Homogeneous Differential Equation

Q.1. Solve the differential equation : $(x^2 + y^2)\,dx - 2xydy = 0$, given that y = 0, when x = 1.

Solution : 1

We have $(x^2 + y^2)\,dx - 2xy\,dy = 0$ & y = 0 when x = 1.

The equation may be written as –

$dy/dx = (x^2 + y^2)/2xy$.

This is homogeneous equation . Put y = vx

Then dy/dx = v + x dv/dx

The equation reduces to ,

$v + x\,dv/dx = (x^2 + v^2 x^2)/2x^2v = (1 + v^2)/2v$

Or, $x\,dv/dx = (1 + v^2)/2v - v = (1 - v^2)/2v$

Or, $2v/(1 - v^2)\,dv = dx/x$

Integrating both sides , we get

$\int[2v/(1 - v^2)]\,dv = \int dx/x$

Or, $-\log[1 - v^2] = \log x + c$

Or, $-\log[1 - (y/x)^2] = \log x + c$

Putting x = 1 and y = 0 , we get c = 0 .

Then $1/[1 - y^2/x2] = x$

Or, $x = x^2 - y^2$.

Q.2. Solve the differential equation : $x\,dy/dx - y = \sqrt{(x^2 + y^2)}$.

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Solution : 2

We have $x\, dy/dx - y = \sqrt{(x^2 + y^2)}$

Or, $dy/dx - y/x = \sqrt{\{(x^2 + y^2)/x^2\}}$ ------------- (1)

Let $y = vx \Rightarrow dy/dx = v + dv/dx$

Putting in (1) we get $v + x dv/dx - v = \sqrt{(x^2 + v^2x^2)}/x$

Or, $x dv/dx = \sqrt{(1 + v^2)}$

Integrating we get , $\int dv/\sqrt{(1 + v^2)} = \int dx/x$ + const.

Or, $\log [v + \sqrt{(1 + v^2)} = \log x + \log c$

Or, $\log[y/x + \sqrt{(x^2 + y^2)}/x] = \log x + \log c$

Or, $\log[\{y + \sqrt{(x^2 + y^2)}\}/x^2] = \log c$

Or, $[y + \sqrt{(x^2 + y^2)}] = cx^2$.

Q.3. Solve the differential equation : $2dy/dx = y/x + y^2/x^2$ by substituting $y = vx$.

Solution : 3

We have, $2\, dy/dx = y/x + y^2/x^2$ --------------- (1)

Putting $y = vx$, we get $dy/dx = v + dv/dx$

Putting in (1), we get

$2(v + dv/dx) = v + v^2$

Or, $2dv/dx = v + v^2 - 2v = v^2 - v$

Integrating both sides, we get

$\int 1/(v2 - v)dv = 1/2 \int dx/x$

Or, $\int dv/(v^2 - v + 1/4 - 1/4) = 1/2 \int dx/x$

Or, $\int dv/[(v - 1/2)^2 - (1/2)^2] = 1/2 \int dx/x$

Or, $1/(2\times 1/2)\log[\,|v - 1/2 - 1/2\,| / |v - 1/2 + 1/2\,|\,] = 1/2 \log x$

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Or, $\log [(v - 1)/v] = 1/2 \log x$

Or, $\log [(y - x)/y] = 1/2 \log x + c$.

Q.4. Solve : $x\, dy/dx - y = \sqrt{(x^2 + y^2)}$.

Solution : 4

We are given $x\, dy/dx - y = \sqrt{(x^2 + y^2)}$

Or, $dy/dx = y/x + \{\sqrt{(x^2 + y^2)}\}/x$

Or, $dy/dx = y/x + \sqrt{\{(x^2 + y^2/x^2)\}}$

Or, $dy/dx = y/x + \sqrt{\{1 + (y/x)^2\}}$

Putting $y = vx$ we get

$dy/dx = v + x\, dv/dx$

and our equation reduces to

$v + x\, dv/dx = v + \sqrt{(1 + v^2)}$

Or, $x\, dv/dx = \sqrt{(1 + v^2)}$

Or, $dv/\sqrt{(1 + v^2)} = dx/x$

Integrating both sides we get

$\int dv/\sqrt{(1 + v^2)} = \int dx/x$

Or, $\log [v + \sqrt{(1 + v^2)}] = \log x + \log c$

Or, $\log[y/x + \sqrt{\{1 + (y/x)^2\}}] = \log x + \log c$

Or, $\log[y + \sqrt{(x^2 + y^2)}] - \log x = \log x + \log c$

Or, $\log [y + \sqrt{(x^2 + y^2)}] = 2 \log x + \log c$

Or, $\log [y + \sqrt{(x^2 + y^2)}] = \log x^2 + \log c$

Or, $\log [y + \sqrt{(x^2 + y^2)}] = \log cx^2$

Therefore, $y + \sqrt{(x^2 + y^2)} = cx^2$.

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Q.5. Solve : $x(x - y)\,dy + y^2dx = 0$.

Solution : 5

We are given,

$x(x - y)\,dy + y^2dx = 0$

Or, $dy/dx = - y^2/(x^2 - xy) = y^2/(xy - x^2)$

Putting $y = v\,x$,

we get $dy/dx = v + xdv/dx$

Or, $v + xdv/dx = v^2x^2/(x.vx - x^2) = v2x2/(vx^2 - x^2)$

Or, $v + xdv/dx = v^2/(v - 1)$

Or, $x\,dv/dx = v^2/(v - 1) - v = (v^2 - v^2 + v)(v - 1\ )$

Or, $x\,dv/dx = v/(v - 1)$

Or, $[(v - 1)/v]dv = (1/x)\,dx$

Integrating, we get

$\int(1 - 1/v)\,dv = \int(1/x)\,dx + c$

Or, $v - \log v = \log x + c$

Or, $y/x - \log(y/x) = \log x + c$

Or, $y/x - [\log y/x + \log x] = c$

Or, $y/x - [\log y - \log x + \log x] = c$

Or, $y/x - \log y = c$

Or, $y = x \log y = c\,x$.

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