Derivatives of Logarithmic Function

Q.1. If $y = e^x \log x$, find dy/dx at x = 1.

Solution : 1

We have, $y = e^{x} \log x$

Therefore, $dy/dx = e^{x}. d/dx(\log x) + \log x. d/dx(e^{x})$

$= ex . 1/x + \log x . e^{x}$

$= e^{x} (1/x + \log x)$

Therefore, $[dy/dx]$ x =1 = e1 (1/1 + log 1)

= e.

Q.2. If $y = e^x \log \tan 2x$, find dy / dx .

Solution : 2

We have, $y = e^{x}. \log (\tan 2x)$,

$dy /dx = e^{x} . d/dx\{\log (\tan 2x)\}+ \log (\tan 2x) . d/dx (e^{x})$

$= e^{x} \{1/\tan 2x . \sec2 2x . 2\}+ \log (\tan 2x) . e^{x}$

$= e^{x} \{(2 \sec 2 2x) / \tan 2x\}+ e^{x} . \log (\tan 2x)$

$= 2 e^{x} \{1 / (\sin 2x \cos 2x)\}+ e^{x} \log (\tan 2x)$

$= 2 e^{x} \{2 / (2 \sin 2x \cos 2x)\}+ e^{x} \log (\tan 2x)$

$= 4 e^{x} / \sin 4x + e^{x} \log (\tan 2x)$

$= e^{x} \{4 \text{cosec } 4x + \log (\tan 2x)\}$.

Q.3. If $y = \log \sqrt{(1 - \cos x) / (1 + \cos x)}$, find dy / dx.

Solution : 3

$y = \log \sqrt{(1 - \cos x) / (1 + \cos x)}$

$= (1/2) \log \{(1 - \cos x) / (1 + \cos x)\}$

$= (1/2) \{\log (1 - \cos x) - \log (1 + \cos x)$

Hence, $dy/dx = (1/2) [\sin x / (1 - \cos x) + \sin x / (1 + \cos x)]$

$= (\sin x / 2) [1 / (1 - \cos x) + 1/ (1 + \cos x)]$

$= (\sin x / 2) [(1 + \cos x + 1 - \cos x) / (1 - \cos^2 x)]$

$= (\sin x / 2)[2 / (1 - \cos^2 x)]$

$= (\sin x / 2).(2/\sin^2 x)$

$= 1 / \sin x = \operatorname{cosec} x$.

Q.4. If $y = - \cot^2 (x/2) - 2 \log \sin (x/2)$, prove that : $dy/dx = \cot^3 (x/2)$.

Solution : 4

$y = - \cot^2 (x/2) - 2 \log \sin (x/2)$

$= - \{\cot (x/2)\}^2 - 2 \log \sin (x/2)$

Hence , $dy / dx = - 2 \cot (x/2) \{- \operatorname{cosec}^2 (x/2)\} \times 1/2 - 2 / \sin (x/2) \times \cos (x/2) \times 1/2$

$= \cot (x/2) \operatorname{cosec}^2 (x/2) - \cot (x/2)$

$= \cot (x/2) [\operatorname{cosec}^2 (x/2) - 1]$

$= \cot (x/2) . \cot^2 (x/2) = \cot^3 (x/2)$. **[Proved.]**

Q.5. If $y = \log (\tan x)$, find dy / dx.

Solution : 5

$y = \log(\tan x) \Rightarrow dy / dx = (1/\tan x) . d / dx (\tan x)$

$= (1 / \tan x) . \sec^2 x$

$= \text{cosec } x \times \sec x.$

Q.6. If $y = \log(\sec x + \tan x)$, find dy / dx .

Solution : 6

$y = \log(\sec x + \tan x) \Rightarrow dy / dx = 1 / (\sec x + \tan x). d / dx (\sec x + \tan x)$

$= (\sec x \tan x + \sec^2 x) / (\sec x + \tan x)$

$= \{\sec x (\sec x + \tan x)\}/ (\sec x + \tan x)$

$= \sec x.$

Q.7. If $y = \log \tan (\pi / 4 + x / 2)$, find dy / dx.

Solution : 7

$y = \log \tan (\pi / 4 + x / 2)$

Then $dy / dx = d / dx [\log \tan (\pi / 4 + x / 2)]$

$= [1 / \tan (\pi / 4 + x / 2)] . d / dx [\tan (\pi / 4 + x / 2)]$

$= [1 / \tan (\pi /4 + x / 2)] . \sec 2 (\pi / 4 + x / 2) . d / dx (\pi / 4 + x / 2)$

$= [\sec^2 (\pi / 4 + x / 2) / \tan (\pi / 4 + x / 2)] . (1/2)$

$= 1/ [2 \sin (\pi / 4 + x / 2) \cos (\pi / 4 + x / 2)$

$= 1/ \sin (\pi / 2 + x) = 1 / \cos x = \sec x .$

Q.8. Differentiate $x \tan x \log_5 x$ w.r.t. x.

Solution : 8

Let $y = x \tan x \log_5 x$,

Then $dy/dx = x \tan x.\ d/dx (\log_5 x) + \log_5 x\ .d/dx (x \tan x)$

$= x \tan x\ .\ 1/x \log_5 + \log_5 x\ (x\ .\ \sec^2 x + \tan x\ .1)$

$= \tan x/\log_5 + (x \sec^2 x + \tan x) \log_5 x.$