

Ellipse

Q.1. Find the equation of the normal to the ellipse $5x^2 + 3y^2 = 137$ at the point where the ordinate is 2.

Solution : 1

Equation of the ellipse is :

$$5x^2 + 3y^2 = 137 \text{ ----- (1)}$$

$$\text{If } y = 2, 5x^2 + 3(2)^2 = 137$$

$$\text{Or, } 5x^2 = 125 \Rightarrow x^2 = 25$$

$$\text{Or, } x = \pm 5.$$

Differentiating (1) we get $dy/dx = 10x/6y = 50/12 = 25/6$ [at (5,2)] = m .

Equation of normal at (5,2) is , $y - 2 = 6/25 (x - 5)$

$$\text{Or, } 6x - 25y + 20 = 0$$

And equation at (- 5 ,2) is , $6x + 25y - 20 = 0$

Q.2. Find the equation of the ellipse whose minor axis is 4 and which has a distance of 6 units between its foci.

Solution : 2

We have, $2b = 4 \Rightarrow b = 2$. $FF' = 6$.

$$2ae = 6 \Rightarrow ae = 3 \text{ ----- (i)}$$

$$\text{and } b = a\sqrt{1 - e^2} = 2 \text{ ----- (ii)}$$

dividing (i) by (ii) we get

$$ae/a\sqrt{1 - e^2} = 3/2$$

$$\text{Or, } 4e^2 = 9(1 - e^2)$$

$$\text{Or, } 13e^2 = 9 \Rightarrow e = 3/\sqrt{13}$$

$$\text{From (ii) } a\sqrt{1 - 9/13} = 2$$

$$\text{Or, } 2a/\sqrt{13} = 2 \Rightarrow a = \sqrt{13}.$$

Therefore, equation of the ellipse is $x^2/a^2 + y^2/b^2 = 1$

$$\text{Or, } x^2/13 + y^2/4 = 1.$$

Q.3. Find the equation of the ellipse whose eccentricity is $1/2$ and whose foci are the points $(\pm 2, 0)$.

Solution : 3

We have, $e = 1/2$, and focus is $(\pm 2, 0)$.

$$\text{Thus, } ae = \pm 2$$

$$\text{Or, } a \times 1/2 = \pm 2 \Rightarrow a = \pm 4.$$

$$\text{Also, } b = a\sqrt{1 - e^2} = \pm 4\sqrt{1 - 1/4} = \pm 4 \times \sqrt{3/4} = \pm 2\sqrt{3}.$$

Equation of the ellipse is $x^2/a^2 + y^2/b^2 = 1$.

$$\text{Or, } x^2/(\pm 4)^2 + y^2/(\pm 2\sqrt{3})^2 = 1$$

$$\text{Or, } x^2/16 + y^2/12 = 1.$$

Q.4. The directrix of a conic section is $3x + 4y = 1$ and focus is $(2, 3)$. Find the equation of the conic section if its eccentricity is $1/\sqrt{2}$.

Solution : 4

We have, focus $F(2, 3)$, $e = 1/\sqrt{2}$, and directrix $3x + 4y = 1$.

Let $P(x, y)$ be any point on the ellipse and $|MP|$ be the perpendicular distance from the directrix $3x + 4y = 1$, then by definition,

$$|FP| = e|MP|$$

$$\sqrt{[(x - 2)^2 + (y - 3)^2]} = (1/\sqrt{2}) \cdot |3x + 4y - 1|/\sqrt{3^2 + 4^2} = |3x + 4y - 1|/5\sqrt{2}.$$

Squaring both sides we get,

$$50 \{(x - 2)^2 + (y - 3)^2\} = (3x + 4y - 1)^2$$

$$\text{Or, } 50(x^2 - 4x + 4 + y^2 - 6y + 9) = 9x^2 + 16y^2 + 1 + 24xy - 6x - 8y$$

$$\text{Or, } 41x^2 - 24xy + 34y^2 - 194x - 292y + 649 = 0.$$

Which is the required equation of the conic.

$$\text{Here } a = 41, h = -12, b = 34$$

$$\text{and } h^2 - ab = (-12)^2 - 41 \times 34 = 144 - 1394 = -1250 < 0$$

The conic section is an ellipse.

Q.5. Find the eccentricity of the ellipse $(x - 3)^2/8 + (y - 4)^2/6 = 1$.

Solution : 5

$$\text{We have, } (x - 3)^2/8 + (y - 4)^2/6 = 1.$$

Shifting origin to the point (3, 4), we have the ellipse as

$$X^2/8 + Y^2/6 = 1.$$

$$a^2 = 8 \Rightarrow a = 2\sqrt{2}; b^2 = 6 \Rightarrow b = \sqrt{6}.$$

$$\text{Now, } b^2 = a^2(1 - e^2)$$

$$\text{Or, } 6 = 8(1 - e^2) \Rightarrow 6/8 = 1 - e^2 \Rightarrow e^2 = 1 - 6/8 = 2/8 = 1/4$$

Therefore, $e = 1/2$.

Q.6. Find the equation of the ellipse whose latus rectum is 10 and whose eccentricity is $2/3$.

Solution : 6

$$\text{We have, latus rectum} = 2b^2/a = 10, \text{ and } e = 2/3.$$

$$\text{Now, } b^2 = a^2 (1 - e^2) = a^2 (1 - 4/9)$$

$$\text{Or, } b^2/a = a^2/a\{(9 - 4)/9\}$$

$$\text{Or, } 10/2 = a \cdot 5/9$$

$$\text{Or, } 1 = a/9 \Rightarrow a = 9.$$

$$\text{And } b^2 = a^2 (1 - e^2) = 81 (1 - 4/9) = 45$$

$$\text{Therefore, } b = 3\sqrt{5}.$$

Thus the equation of ellipse is

$$x^2/9^2 + y^2/(3\sqrt{5})^2 = 1$$

$$\text{Or, } x^2/81 + y^2/45 = 1.$$

Q.7. Find the equation of the ellipse whose foci are at $(-2, 4)$ and $(4, 4)$ and whose major and minor axes are 10 and 8 respectively. Also find the eccentricity of the ellipse.

Solution : 7

$$2ae = 4 - (-2) = 6 \Rightarrow ae = 3 \Rightarrow e = 3/a.$$

$$\text{Again } 2a = 10 \Rightarrow a = 5 \text{ and } 2b = 8 \Rightarrow b = 4.$$

$$\text{Thus } e = 3/a = 3/5. \text{ Centre of the ellipse is } [(-2 + 4)/2, (4 + 4)/2] = (1, 4)$$

herefore, equation of the ellipse is

$$(x - 1)^2/25 + (y - 4)^2/16 = 1.$$

Q.8. Find the equation of the ellipse whose foci are $(-1, 5)$ and $(5, 5)$ and whose major axis is 10.

Solution : 8

$$2ae = 5 - (-1) = 6 \Rightarrow ae = 3 \Rightarrow e = 3/a$$

$$\text{Also } 2a = 10 \Rightarrow a = 5$$

$$\text{Thus } e = 3/a = 3/5,$$

Using $b^2 = a^2 (1 - e^2)$ we get, $b^2 = 25 (1 - 9/25) = 16 \Rightarrow b = 4$.

Centre of the ellipse is $[(-1 + 5)/2, (5 + 5)/2] = (2, 5)$

Therefore, equation of the ellipse is

$$(x - 2)^2/25 + (y - 5)^2/16 = 1.$$

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