

Derivatives of Trigonometric Functions

Q.1. If $y = \sqrt{[(1 - \cos x) / (1 + \cos x)]}$, find dy / dx .

Solution : 1

We are given : $y = \sqrt{[(1 - \cos x) / (1 + \cos x)]}$

Or, $y = \sqrt{[(1 - \cos x)(1 + \cos x) / (1 - \cos^2 x)]}$

Or, $y = (1 - \cos x) / \sin x$

Or, $y = \operatorname{cosec} x - \cos x$

Or, $dy / dx = -\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x$

$= -\operatorname{cosec} x [\cot x - \operatorname{cosec} x]$.

Q.2. If $\sin y = x \sin (a + y)$, show that $dy / dx = \sin^2 (a + y) / \sin a$.

Solution : 2

We are given : $\sin y = x \sin (a + y)$

Or, $x = \sin y / \sin (a + y)$

Differentiating both sides with respect to y we get

$dx / dy = [\sin (a + y) \cdot \cos y - \sin y \cos (a + y)] / \sin^2 (a + y)$

$= \sin (a + y - y) / \sin^2 (a + y)$

$= \sin a / \sin^2 (a + y)$

Hence, $dy / dx = \sin^2 (a + y) / \sin a$. **[Proved.]**

Q.3. $\sin x^2$ If $y = e$, find dy / dx .

Solution : 3

We are given, $\sin x^2$.

$$y = e \sin x^2$$

$$dy / dx = e d / dx (\sin x^2)$$

$$\sin x^2 = e \times \cos x^2 \times d / dx (x^2) \sin x^2$$

$$= 2x \cos x^2 e$$

Q.4. If $\sin (xy) + \cos (xy) = 1$ and $\tan (xy) \neq 1$, then show that $dy / dx = - y / x$.

Solution : 4

We are given,

$$\sin (xy) + \cos (xy) = 1,$$

Differentiating both sides with respect to x we get,

$$\cos (xy). d/dx (xy) - \sin (xy). d/dx (xy) = 0$$

$$\text{Or, } d/dx (xy) (\cos xy - \sin xy) = 0$$

$$\text{Or, } (x dy/dx + y) (\sin xy - \cos xy) = 0$$

$$\text{Either } \sin xy - \cos xy = 0 \text{ Or, } x dy/dx + y = 0$$

We get $\sin xy - \cos xy = 0 \Rightarrow \sin xy = \cos xy \Rightarrow \tan xy = 1$ but $\tan xy \neq 1$.

Hence, $x dy/dx + y = 0 \Rightarrow x dy/dx = - y \Rightarrow dy/dx = - y/x$. **[Proved.]**

Q.5. If $x = a \sin^3 t$ and $y = a \cos^3 t$, find dy/dx .

Solution : 5

We have, $x = a \sin^3 t \Rightarrow dx / dt = 3a \sin^2 t. \cos t$ --- (i)

Also $y = a \cos^3 t \Rightarrow dy / dt = 3a \cos^2 t. (- \sin t)$ --- (ii)

Dividing (ii) by (i) we get, $(dy/dt)/(dx/dt) = - 3a \cos^2 t. \sin t / 3a \sin^2 t. \cos t$

Or, $dy / dx = -\cot t$.

Q.6. If $y = \sin \sqrt{(\sin x + \cos x)}$, find dy / dx .

Solution : 6

Using chain rule,

$$\begin{aligned} dy / dx &= \cos \sqrt{(\sin x + \cos x)} \cdot d/dx [\sqrt{(\sin x + \cos x)}] \\ &= \cos \sqrt{(\sin x + \cos x)} \cdot \frac{1}{2\sqrt{(\sin x + \cos x)}} \cdot d/dx (\sin x + \cos x) \\ &= \left[\frac{\cos \sqrt{(\sin x + \cos x)}}{2\sqrt{(\sin x + \cos x)}} \right] \cdot (\cos x - \sin x) \\ &= \left[\cos \sqrt{(\sin x + \cos x)} \cdot (\cos x - \sin x) \right] / [2\sqrt{(\sin x + \cos x)}] \end{aligned}$$

Q.7. If $y = \sqrt{\{(1 - \tan x)/(1 + \tan x)\}}$, find dy/dx .

Solution : 7

Using chain rule,

$$\begin{aligned} dy / dx &= \frac{1}{2\sqrt{\{(1 - \tan x)/(1 + \tan x)\}}} \cdot d/dx \left\{ \frac{1 - \tan x}{1 + \tan x} \right\} \\ &= \frac{1}{2\sqrt{\{(1 - \tan x)/(1 + \tan x)\}}} \cdot \left[(1 + \tan x) \cdot d/dx (1 - \tan x) - (1 - \tan x) \times d/dx (1 + \tan x) \right] \\ &= \frac{1}{2\sqrt{\{(1 - \tan x)/(1 + \tan x)\}}} \cdot \left[\{(1 + \tan x)(-\sec^2 x) - (1 - \tan x)(\sec^2 x)\} \right] / [(1 + \tan x)^2] \\ &= -\sec^2 x / \{(1 + \tan x)^2\} \cdot \sqrt{(1 + \tan x)}/\sqrt{(1 - \tan x)} \\ &= 0 - \sec^2 x / [(1 + \tan x)^{3/2}(1 - \tan x)^{1/2}] \end{aligned}$$

Q.8. If $y = \sin [\sqrt{\{\sin \sqrt{x}\}}]$, find dy / dx .

Solution : 8

Using chain rule,

$$dy / dx = \cos [\sqrt{\{\sin \sqrt{x}\}}] \cdot d/dx [\sqrt{\{\sin \sqrt{x}\}}]$$

$$\begin{aligned}
&= \cos [\sqrt{\sin \sqrt{x}}] \cdot \frac{1}{[2 \sin \sqrt{x}]} \cdot \frac{d}{dx} [\sin \sqrt{x}] \\
&= \cos [\sqrt{\sin \sqrt{x}}] \cdot \frac{1}{[2 \sin \sqrt{x}]} \cdot \cos \sqrt{x} \cdot \frac{d}{dx} (\sqrt{x}) \\
&= \cos [\sqrt{\sin \sqrt{x}}] \cdot \frac{1}{[2 \sin \sqrt{x}]} \cdot \cos \sqrt{x} \cdot \frac{1}{2\sqrt{x}} \\
&= (\cos [\sqrt{\sin \sqrt{x}}] \cdot \cot \sqrt{x}) / 4 \sqrt{x} .
\end{aligned}$$

Q.9. If $y = \sqrt{[(1 - \sin 2x)/(1 + \sin 2x)]}$, show that $dy/dx + \sec 2(\pi/4 - x) = 0$.

Solution : 9

We have , $y = \sqrt{[(1 - \sin 2x)/(1 + \sin 2x)]}$

$$= \sqrt{[(\cos x - \sin x)^2/(\cos x + \sin x)^2]}$$

$$= (\cos x - \sin x)/(\cos x + \sin x)$$

$$= (1 - \tan x)/(1 + \tan x) = \tan (\pi/4 - x)$$

Differentiating w. r. t. x , we get

$$dy/dx = \sec 2(\pi/4 - x) \cdot (-1)$$

$$\text{Or, } dy/dx + \sec 2(\pi/4 - x) = 0 \text{ . } \mathbf{[Proved.]}$$

Q.10. Find dy/dx , when : $x = a(1 - \cos \theta)$; $y = a(\theta + \sin \theta)$.

Solution : 10

We have , $x = a(1 - \cos \theta)$; $y = a(\theta + \sin \theta)$

Then $dx/d\theta = a \sin \theta$; $dy/d\theta = a(1 + \cos \theta)$

Therefore , $dy/dx = (dy/d\theta)/(dx/d\theta) = a(1 + \cos \theta)/a \sin \theta$

$$= \{2 \cos 2(\theta/2)\}/[2 \sin (\theta/2) \cos (\theta/2)]$$

$$= \cot (\theta/2) .$$

Q.11. Find dy/dx , when $y = a(\theta + \sin \theta)$; $x = a(1 + \cos \theta)$.

Solution : 11

We have , $y = a (\theta + \sin \theta)$; $x = a (1 + \cos \theta)$

Then $dy/d\theta = a (1 + \cos \theta) = 2 a \cos^2 (\theta/2)$;

$dx/d\theta = - a \sin \theta = - 2a \sin (\theta/2) \cos (\theta/2)$.

Therefore , $dy/dx = (dy/d\theta)/(dx/d\theta) = - \cot \theta/2$.