ICSE 2025 EXAMINATION

Sample Question Paper - 2

Mathematics

Time Allowed: 2 hours and 30 minutes **Maximum Marks: 80**

General Instructions:

- Answers to this Paper must be written on the paper provided separately.
- You will not be allowed to write during the first 15 minutes.
- This time is to be spent reading the question paper.
- The time given at the head of this Paper is the time allowed for writing the answers.
- **Attempt all questions from Section A and any four questions from Section B**.
- All work, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answers.
- Omission of essential work will result in a loss of marks.
- The intended marks for questions or parts of questions are given in brackets [ ]
- Mathematical tables are provided.

Section A

1. **Question 1 Choose the correct answers to the questions from the given options:** **[15]**

(a) If the cost of an article is ₹ 25,000 and CGST paid by the owner is ₹ 2250, the rate of GST is **[1]**

a) 18% b) 15%

c) 9% d) 10%

(b) From a group of Saras birds, one-fourth of the number are moving about in lotus plants, one-ninth coupled with one-fourth as well as 7 times the square root of the total number are moving on a hill, while 56 birds are sitting in the Bakula trees. Then, what is the total number of birds? **[1]**

a) 629 b) 675

c) 576 d) 567

(c) If x + 1 is a factor of $3x^3 + kx^2 + 7x + 4$, then the value of k is **[1]**

a) 14 b) 0

c) 6 d) -6

(d) If $A = \begin{bmatrix} 5 & 5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 5 & 5 \end{bmatrix}$ and $A^n = \begin{bmatrix} 5^{200} & 5^{200} \\ 0 & 0 \end{bmatrix}$, then the value of n is **[1]**

a) 100 b) 75

c) 25 d) 50

(e) If a, b and c are respectively the pth, qth and rth terms of a GP, then the value of **[1]** (q - r) log a + (r - p) log P + (p - g) log c is

a) log abc
b) log bc
c) log ab
d) 0

(f) A point M is reflected in X-axis to M'(4, -5). M" is the image of M, when reflected in the Y-axis. The coordinates of M'" when M" is reflected in the origin, is **[1]**

a) (-4, -5)
b) (-4, 5)
c) (4, 5)
d) (4, -5)

(g) Diagonal AC of a rectangle ABCD is produced to the point E such that AC : CE = 2 : 1, AB = 8 cm and BC = 6 m. The length of DE is **[1]**

a) $3\sqrt{17}$ cm
b) 15 cm
c) 13 cm
d) $2\sqrt{19}$ cm

(h) A hollow cone of radius 6 cm and height 8 cm is vertical standing at the origin, such that the vertex of the cone is at the origin. Some pipes are hanging around the circular base of the cone, such that they touch the surface of the graph paper. Then, the total surface area of the formed by the figure will be **[1]**

a) 494.68 cm^2
b) 484.98 cm^2
c) 489.84 cm^2
d) 948.84 cm^2

(i) Find the range of values of x which satisfy the inequation, $(x + 1)^2 - (x - 1)^2 < 6$. **[1]**

a) $\left(-\infty, \frac{3}{2}\right)$
b) $\left(\frac{3}{2}, \infty\right)$
c) $\left(-\infty, -\frac{3}{2}\right)$
d) $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

(j) The probability that the minute hand lies from 5 to 15 min in the wall clock, is **[1]**

a) $\frac{1}{6}$
b) $\frac{5}{6}$
c) $\frac{1}{5}$
d) $\frac{1}{10}$

(k) If $\begin{bmatrix} a^x \\ a^{-x} \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} = \begin{bmatrix} p & a^{-2} \\ q & \log_2 2 \end{bmatrix}$, a > 0, then a^{p-q} is equal to **[1]**

a) $4^{\frac{3}{2}}$
b) 1
c) $2^{\frac{-3}{2}}$
d) $2^{\frac{3}{2}}$

(l) Suppose PQ be a pole, whose coordinates are P(1, 3) and 0(3, 3) and A be the position of a man whose coordinates are (1, 1). **[1]**

i. If a pole makes an angle of elevation to the point A, then the angle θ is
ii. Also, if we shift the origin at (1, 1), then the angle θ is

a) 75, 45°
b) 45°, 60°
c) 45°, 90
d) 45°, 45°

(m) If P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. Then, the perimeter of the quadrilateral PQSR will be **[1]**

a) 2($\sqrt{3}$ + 1)r
b) $2\sqrt{3} + r$

c) 2r d) $2\sqrt{3}r$

(n) If the ratio of mode and median of a certain data is 6 : 5, then the ratio of its mean and median is **[1]**

a) 10 : 9 b) 9 : 10

c) 10 : 8 d) 8 : 10

(o) **Assertion (A):** $a_n - a_{n-1}$ is not independent of n then the given sequence is an AP. **[1]**

Reason (R): Common difference $d = a_n - a_{n-1}$ is constant or independent of n.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

2. **Question 2** **[12]**

(a) Mr. Gupta opened a recurring deposit account in a bank. He deposited ₹ 2,500 per month for 2 years. At the time of maturity, he got ₹67,500. Find: **[4]**

i. the total interest earned by Mr. Gupta

ii. the rate of interest per annum.

(b) Find the third proportional to **[4]**

i. 16 and 36

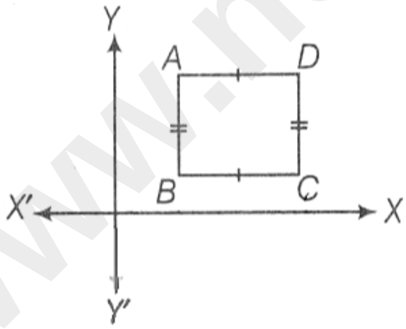
ii. $(x^2 + y^2 + xy)^2$ and $(x^3 - y^3)$

(c) Prove that: $\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\text{cosec}^3\theta} = \sin^2\theta\cos^2\theta$ **[4]**

3. **Question 3** **[13]**

(a) The internal and external diameters of a hollow hemispherical vessel are 7cm and 14 cm, respectively. The cost of silver plating of 1 sq cm surface is ₹ 0.60. Find the total cost of silver plating the vessel all over. **[4]**

(b) The side AB of a square ABCD is parallel to the Y-axis as shown in the given figure. **[4]**



Calculate

i. the slope of AD.

ii. the slope of BD.

iii. the slope of AC. [Given, $\tan(90^o + 0) = -\cot\theta$]

(c) Use graph paper to answer this question: **[5]**

i. The point P(2, - 4) is reflected about the line x = 0 to get the image Q. Find the coordinates of Q.

ii. Point Q is reflected about the line y = 0 to get the image R. Find the coordinates of R.

iii. Name the figure PQR.

iv. Find the area of figure PQR.

Section B

Attempt any 4 questions

4. **Question 4** **[10]**

(a) A shopkeeper bought an article with market price ₹1200 from the wholesaler at a discount of 10%. The shopkeeper sells this article to the customer on the market price printed on it. If the rate of GST is 6%, then find: **[3]**

i. GST paid by the wholesaler.

ii. Amount paid by the customer to buy the item.

(b) The sum of the squares of two consecutive odd positive integers is 290. Find them. **[3]**

(c) Draw a Histogram for the given data, using a graph paper: **[4]**

Weekly Wages (in ₹)	No. of People
3000-4000	4
4000-5000	9
5000-6000	18
6000-7000	6
7000-8000	7
8000-9000	2
9000-10000	4

Estimate the mode from the graph.

5. **Question 5** **[10]**

(a) Evaluate, $\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$. **[3]**

(b) O is the circumcentre of the $\triangle ABC$ and D is mid-point of the base BC. Prove that $\angle BOD = \angle A$. **[3]**

(c) Use factor theorem to factorise $6x^3 + 17x^2 + 4x - 12$ completely. **[4]**

6. **Question 6** **[10]**

(a) Calculate the ratio in which the line joining A (-4, 2) and B(3, 6) is divided by P(x, 3). Also, find **[3]**

i. x

ii. length of AP

(b) Prove that: $1 - \frac{\cos^2\theta}{1+\sin\theta} = \sin\theta$ **[3]**

(c) Sum of the first n terms of an AP is $5n^2 - 3n$. Find the AP and also find its 16th term. **[4]**

7. **Question 7** **[10]**

(a) A grassy land is in the shape of a right triangle. The hypotenuse of the land is 1 m more than twice the shortest side. If the third side is 7 m more than the shortest side, find the sides of the grassy land. **[5]**

(b) The marks obtained by 100 students in a Mathematics test are given below **[5]**

Marks	Number of students
0-10	3
10-20	7
20-30	12

30-40	17
40-50	23
50-60	14
60-70	9
70-80	6
80-90	5
90-100	4

Draw an ogive for the given distribution on a graph sheet, (use a scale of 2 cm = 10 units on both axes). Use the ogive to estimate the

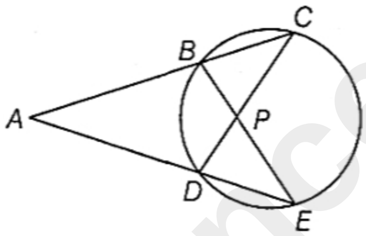
i. median.
ii. lower quartile.
iii. number of students who obtained more than 85% marks in the test.
iv. number of students who did not pass in the test, if the pass percentage was 35.

8. **Question 8** **[10]**

(a) A number is selected at random from first 50 natural numbers. Find the probability that it is a multiple of 3 and 4. **[3]**

(b) How many solid spheres of diameter 6 cm are required to be melted to form a cylindrical solid of height 45 cm and diameter 4 cm? **[3]**

(c) In the given figure, AC = AE. **[4]**



Show that

i. CP = EP
ii. BP = DP

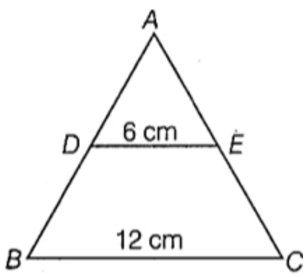
9. **Question 9** **[10]**

(a) Solve the following inequation and represent the solution set on the number line. **[3]**

$\frac{3x}{5} + 2 < x + 4 \leq \frac{x}{2} + 5, x \in R$

(b) Mode and mean of a data are 12k and 15k respectively. Find the median of the data. **[3]**

(c) In the given figure, if DE || BC, find the ratio of ar (△ADE) and ar (DECB). **[4]**



10. **Question 10** **[10]**

(a) Find the fourth proportional to $(a^3 + 8)$, $(a^4 - 2a^3 + 4a^2)$ and $(a^2 - 4)$. **[3]**

(b) Use a ruler and a pair of compasses to construct a $\triangle ABC$, in which BC = 4.2 cm, $\angle ABC = 60^o$ and AB = 5 cm. Construct a circle of radius 2 cm to touch both the arms of $\angle ABC$ of $\angle ABC$. **[3]**

(c) A man observes the angle of elevation of the top of a building to 30^o. He walks towards it in a horizontal line though its base. On covering 60 m, the angle of elevation changes to 60^o. Find the height of the building correct to the nearest metre. **[4]**

Solution

Section A

1. Question 1 Choose the correct answers to the questions from the given options:

(i) **(a)** 18%

Explanation: {

C.P. = ₹ 25,000, CGST = ₹ 2250

$\therefore$ GST = 2 $\times$ ₹ 2250 = ₹ 4500

let rate of GST = r%

$\therefore$ r% of ₹ 25,000 = ₹ 4500

$\Rightarrow$ r = (4500 $\times$ 100) $\div$ 25000 = 18%

(ii) **(c)** 576

Explanation: {

Let the total number of Saras birds be x.

Then, number of Saras birds moving in lotus plants = $\frac{x}{4}$

Number of Saras birds moving on a hill = $\frac{x}{9}$ + $\frac{x}{4}$ + 7 $\sqrt{x}$

Number of Saras birds sitting on the Bakula trees = 56

According to the question,

$\frac{x}{4} + \frac{x}{9} + \frac{x}{4} + 7\sqrt{x} + 56 = x$

$\Rightarrow 7\sqrt{x} = x - \frac{x}{4} - \frac{x}{9} - \frac{x}{4} - 56$

$\Rightarrow 7\sqrt{x} = \frac{36x-9x-4x-9x}{36} = 56$

$\Rightarrow 7\sqrt{x} = \frac{7x}{18} - 56 \Rightarrow \sqrt{x} = \frac{x}{18} - 8$

$\Rightarrow x = \frac{x^2}{324} + 64 - \frac{8x}{9}$ [squaring on both sides]

$\Rightarrow x = \frac{x^2+20736-288x}{324}$

$\Rightarrow 324x = x^2 + 20736 - 288x$

$\Rightarrow x^2 - 612x + 20736 = 0$

$\Rightarrow x^2 - 36x - 576x + 20736 = 0$ [splitting the middle term]

$\Rightarrow$ x(x - 36) - 576(x - 36) = 0

$\Rightarrow$ (x - 36)(x - 576) = 0

$\Rightarrow$ x - 36 = 0 or x - 576 = 0

$\Rightarrow$ x = 576 or x = 36

Here, x = 36 is not possible, because if there are only 36 birds, then 56 cannot be on the trees.

Thus, total number of Saras birds is 576.

(iii) **(c)** 6

Explanation: {

Let $f(x) = 3x^3 + kx^2 + 7x + 4$

As x + 1 is a factor of f(x), f(-1) = 0

$\Rightarrow 3(-1)^3 + k(-1)^2 + 7(-1) + 4 = 0$

$\Rightarrow$ -3 + k - 7 + 4 = 0

$\Rightarrow$ k = 6

(iv) **(a)** 100

Explanation: {

We have, $A = \begin{bmatrix} 5 & 5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 25 & 25 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 5^2 & 5^2 \\ 0 & 0 \end{bmatrix}$

$\therefore A^2 = A \cdot A = \begin{bmatrix} 5^2 & 5^2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 5^2 & 5^2 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 5^4 & 5^4 \\ 0 & 0 \end{bmatrix}$

$A^n = \begin{bmatrix} 5^{2n} & 5^{2n} \\ 0 & 0 \end{bmatrix}$

Thus, $\begin{bmatrix} 5^{2n} & 5^{2n} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 5^{200} & 5^{200} \\ 0 & 0 \end{bmatrix}$

$\Rightarrow 5^{2n} = 5^{200}$

$\Rightarrow$ 2n = 200

$\Rightarrow$ n = 100

(v) **(d)** 0

Explanation: {

Let A be the first term and R be the common ratio of the given GP.

Then, a = pth term $\Rightarrow$ a = AR^{p-1}

$\Rightarrow$ log a = log A + (p - 1)log R ...(i)

b = qth term $\Rightarrow$ b = AR^{q-1}

$\Rightarrow$ log b = log A + (q - 1) log R ...(ii)

c = rth term $\Rightarrow$ c = AR^{r-1}

$\Rightarrow$ log c = log A + (r - 1) log R ...(iii)

$\Rightarrow$ Now, consider {q - r) log a + (r - p)log b + (p - q)log c

=(q - r){log A + (p - 1)log R} + (r - p){log A + (q - 1)log R} [from Eqs. (i), (ii) and (iii)]

= log A{(q - r)+ (r - p) + (p - q)} + log R{(p - 1) (q - r) + (q - 1)(r - p) + (r - 1)(p - q)}

= (log A) 0 + {p(q - r) + q (r - p)+ r(p - q) - (q - r) - (r - p) - (p - q)} log R|

= (log A)0 + (log R) 0 = 0

(vi) **(d)** (4, -5)

Explanation: {

Since, the image of any point (x, y) under X-axis is (x, - y).

$\therefore$ Coordinate of M $\equiv$ (4, 5)

Since, the image of any point (x, y) under Y-axis is (-x, y).

$\therefore$ Coordinate of M" $\equiv$ (4, -5)

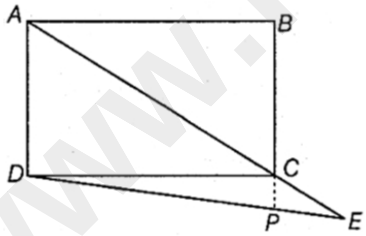
Since, the image of any point (x, y) under origin is (-x, -y).

$\therefore$ Coordinate of M''' = (4, - 5)

(vii) **(a)** $3\sqrt{17}$ cm

Explanation: {

Given AB = 8 cm and BC = 6 cm



$\therefore$ AC = $\sqrt{8^2 + 6^2}$ = 10 cm

Also, given AC : CE = 2 : 1

Now, produce BC to meet DE at the point P as CP is parallel to AD,

$\triangle ECP \sim \triangle EAD$ ...(i)

$\Rightarrow \frac{CP}{AD} = \frac{CE}{AE} \Rightarrow \frac{CP}{6} = \frac{1}{3}$ ...(ii)

$\Rightarrow$ CP = 2 cm

Also, $\triangle$CPD is right triangle.

$\therefore$ DP = $\sqrt{CD^2 + CP^2}$

$= \sqrt{68} = 2\sqrt{17}$ cm

But DP = PE = 2 : 1 [from Eq.(i)]

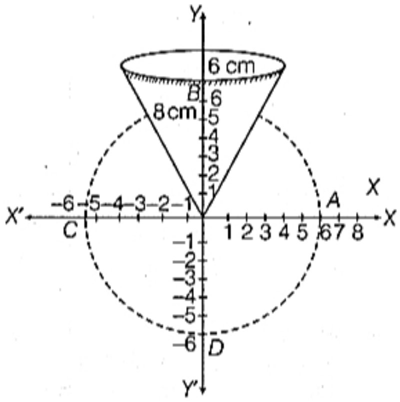
$\therefore$ PE = $\sqrt{17}$ cm

Thus, DE = DP + PE = $2\sqrt{17} + \sqrt{17} = 3\sqrt{17}$ cm

(viii) **(c)** 489.84 cm^2

Explanation: {

According to the given information, a shape of figure is shown below



When the hanging pipes touches the surface paper, a circular shape ABCD is formed on the graph paper. The size of circle ABCD is equal to the size of circular base of the cone.

∴ Radius of the circle ABCD is 6 cm.

Hence, the coordinates of A, B, C and Dare (6, 0), (0, 6), (-6, 0) and (0, -6), respectively.

The figure formed in the given information is cylindrical in outer surface and conical in the inner surface. Now, total surface area of the figure

= Curved surface area of the cylinder + Curved surface area of the cone

= $2\pi rh + \pi rl = \pi r(2h + l)$

= $\pi r(2h + \sqrt{r^2 + h^2})$

= $3.14 \times 6\left(2 \times 8 + \sqrt{6^2 + 8^2}\right)$

= $18.84(16 + \sqrt{36 + 64})$

= $18.84(16 + \sqrt{100}) = 18.84(16 + 10)$

= $18.84 \times 26 = 489.84$ cm^2

(ix) **(a)** $\left(-\infty, \frac{3}{2}\right)$

Explanation: {

We have, $(x + 1)^2 - (x - 1)^2 < 6$

$\Rightarrow (x^2 + 1 + 2x) - (x^2 + 1 - 2x) < 6$ $[\because (a \pm b)^2 = a^2 + b^2 \pm 2ab]$

$\Rightarrow 4x < 6$

$\Rightarrow x < \frac{6}{4}$

$\Rightarrow x < \frac{3}{2}$

$\Rightarrow x \in \left(-\infty, \frac{3}{2}\right)$

(x) **(a)** $\frac{1}{6}$

Explanation: {

In a wall clock, the minute hand cover the 60 min in on complete round.

∴ Total number of possible outcomes = 60

The minute hand cover the time from 5 to 15 min,

Number of outcomes favourable to E

= Distance from 5 to 15 min = 10

∴ Required probability = $\frac{10}{60} = \frac{1}{6}$

(xi) **(c)** $2^{\frac{-3}{2}}$

Explanation: {

We have, $\begin{bmatrix} a^x \\ a^{-x} \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} = \begin{bmatrix} p & a^{-2} \\ q & \log_2 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^x & 2a^x \\ a^{-x} & 2a^{-x} \end{bmatrix} = \begin{bmatrix} p & a^{-2} \\ q & 1 \end{bmatrix}$ $\left[\because \log_2 2 = \frac{\log 2}{\log 2} = 1\right]$

On comparing the corresponding elements both sides, we get

$\Rightarrow a^x = p$ ...(i)

$\Rightarrow 2a^x = a^{-2}$ ...(ii)

$\Rightarrow a^{-x} = q$ ...(iii)

and $2a^{-x} = 1$ ...(iv)

On multiplying Eqs. (ii) and (iv), we get

$4a^{x-x} = a^{-2}$

$\Rightarrow 4a^0 = a^{-2} \Rightarrow 4 = a^{-2} \Rightarrow 4 = \frac{1}{a^2}$

$\Rightarrow a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}$ [$\because a > 0$]

Now, $a^{p-q} = a^{a^x - a^{-x}}$ [from Eqs. (i) and (iii)]

$= a^{\frac{1}{2}a^{-2} - \frac{1}{2}} = a^{2-\frac{1}{2}}$ [from Eqs. (ii) and (iv)]

$= a^{\frac{1}{2}\cdot 4 - \frac{1}{2}}$ [$\because a^{-2} = 4$]

$= \left(\frac{1}{2}\right)^{\frac{3}{2}} = 2^{\frac{-3}{2}}$

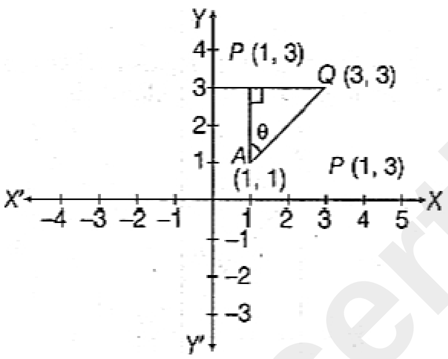
(xii) **(d)** 45°, 45°

Explanation: {

Given, coordinates of pole be P(1, 3) and Q (3, 3) and A(1, 1) be the position of man

i. Now, AP = $\sqrt{(1-1)^2 + (3-1)^2}$ [$\because$ distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$]

= $\sqrt{0^2 + 2^2}$ = 2 units



and PO = $\sqrt{(3-1)^2 + (3-3)^2} = \sqrt{2^2 + 0^2}$ = 2 units

Now, in $\triangle$APQ, we have

$\tan\theta = \frac{PQ}{AP} \Rightarrow \tan\theta = \frac{2}{2} = 1$

$\Rightarrow \theta = 45^o$ [$\because \tan 45^o = 1$]

ii. When we shift the origin at (1, 1), then the angle will remain same, i.e. θ = 45°.

(xiii) **(a)** $2(\sqrt{3} + 1)r$

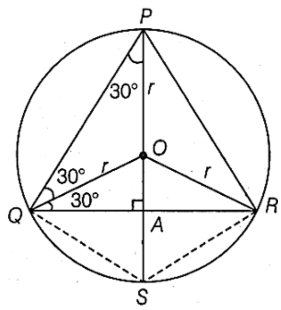
Explanation: {

As PQR is an equilateral triangle, hence PS will be perpendicular to QP and will divide it into 2 equal parts.

Since, $\angle$P and $\angle$S will be supplementary, so

$\angle S = 120^o$ and $\angle QSA = \angle RSA = 60^o$

Now, PA = PQ cos 30^o and OA = OQ sin 30^o = $\frac{r}{2}$



$\Rightarrow$ AS = OA = $\frac{r}{2}$ and PA = PO + OA = r + $\frac{r}{2}$

Hence, PQ = $\frac{PA}{\cos 30^\circ} = \frac{r+\frac{r}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{3}r$

In $\triangle$QAS, AS = QS cos $60^o \Rightarrow$ QS = $\frac{\frac{r}{2}}{\frac{1}{2}}$ = r

Since, AQ = AR, AS is common and $\angle$QAS = $\angle$RAS = 90^o

So, QS = RS.

$\therefore$ Perimeter of PQSP = 2(PQ + QS) = 2($\sqrt{3}$ + 1)r

(xiv) **(b)** 9 : 10

Explanation: {

We know that,

Mode = 3 Median - 2 Mean

On dividing both sides by median, we get

$\frac{\text{Mode}}{\text{Median}} = 3 - 2\frac{\text{Mean}}{\text{Median}}$

$\Rightarrow \frac{6}{5}$ = 3 - 2 $\frac{\text{Mean}}{\text{Median}}$ [$\because \frac{\text{mode}}{\text{median}} = \frac{6}{5}$, given]

$\Rightarrow \frac{6}{5}$ - 3 = -2 $\frac{\text{Mean}}{\text{Median}}$

$\Rightarrow \frac{6-15}{5}$ = -2 $\frac{\text{Mean}}{\text{Median}}$

$\Rightarrow \frac{-9}{5}$ = -2 $\frac{\text{Mean}}{\text{Median}}$

$\Rightarrow \frac{\text{Mean}}{\text{Median}} = \frac{9}{10}$

(xv) **(d)** A is false but R is true.

Explanation: {

We have, common difference of an AP

d = a_n - a_{n-1} is independent of n or constant.

So, A is false but R is true.

2. Question 2

(i) i. P = ₹ 2,500

n = 24 months

r = ?

M.V. = ₹ 67,500

Total money deposited in 2 years

= P $\times$ n

= 2,500 $\times$ 24

= 60,000

Total interest earned by Mr. Gupta = Maturity value - Money deposited

= 67,500 - 60,000

= ₹ 7,500

ii. M.V. = P $\times$ n + $\frac{P\times n(n+1)\times r}{2400}$

67,500 = 2500 $\times$ 24 + $\frac{2500\times 24\times 25\times r}{2400}$

67,500 = 60,000 + 625r

67,500 - 60,000 = 625r

7,500 = 625r

$\frac{7500}{625}$ = r

r = 12% p.a.

(ii) i. 16 and 36

Let the thid proportional to 16 and 36 be x.

$\Rightarrow$ 16, 36 and x ins continous proportion.

$\Rightarrow$ 16 : 36 = 36 : x

$\Rightarrow$ 16 $\times$ x = 36 $\times$ 36

$\Rightarrow x = \frac{36 \times 36}{16}$

$\Rightarrow$ x = 81

ii. $(x^2 + y^2 + xy)^2$ and $x^3 - y^3$

Let third proportional to $(x^2 + y^2 + xy)^2$ and $x^3 - y^3$ be x.

$\Rightarrow (x^2 + y^2 + xy)^2$, $x^3 - y^3$ and x are in continous proportion.

$\Rightarrow (x^2 + y^2 + xy)^2 : x^3 - y^3 = x^3 - y^3 : x$

$$x = \frac{(x^3 - y^3)^2}{(x^2 + y^2 + xy)^2}$$

$$x = \frac{(x - y)^2 (x^2 + y + xy)^2}{(x^2 + y^2 + xy)^2} \quad [\because x^3 - y^3 = (x - y)(x^2 + y^2 + xy)]$$

$x = (x - y)^2$

(iii) L.H.S. $= \frac{(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta)}{\sec^3\theta - \operatorname{cosec}^3\theta}$

$$= \frac{\left(1 + \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)(\sin\theta - \cos\theta)}{\frac{1}{\cos^3\theta} - \frac{1}{\sin^3\theta}}$$

$$= \frac{\left(\frac{1 + \cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}\right)(\sin\theta - \cos\theta)}{\frac{\sin^3\theta - \cos^3\theta}{\sin^3\theta\cos^3\theta}} \quad \{\sin^2\theta + \cos^2\theta = 1\}$$

$$= \frac{(\sin\theta\cos\theta + 1)(\sin\theta - \cos\theta)(\sin^3\theta\cos^3\theta)}{(\sin^3\theta - \cos^3\theta)\sin\theta\cos\theta}$$

Since, we know,

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= \frac{(\sin\theta\cos\theta + 1)(\sin\theta - \cos\theta)(\sin^3\theta\cos^3\theta)}{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta(sin\theta\ cos\theta))}$$

$$= \frac{(\sin\theta\cos\theta + 1)(\sin^2\theta\cos^2\theta)}{(1 + \sin\theta\cos\theta)} \quad \{\because \sin^2\theta + \cos^2\theta = 1\}$$

$= \sin^2\theta\cos^2\theta$ = RHS proved

3. Question 3

(i) Given, internal diameter of hollow hemispherical vessel = 7 cm

external diameter of a hollow hemispherical vessel = 14 cm

$r_1 = \frac{7}{2}$ cm

$r_2 = \frac{14}{2}$ = 7cm

Area of Ring = $\pi r_2^2 - \pi r_1^2$

Total area to be painted

$= 2\pi r_2^2 + 2\pi r_1^2 + (\pi r_2^2 - \pi r_1^2)$

$= 3\pi r_2^2 + \pi r_1^2$

$= \pi (3\pi r_2^2 + r_1^2)$

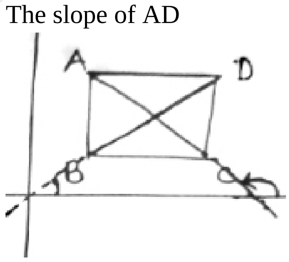
$= \pi [3 \times 49 + (3.5)^2]$ = 3.14 $\times$ (147 + 12.25)

= 500.045 cm^2

Hence, the total cost of silver painting the vessel = 500.045 $\times$ 0.6

= ₹ 300.027

(ii) i. The slope of AD



We know that the slope of any line parallel to x-axis is 0.

$\therefore$ The slope of AD = 0

ii. The slope of BD.

As ABCD in a square

$\therefore$ the diagonal BD makes an angle of 45^o with +ve direction of x-axis

$\therefore$ Slope of BD = tan 45^o = 1

iii. The diagonal AC make augle of 135^o with positive direction of x-axis.

$\therefore$ Slope of AC = tan 135^o

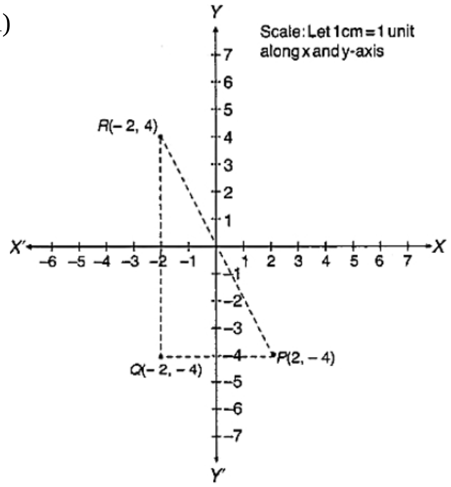
= tan (90 $\times$ 2 - 45)

= tan (-45)

= -tan 45

= -1

(iii)



i. The coordinates of Q are (- 2, - 4)

ii. The coordinates of R are (- 2, 4)

iii. PQR is Right angle Triangle

iv. Area of $\triangle$PQR = $\frac{1}{2} \times$ Base $\times$ height

$= \frac{1}{2} \times 4 \times 8$

= 2×8

= 16 sq. unit

Section B

4. Question 4

(i) i. C.P for the shopkeeper

= 1200 $\times \frac{90}{100}$ = ₹1080

GST paid by the wholesaler

= 1080 $\times \frac{60}{100}$ = ₹64.80

ii. S.P of the article = ₹1200

GST paid by the customer

= 1200 $\times \frac{6}{100}$ = ₹72

Amount paid by the customer

= S.P. + GST = 1200 + 72 = ₹1272

(ii) Let the two consecutive odd no. be x and x + 2.

A/c question

$x^2 + (x + 2)^2 = 290$

$\Rightarrow x^2 + x^2 + 4x + 4 = 290$

$\Rightarrow 2x^2 + 4x - 286 = 0$

$\Rightarrow 2(x^2 + 2x - 143) = 0$

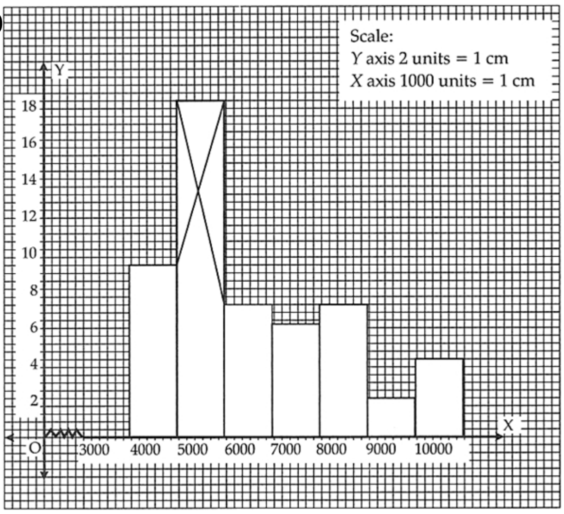
$\Rightarrow x^2 + 2x - 143 = 0$

$\Rightarrow x^2 + 13x - 11x - 143 = 0$

$\Rightarrow x(x + 13) - 11(x + 13) = 0$

x = 11 or x = -13 rejected

(iii)



In the given histogram, inside the highest rectangle, which represents the maximum frequency.

$\therefore$ Modal class = 5000 - 6000

Then, mode = 5500.

5. Question 5

(i) $\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

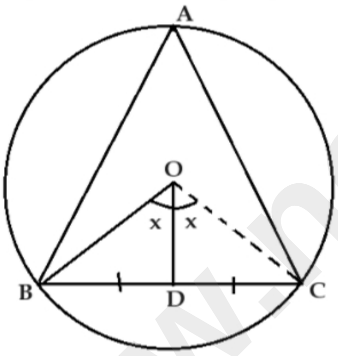
= $\begin{bmatrix} 4\times\frac{1}{2} & 2\times\frac{1}{2} \\ 1 & 2\times 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$[$\therefore$ sin 30° = cos 60° = $\frac{1}{2}$ and sin 90° = cos 0° = 1]

= $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

= $\begin{bmatrix} 2\times 4+1\times 5 & 2\times 5+1\times 4 \\ 1\times 4+2\times 5 & 1\times 5+2\times 4 \end{bmatrix}$

= $\begin{bmatrix} 8+5 & 10+4 \\ 4+10 & 5+8 \end{bmatrix}$ = $\begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$.

(ii)



To prove $\angle BOD = \angle A$

In $\triangle BDO$ and $\triangle CDO$

BD = DC (given)

OD = OD (common)

BO = OC (Radius)

By SSS

$\triangle BDO \cong \triangle CDO$

$\angle BOD = \angle COD$ = x

Now,

$\angle BAC = \frac{1}{2}\angle BOC$ (angle made by same arc)

$\angle BAC = \frac{1}{2}\times(2x)$

$\angle BAC$ = x

$\angle BAC$ = $\angle BOD$

$\angle A = \angle BOD$

Hence proved.

(iii) Let p(x) = $6x^3 - 17x^2 + 4x - 12$

Remainder p(- 2) = $6(-2)^3 + 17(-2)^2 + 4(-2) - 12$

= -48 + 68 - 8 - 12

= 68 - 68 = 0

∴ (x + 2) is a factor of given polynomial p(x)

$$
\begin{array}{r|l}
x+2 & 6x^3+17x^2+4x-12 \;\big(\; 6x^2+5x-6 \\
 & 6x^3+12x^2 \\
 & (-)\quad(-) \\ \hline
 & 5x^2+4x-12 \\
 & 5x^2+10x \\
 & (-)\quad(-) \\ \hline
 & -6x-12 \\
 & -6x-12 \\
 & (+)\quad(+) \\ \hline
 & 0
\end{array}
$$

∴ $6x^3 + 17x^2 + 4x - 12$

= $(x + 2)(6x^2 + 5x - 6)$

= $(x + 2)\{6x^2 + 9x - 4x - 6\}$

= (x + 2) {3x(2x + 3) - 2(2x + 3)}

= (x + 2)(2x + 3)(3x - 2)

6. Question 6

(i) i. By using section formula,

Let the ratio be k : 1

$y = \frac{m_2y_1 + m_1y_2}{m_1 + m_2}$

$3 = \frac{1\times 2 + k\times 6}{k+1}$

3k + 3 = 2 + 6k

3k - 6k = 2 - 3

-3k = -1

k = $\frac{1}{3}$

Ratio = 1 : 3

$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$

$x = \frac{1\times 3 + 3\times -4}{1+3}$

$x = \frac{3-12}{4}$

$x = \frac{-9}{4}$

ii. Here coordinate of P is $\left(\frac{-9}{4}, 3\right)$

Length of, AP = $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$= \sqrt{(3-2)^2 + \left(-\frac{9}{4} - (-4)\right)^2}$

$= \sqrt{(1)^2 + \left(\frac{-9}{4} + 4\right)^2}$

$= \sqrt{(1)^2 + \left(\frac{-9+16}{4}\right)^2}$

$= \sqrt{(1)^2 + \left(\frac{7}{4}\right)^2}$

$= \sqrt{1 + \frac{49}{16}}$

$= \sqrt{\frac{65}{16}}$

$= \frac{\sqrt{65}}{4}$

(ii) LHS = $\frac{1}{1} - \frac{\cos^2\theta}{1+\sin\theta} = \frac{1+\sin\theta-\cos^2\theta}{1+\sin\theta}$

= $\frac{1+\sin\theta-(1-\sin^2\theta)}{1+\sin\theta}$

= $\frac{1+\sin\theta-1+\sin^2\theta}{1+\sin\theta}$

= $\frac{\sin\theta(1+\sin\theta)}{(1+\sin\theta)}$

= $\sin\theta$

= RHS

Hence Proved

(iii) $S_n = 5n^2 - 3n$

$S_1 = a_1$

$= 5(1)^2 - 3(1)$

$= 5 - 3$

$= 2$

$a_1 = 2$

$S_2 = a_1 + a_2$

$a_1 + a_2 = 5(1)^2 - 3(2)$

$= 20 - 6$

$= 14$

$\therefore a_1 + a_2 = 14$

$2 + a_2 = 14$

$a_2 = 14 - 2$

$a_2 = 12$

Again, $S_3 = a_1 + a_2 + a_3$

$a_1 + a_2 + a_3 = 5(3)^2 - 3(3)$

$= 45 - 9$

$= 36$

$a_1 + a_2 + a_3 = 36$

$14 + a_3 = 36$

$a_3 = 36 - 14$

$a_3 = 22$

$a_1 = 2, a_2 = 12, a_3 = 22$

Hence square becomes 2, 12, 22, ...

$a_1 = a = 2$

$d = 12 - 2 = 10$

$a_{16} = a + (16 - 1)d$

$= 2 + 15 \times 10$

$= 2 + 150$

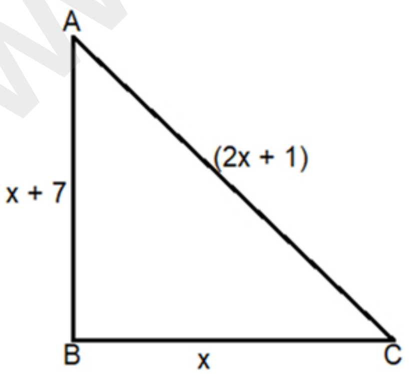
$a_{16} = 152$

7. Question 7

(i) Let the shortest side be x m.

hypotenuse = (2x + 1)m

3rd side = (x + 7)m



$(2x + 1)^2 = (x + 7)^2 + x^2$ {by Pythagoas theorem}

$\Rightarrow 4x^2 + 1 + 4x = x^2 + 49 + 14x + x^2$

$\Rightarrow 2x^2 - 10x - 48 = 0$

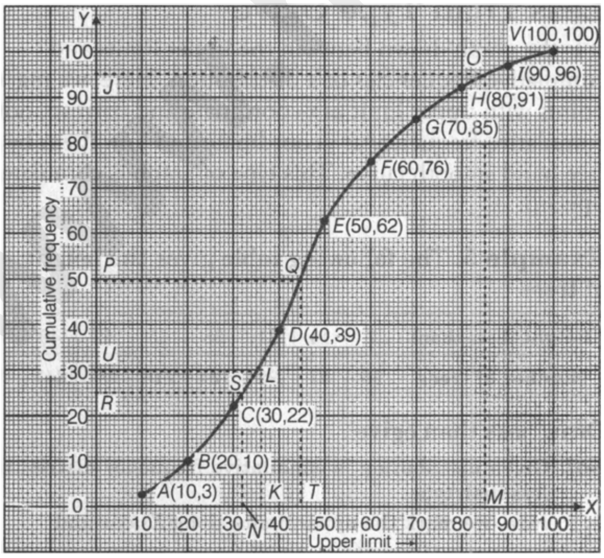
$\Rightarrow x^2 - 5x - 24 = 0$

$\Rightarrow x^2 - 8x + 3x - 24 = 0$
$\Rightarrow x(x - 8) + 3(x - 8) = 0$
$\Rightarrow (x + 3)(x - 8) = 0$
x = -3, 8
x = -3 rejected ($\because$ length can never be -ve)
$\therefore$ x = 8
hypotenuse i.e. AC
= 2x + 1
$= 2 \times 8 + 1 = 17$
BC = x = 8m
AB = x + 7
= 8 + 7
AB = 15 m

(ii) The cumulative frequency table for the given continuous distribution is given below

Marks	Number of students	Cumulative frequency (cf)
0-10	3	3
10-20	7	10
20-30	12	22
30-40	17	39
40-50	23	62
50-60	14	76
60-70	9	85
70-80	6	91
80-90	5	96
90-100	4	100

On the graph paper, we plot the following points A (10, 3), B (20, 10), C (30, 22), D (40, 39), E (50, 62), F (60, 76), G (70, 85), H (80, 91), 7(90, 96) and V(100, 100). Join all these points by a free hand drawing. The required ogive is shown on the graph paper given below



Here, number of students (n) = 100, which is even.

i. Let P be the point on Y-axis representing frequency
$= \frac{n}{2} = \frac{100}{2} = 50$
Through P, draw a horizontal line to meet the ogive at point Q. Through Q, draw a vertical line to meet the X-axis at T. The abscissa of the point T represents 43 marks. Hence, the median marks is 43.

ii. Let R be the point on Y-axis representing frequency

$= \frac{n}{4} = \frac{100}{4}$ = 25.

Through R, draw a horizontal line to meet the ogive at point S. Through S, draw a vertical line to meet the X-axis at N. The abscissa of the point N represents 31 marks. Hence, the lower quartile = 31 marks.

iii. 85% marks = 85% of 100 = 85 marks.

Let the point M on X-axis represents 85 marks. Through M, draw a vertical line to meet the ogive at the point O. Through O draw a horizontal line to meet the Y-axis at point J. The ordinate of point J represents 95 students.

∴ Number of students who obtained more than 85% in the test = 100 - 95 = 5

iv. 35% marks = 35% of 100 = 35

Let the point K on X-axis represents 35 marks. Through K, draw a vertical line to meet the ogive at the point L. Through L, draw a horizontal line to meet the Y-axis at point U. The ordinate of point U represents 30 students on Y-axis. Hence, the number of students, who did not pass in the test is 30.

8. Question 8

(i) n(s) = 50

n(multiple of 3 and 4) = {12, 24, 36, 48}

i.e multiple of 12

(multiple of 3 and 4) = $\frac{4}{50} = \frac{2}{25}$

(ii) Given, diameter of solid sphere, d_1 = 6 cm

∴ Radius of sphere, $r_1 = \frac{6}{2}$ = 3 cm

Also, given diameter of cylinder, d_2 = 4 cm

∴ Radius of cylinder, $r_2 = \frac{4}{2}$ = 2 cm

∵ Height of cylinder, h = 45 cm [given]

Let the required number of spheres be N.

∴ N × Volume of sphere = Volume of cylinder

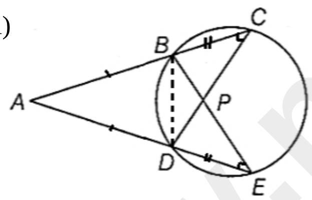
⇒ $N \times \frac{4}{3}\pi r_1^3 = \pi r_2^2 h$

⇒ $N \times \frac{4}{3}\pi \times (3)^3 = \pi \times (2)^2 \times 45$

∴ $N = \frac{2\times2\times45}{4\times3\times3} = 5$

Hence, the required number of solid spheres is 5.

(iii)



In $\triangle$ADC and $\triangle$ABE

$\angle$ACD = $\angle$AEB (angle in the same segment BD)

AC = AE (given)

$\angle$A = $\angle$A (common)

∴ $\triangle$ADC ≅ $\triangle$ ABE (ASA Còng rule)

⇒ AB = AD (CPCT)

But AC = AE

∴ AC - AB = AE - AD

⇒ BC = DE

In $\triangle$BPC and $\triangle$DPE

$\angle$C = $\angle$E (angle in the same segment)

BC = DE

$\angle$CBP = $\angle$CDE (angle on the same segment)

∴ $\triangle$BPC ≅ $\triangle$DPE (ASA cong rule)

⇒ BP = DP and CP = PE (CPCT)

9. Question 9

(i) $\frac{3x}{5} + 2 < x + 4$

⇒ $\frac{3x+10}{5} < x + 4$

$\Rightarrow$ 3x + 10 < 5(x + 4)
$\Rightarrow$ 3x + 10 < 5x + 20
$\Rightarrow$ 10 - 20 < 5x - 3x
$\Rightarrow$ -10 < 2x
$\Rightarrow$ -5 < x

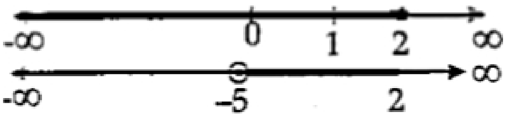
And

x + 4 $\leq \frac{x}{2}$ + 5

$\Rightarrow$ x + 4 $\leq \frac{x+10}{2}$

$\Rightarrow$ 2x + 8 $\leq$ x + 10

$\Rightarrow$ x $\leq$ 2



Solution Set = {x: - 5 < x $\leq$ 2, x $\in$ R}

(ii) Given:

mode = 12 k, mean = 15 k, median = ?

Using Empirical relation;

Mode = 3 median - 2 mean

12k = 3 median - 2(15 k)

3 median = 12k + 30k

3 median = 42k

median = $\frac{42k}{3}$

median = 14k

(iii) Given, DE || BC, DE = 6 cm and BC = 12 cm.

In $\triangle$ABC and $\triangle$ADE,

$\angle$ABC = $\angle$ADE [corresponding angles]

$\angle$ACB = $\angle$AED [corresponding angles]

and $\angle$A = $\angle$A [common angle]

$\therefore \triangle ABC \sim \triangle ADE$ [by AAA similarity criterion]

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$\therefore \frac{\text{ar}(\Delta ADE)}{\text{ar}(\triangle ABC)} = \frac{(DE)^2}{(BC)^2} = \frac{(6)^2}{(12)^2} = \left(\frac{1}{2}\right)^2$

$\Rightarrow \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle ABC)} = \frac{1}{4}$

Let ar ($\triangle$ADE) = k, then ar ($\triangle$ABC) = 4k

Now, ar (DECB) = ar ($\triangle$ABC) - ar ($\triangle$ADE)

= 4k - k = 3k

$\therefore$ Required ratio = ar ($\triangle$ADE) : ar (DECB)

= k : 3k = 1 : 3

10. Question 10

(i) Let fourth proportional be x.

Then, $(a^3 + 8) : (a^4 - 2a^3 + 4a^2) :: (a^2 - 4) : x$

$\Rightarrow \frac{a^3+8}{a^4-2a^3+4a^2} = \frac{a^2-4}{x}$

$\Rightarrow x(a^3 + 8) = (a^2 - 4)(a^4 - 2a^3 + 4a^2)$ [by cross-multiplication]

$\therefore$ x = $\frac{(a^2-2^2)\times a^2(a^2-2a+4)}{(a^3+2^3)}$

= $\frac{a^2(a-2)(a+2)(a^2-2a+4)}{(a+2)(a^2-2a+4)}$ = $a^2(a - 2)$

Hence, the required value of fourth proportional is $a^2(a - 2)$.

(ii) i. Draw a line BC = 4.2 cm.

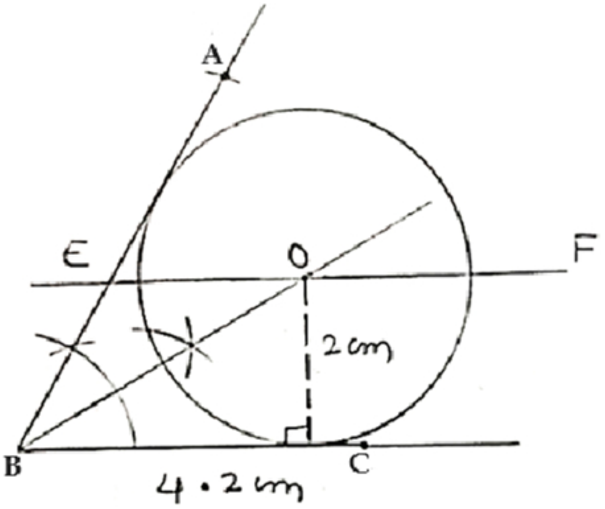
ii. At B, draw an angle of 60° using compass.

iii. Cut an arc of 5 cm on the angle arm at B and name this point as A.

iv. Draw angle bisector of angle ABC.

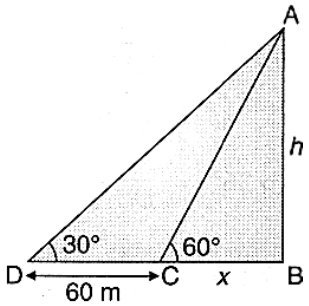
v. Draw a line EF||BC at a distance of 2 cm which cuts the angle bisector at O.

vi. Take O as centre and 2 cm as radius, draw a circle which touches both the arms.of the angle.



(iii)Let the height of the building be h m and D be the position of a man.

Here, BC = x m



Now, In $\triangle$ABC,

tan 60^o = $\frac{AB}{BC}$

$\sqrt{3} = \frac{h}{x}$

x = $\frac{h}{\sqrt{3}}$ ...(i)

In $\triangle$ABD,

tan 30^o = $\frac{AB}{BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{60+x}$

60 + x = $h\sqrt{3}$

$\sqrt{3}h$ = 60 + $\frac{h}{\sqrt{3}}$ [From eq. (i)]

$\frac{\sqrt{3}h}{1} - \frac{h}{\sqrt{3}}$ = 60

$\frac{3h-h}{\sqrt{3}}$ = 60

2h = $60\sqrt{3}$

h = $\frac{60\sqrt{3}}{2}$

$\Rightarrow$ h = $30\sqrt{3}$

= 30 $\times$ 1.732

Height of building = 51.96 m = 52 m (Approx).