plane on the three axes =a + b + c

$= \frac{5}{2} + 5 - 5 = \frac{5}{2}$

16. We have, $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$,

$\vec{a} + \vec{b}$ = 1 + 2 $\hat{j} - 3\hat{k} + 3\hat{i} - \hat{j} + 2\hat{k}$

$\Rightarrow \vec{a} + \vec{b}$= 4 $\hat{i} + \hat{j} - \hat{k}$

$\vec{a} - \vec{b}$ = $\hat{i}$ + 2 $\hat{j} - 3\hat{k} - (3\hat{i} - \hat{j} + 2\hat{k})$

$\Rightarrow \vec{a} - \vec{b}$= -2$\hat{i} + 3\hat{j} - 5\hat{k}$

Now $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$ = $(4\hat{i} + \hat{j} - \hat{k}) \cdot (-2\hat{i} + 3\hat{j} - 5\hat{k})$

= (4 × -2) + (1 × 3) + (-1 × -5) = -8 + 3 + 5 = 0

Here,we see that the dot product of two vectors is zero so the vector $(\vec{a}+\vec{b})$ is perpendicular to $(\vec{a}-\vec{b})$

Hence,proved.

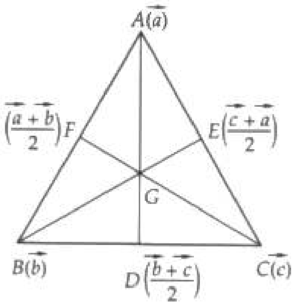
OR

Let ABC be a triangle and let D, E, F be the mid-points of its sides BC, CA and AB respectively. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of A, B and C resectively. Then, the position vectors of D, E and F are $\frac{\vec{b}+\vec{c}}{2}, \frac{\vec{c}+\vec{a}}{2}$ and $\frac{\vec{a}+\vec{b}}{2}$ respectively.

The position vector of a point dividing AD in the ratio 2:1 is

$$\frac{1 \cdot \vec{a}+2\left(\frac{\vec{b}+\vec{c}}{2}\right)}{1+2}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

Similarly, position vectors of points dividing BE and CF in the ratio 2: 1 are each equal to $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$.



Thus, the point dividing AD in the ratio 2: 1 also divides BE and CF in the same ratio. Hence, the median of a triangle are concurrent and the position vector of the centroid is $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

17. Equation of one plane is $\vec{r}.\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=5 \ldots$(i)

Comparing this equation with $\vec{r}.\vec{n}_1=d_1$, we have

Normal vector to plane (i) is $\vec{n}_1=2\hat{i}+2\hat{j}-3\hat{k}$

Again, equation of second plane is $\vec{r}.\left(3\hat{i}-3\hat{j}+5\hat{k}\right)=3 \ldots$(ii)

Comparing this equation with $\vec{r}.\vec{n}_2=d_2$, we have

Normal vector to plane (i) is $\vec{n}_2=3\hat{i}-3\hat{j}+5\hat{k}$

Let θ be the acute angle between planes (i) and (ii).

$\therefore$ angle between normals $\vec{n}_1$ and $\vec{n}_2$ to planes (i) and (ii) is also θ

$\therefore \cos\theta=\frac{|\vec{n}_1.\vec{n}_2|}{|\vec{n}_1|.|\vec{n}_2|} = \frac{|2(3)+2(-3)+(-3)5|}{\sqrt{4+4+9}\sqrt{9+9+25}}$

$=\frac{|6-6-15|}{\sqrt{17}\sqrt{13}}$

$=\frac{|-15|}{\sqrt{731}}=\frac{15}{\sqrt{731}}$

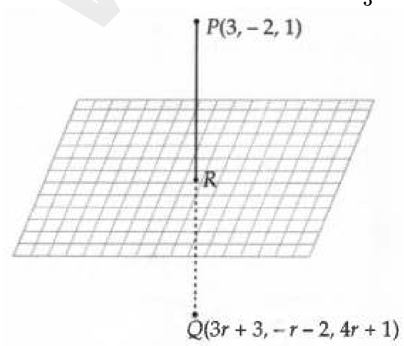
$\Rightarrow \theta=\cos^{-1}\frac{15}{\sqrt{731}}$

OR

Let Q be the image of the point P (3, - 2 ,1) in the plane 3x - y + 4z = 2.

Then, PQ is normal to the plane. Therefore, direction ratios of PQ are proportional to 3, - 1, 4. Since PQ passes through P (3, - 2,1) and has direction ratios proportional to 3, -1,4.

Therefore, the equation of PQ is $\frac{x-3}{3}=\frac{y+2}{-1}=\frac{z-1}{4}=r$



Let the coordinates of Q be (3 r+3,-r-2,4 r+1). Let R be the mid-point of PQ .

Then, R lies on the plane 3x - y + 4z = 2. The coordinates of R are

$\left(\frac{3r+3+3}{2}, \frac{-r-2-2}{2}, \frac{4r+1+1}{2}\right)$ or, $\left(\frac{3r+6}{2}, \frac{-r-4}{2}, 2r+1\right)$

Since R lies on the plane 3x - y + 4z = 2

$3\left(\frac{3r+6}{2}\right)-\left(\frac{-r-4}{2}\right)+4(2r+1)=2 \Rightarrow 13r=-13 \Rightarrow r=-1$

Putting r = -1 in (3r + 3, -r -2, 4r + 1), we obtain the coordinates of Q as (0, -1, -3)

Hence, the image of (3, -2, 1) in the plane 3x - y + 4z = 2 is (0, -1, -3).

18. To find area of region

$\{(x, y): 0 \leq y \leq x^2+3; 0 \leq \ y \leq 2x+3; 0 \leq x \leq 3\}$

$\Rightarrow$ y = x^2 + 3 ...(i)

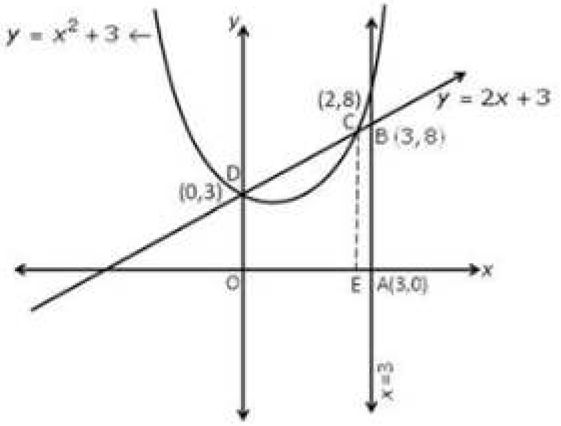
y = 2x +3 ...(ii)

and x = 0, x = 3

Equation (i) represents a parabola with vertex (3, 0) and axis as y-axis.

Equation (ii) represents a line a passing through (0, 3) and $\left(-\frac{3}{2}, 0\right)$.

A rough sketch of curve is as under:-



Thus Required area of the region= Area of the bounded Region ABCDOA

A = Region ABCEA + Region ECDOE

$=\int_2^3 y_1 dx+\int_0^2 y_z dx$

$=\int_2^3(2x+3) dx+\int_0^2\left(x^2+3\right) dx$

$=\left(x^2+3x\right)_2^3+\left(\frac{x^3}{3}+x\right)_0^2$

$=[(9+9)-(4+6)]+\left[\left(\frac{8}{3}+2\right)-(0)\right]$

$=[18-10]+\left[\frac{14}{3}\right]$

$=8+\frac{14}{3}$

$A=\frac{38}{3}$ sq. units.

SECTION C - 15 MARKS

19. In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) **(d)** 1229

Explanation: {

Given R(x) = $120x^2$ + 300 - x

Therefore, AR(x) = $\frac{R(x)}{x}=\frac{120x^2+300-x}{x}$

AR(10) = $\frac{120(10)^2+3(0)-(10)}{10}$

= $\frac{12000+300-10}{10}$

At x = 10, = $\frac{12290}{10}$ = 1229

(ii) **(c)** linear function

Explanation: {

linear function

(iii) Given $\bar{x}$ = 18, $\bar{y}$ = 100, σ_x = 14, σ_y = 20, r = 0.8

$b_{xy}=r \cdot \frac{\sigma_y}{\sigma_x}=0.8 \times \frac{20}{14}=\frac{8}{7}$

Regression equation y on x

y - $\bar{y}$ = $b_{yx} \cdot (x - \bar{x})$
y - 100 = $\frac{8}{7}$(x - 18)
7y - 700 = 8x - 144
8x - 7y + 556 = 0

(iv) Let x be the number of items produced and sold. Let C(x) be the total cost incurred in producing x items. Then,
C(x) = Fixed cost + Variable cost = 26,000 + 30x
Let R(x) be the total revenue received in selling x items. Then, R(x) = 43x
At the break-even point, we have C(x) = R(x)
$\Rightarrow$ 26,000 + 30x = 43x
$\Rightarrow$ 13x = 26,000
$\Rightarrow$ x = 2000
Hence, the break-even point is 2000 items.

(v) Given p = 1500 - 2x - x^2
$\Rightarrow$ R(x) = px = 1500x - $2x^2$ - x^3
So, MR = $\frac{d}{dx}$(R(x)) = 1500 - 4x - $3x^2$
$[MR]_{x=10}$ = 1500 - 4 $\times$ 10 - 3 $\times$ $(10)^2$ = 1160

20. = 136 - $\frac{40}{100}(x - 100)$, for $\geq$ 100
= 136 - $\frac{4x}{10} + \frac{4}{10} \times 100$
= 136 - $\frac{4x}{10}$ + 40
= 176 - $\frac{2x}{5}$
Amount of money, A = (Number of passengers) $\times$ (Tour operator charges)
A = $x\left(176 - \frac{2x}{5}\right)$
A = $176x - \frac{2x^2}{5}$
$\frac{dA}{dx} = 176 - \frac{4x}{5}$
When $\frac{dA}{dx}$ = 0 we get,
176 - $\frac{4x}{5}$
4x = 176 $\times$ 5
x = $\frac{176 \times 5}{4}$ = 220
$\frac{d^2A}{dx^2}$ = $-\frac{4}{5}$, negative
$\therefore$ The amount of money Is maxImum when the number of passengers is 220.

OR

Let C denote the cost and x the output. It is given that the cost C is a linear function of output x.
$\therefore$ C = ax + b, where a, b are constants.
When x = 250, we have C = 4000
$\therefore$ 4000 = 250 a + b ...(i)
When x = 350, we have C = 5000
$\therefore$ 5000 = 350 a + b ... (ii)
Solving (i) and (ii), we get a = 10 and b = 1500.
Substituting these values in C = ax + b, we get C = 10x + 1500

21. i. Given lines of regression are
4x + 3y + 7 = 0 ...(i)
and 3x + 4y + 8 = 0 ...(ii)
Solve (i) and (ii) simultaneously, we get
x = -$\frac{4}{7}$ and y = $\frac{11}{7}$
Point of intersection of the two lines gives the required mean:
$\bar{x}$ = -$\frac{4}{7}$, $\bar{y}$ = $\frac{11}{7}$
Point of intersection of the two lines gives the required mean:
$\bar{x} = -\frac{4}{7}, \bar{y} = -\frac{11}{7}$

ii. Let equation (i) be the line of regression of y on x and (ii) be the line of regression of x on y.
4x + 3y + 7 = 0

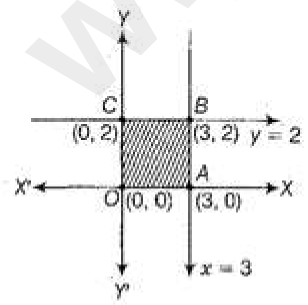
$\Rightarrow 3y = -4x - 7$
$\Rightarrow y = -\frac{4}{3}x - \frac{7}{3}$
$\Rightarrow b_{yx} = -\frac{4}{3}$
From (ii),
$3x + 4y + 8 = 0$
$3x = -4y - 8$
$\Rightarrow x = -\frac{4}{3}y - \frac{8}{3}$
$\Rightarrow b_{yx} = -\frac{4}{3}$
but $r^2 = b_{yx} \cdot b_{xy}$
$= \left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right) > 1$
which is not possible.
$\therefore$ Eq. (ii) is the line of regression of y on x and eq. (i) is the line of regression of x on y.
$3x + 4y + 8 = 0$
$\Rightarrow 4y = -3x - 8$
$\Rightarrow y = -\frac{3}{4}x - \frac{8}{4}$
$\Rightarrow b_{yx} = -\frac{3}{4}$
and $4x + 3y + 7 = 0$
$\Rightarrow 4x = -3y - 7$
$\Rightarrow x = -\frac{3}{4}y - \frac{7}{4}$
$\Rightarrow b_{xy} = -\frac{3}{4}$

iii. Coefficient of correlation,
$|r| = \sqrt{b_{yx} \times b_{xy}} = \sqrt{\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)}$
$r = \pm\frac{3}{4}$
$\therefore r = -\frac{3}{4}$, as r, b_{xy} and b_{yx} all have same signs.

22. Let the man rides to his motorcycle to a distance x km at the speed of 50 km/h and to a distance y km at the speed of 80 km/h. Therefore, cost on petrol is 2x + 3y.
Since, he has to spend Rs 120 at most on petrol.
$\therefore 2x + 3y \leqslant 120$ .... (i)
Also, he has at most 1 hour time.
$\therefore \frac{x}{50} + \frac{y}{80} \leqslant 1$
$\Rightarrow 8x + 5y \leqslant 400$ ..... (ii)
Also, we have $x \geqslant 0, y \geqslant 0$ [non-negative constraints]
Thus, required LPP to travel maximum distance by him is
Maximise Z = x + y, subject to $2x + 3y \leqslant 120, 8x + 5y \leqslant 400, x \geqslant 0, y \geqslant 0$

OR

Maximise Z = 11x + 7y, subject to the constraints $x \leqslant 3, y \leqslant 2, x \geqslant 0, y \geqslant 0$.



The shaded region as shown in the figure as OABC is bounded and the coordinates of corner points are (0, 0), (3, 0), (3, 2), and (0, 2), respectively.

Corner Points	Corresponding value of Z
(0, 0)	0
(3, 0)	33

(3, 2)	47 (Maximum)
(0, 2)	14

Hence, Z is maximise at (3, 2) and its maximum value is 47.