Derivatives of Implicit Function

Q.1. If y = x log y, show that x dy/dx = y^2/(y – x) .

Solution : 1

We have y = x log y,

Differentiating both sides with respect to x we get , dy/dx = x . (1/y) .

dy/dx + log y . 1

Or, (1 – x /y) dy/dx = log y

Or, [(y – x)/y] dy/dx = log y

Or, dy/dx = y log y /(y – x)

Or, x dy/dx = x y log y/(y – x)

= y(x log y)/(y – x) [As, x log y = y]

= y2/(y – x) .

Q.2. If y log x = x – y , prove that dy/dx = log x/(1 + log $x)^2$.

Solution : 2

We have , y log x = x – y ,

Differentiating with respect to x we get ,

y × 1/x + log x × dy/dx = 1 – dy/dx

Or, (1 + log x)dy/dx = 1 – y/x --- --- --- (i)

We are given, y log x = x – y

Or, y log x + y = x

Or, y (1 + log x) = x

Or, y/x = 1/(1 + log x) --- --- --- (ii)

Hence, putting y/x from (ii) to (i) we get,

(1 + log x) dy/dx = 1 – 1/(1 + log x)

= (1 + log x – 1) /(1 + log x) = log x/(1 + log x)

Therefore , dy/dx = log x/$(1 + \log x)^2$. **[Proved.]**

Q.3. If log $(x^2 + y^2) = 2 \tan^{-1} (y/x)$, show that dy/dx = (x +y)/(x – y) .

Solution : 3

We have , log $(x^2 + y^2)$ = 2 tan –1 (y/x) ,

Differentiating with respect to x we get ,

$1/(x^2 + y^2) . d/dx (x^2 + y^2) = 2 . 1/\{1 + (y/x)^2\}. d/dx (y/x)$

Or, $1/(x^2 + y^2) \{d/dx (x^2) + d/dx (y^2)\} = 2x^2/(x^2 + y^2) . [\{x .dy/dx – y .1\}/x^2]$

Or, $1/(x^2 + y^2) (2x + 2y\, dy/dx) = 2/(x^2 + y^2) .\{x\, dy/dx – y\}$

Or, 2 (x + y dy/dx) = 2 (x dy/dx – y)

Or, x + y dy/dx = x dy/dx – y

Or, (y – x)dy/dx = – (x +y)

Or, dy/dx = (x + y)/(x – y) . **[Proved.]**

Q.4. If x √(1 +y) + y √(1 + x) = 0 , prove that dy/dx = – $1/(x +1)^2$.

Solution : 4

We have , x √(1 + y) + y √(1 +x) = 0

x √(1 +y) = – y √(1 +x)

Squaring both sides we get ,

$x^2 (1 + y) = y^2 (1 +x)$

Or, $x^2 – y^2 = y^2 x – x^2 y$

Or, (x + y) (x − y) = − xy (x − y)

Or, x + y = − xy [As x = y does not satisfy the given equation, x − y ≠ 0]

Or, x = − y − xy = > y (1 + x) = − x Or, y = − x /(1 + x)

Therefore , dy/dx = − [{(1 + x) . 1 − x (0 + 1)}/{$(1 + x)^2$}] = − 1 /$(1 + x)^2$. **[Proved.]**

Q.5. If y√(1 − x2) + x√(1 − y2) = 1 , prove that dy/dx = − √{$(1 - y^2)/(1 - x^2)$}.

Solution : 5

Let x = sin θ and y = cos φ , then

y √$(1 - x^2)$ + x √$(1 - y^2)$ = 1 reduces to

cos φ √$(1 - \sin^2\theta)$ + sin θ √$(1 - \cos^2\varphi)$ = 1

Or, cos φ cos θ + sin θ sin φ = 1

Or, cos (θ − φ) = 1

Or, θ − φ = cos −1 1

Or, sin −1 x − cos −1 y = cos −1 1

Or, sin −1 x − cos −1 y = 0

[As, cos −1 1 = 0]

Differentiating we get , 1/√$(1 - x^2)$ − (−1)/√$(1 - y^2)$ dy/dx = 0

Or, 1/√$(1 - y^2)$ dy/dx = −1/√$(1 - x^2)$

Or, dy/dx = − √{$(1 - y^2)/(1 - x^2)$}. **[Proved.]**

Q.6. If sin y = x sin (a + y) , prove that dy/dx = $\sin^2$ (a + y)/sin a .

Solution : 6

We have sin y = x sin (a + y)

Differentiating both sides with respect to y we get ,

$dx/dy = [\sin(a + y) . \cos y - \sin y . \cos(a + y)] / \sin^2 (a + y)$.

$= \sin(a + y - y)/\sin^2 (a + y)$

$= \sin a / \sin^2 (a + y)$

Hence , $dy/dx = \sin^2 (a + y) / \sin a$. **[Proved.]**

Q.7. If y = x sin y , prove that x. dy/dx = y/(1 – x cos y) .

Solution : 7

We have y = x sin y

Differentiating both sides with respect to x we get

dy/dx = x . cos y dy/dx + 1. sin y

Or, dy/dx (1 – x cos y) = sin y

Or, dy/dx = sin y / (1 – x cos y)

Or, x . dy/dx = (x . sin y)/(1 – x cos y) [As, y = x sin y]

= y/(1- x cos y) **[Proved.]**