ICSE Board
Class X Mathematics
Sample Paper 1

Time: 2½ hrs **Total Marks: 80**

General Instructions:

1. Answers to this paper must be written on the paper provided separately.
2. You will **NOT** be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections. Attempt **all** questions from **Section A** and any **four** questions from **Section B**.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
6. All working, including rough work, must be clearly shown and should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
7. Mathematical tables are provided.

SECTION – A (40 Marks)

*(Answer **all questions** from this Section)*

Q. 1.

(a) Solve and graphically represent the solution set of $2x - 9 < 7$ and $3x + 9 \leq 25$, $x \in R$. [3]

(b) Rahul bought a jacket for Rs. 462 which includes a 20% discount on the listed price and 5% sale tax on the reduced price. Find the list price of the jacket. [3]

(c) Rajinder deposited Rs. 150 per month in a bank for 8 months in a Recurring Deposit account. What will be the maturity value of his deposits, if the rate of interest is 8% p.a. and interest is calculated at the end of every month? [4]

Q. 2.

(a) Find three terms of an A.P. whose sum is 3 and product is −8. [3]

(b) ABCD is a rhombus. The co-ordinates of A and C are (3, 6) and (−1, 2) respectively. Write down the equation of BD. [3]

(c) If $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$, the find $-A^2 + 6A$. [4]

Q. 3.

(a) Prove that $\dfrac{\cos(90^\circ - A)}{\sin A} + \dfrac{\tan B}{\cot(90^\circ - B)} = 2$ [3]

(b) Cards numbered from 1 to 18 are put in a box and mixed thoroughly. One card is drawn at a random. Find the probability that the card drawn bears

i. a prime number

ii. a factor of 18

iii. a number divisible by 2 and 3 [3]

(c) The points A(3, 2), B(0, 4) and C(−4, −3) are the vertices of a triangle.

i. Plot the points on a graph paper

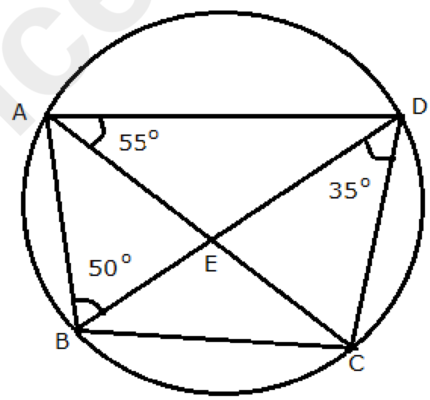
ii. Draw the triangle formed by the reflection of these points at the x-axis

iii. Are the two triangles congruent? [4]

Q. 4.

(a) Determine the value of ‘m’ if (x – m) is a factor of the polynomial, $(x^3 - (m^2 - 1)x + 2)$. Hence find the value of k if 3m + 6 = k. [3]

(b) In the given fig, find ∠CEB and ∠ADB, where E is the point of intersection of chords AC and BD of circle. [3]



(c) If the mean of the distribution is 62.8 and sum of frequencies is 50, find p and q. [4]

Class	Frequency
0-20	5
20-40	P
40-60	10
60-80	Q
80-100	7
100-120	8

SECTION – B

*(Answer **any four questions** from this Section)*

Q. 5.

(a) Solve $2x^2 + 2x - 3 = 0$, giving your answer correct upto one decimal place. [3]

(b) The volume of a right circular cone is 660 cm^3 and the diameter of its base is 12 cm. Calculate

(i) the height of the cone,

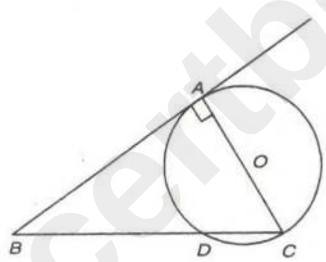
(ii) the slant height of the cone,

(iii) the total surface area of the cone. [4]

(c) If a, b, c, d are in G.P., then show that $(a - b + c)(b + c + d) = ab + bc + cd$. [3]

Q. 6.

(a) In the given figure angle A of the triangle ABC is a right angle. The circle on AC as diameter cuts BC at D. If BD = 9 and DC = 7, calculate the length of AB. [3]



(b) The manufacturer sold a TV to a wholesaler for Rs. 7000. The wholesaler sold it to a trader at a profit of Rs. 1000. If the trader sold it to the customer at a profit of Rs. 1500, find

i. The total VAT collected by the state government at the rate of 5%

ii. The amount that the customer pays for the TV [4]

(c) Prove the identity $\cos^6 A + \sin^6 A = 1 - 3\cos^2 A \sin^2 A$ [3]

Q. 7.

(a) Construct ΔABC, in which AC = 5 cm, BC = 7 cm and AB = 6 cm.

i. Mark D, the mid-point of AB.

ii. Construct the circle which touches BC at C, and passes through D. [3]

(b) Find the equation of the line perpendicular to $5x - 2y = 8$ and which passes through the midpoint of the line segment joining (2, 3) and (4, 5). [3]

(c) The mean of the following frequency table is 50. Find the value of f_1 and f_2. [4]

Class	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	f_1	32	f_2	19	120

Q. 8.

(a) $(x-2)$ is a factor of the expression $x^3 + ax^2 + bx + 6$. When this expression is divided by $(x-3)$, it leaves the remainder 3. Find the values of 'a' and 'b'. [3]

(b) If a, b and c are in continued proportion, then prove that
$(a + b + c)(a - b + c) = a^2 + b^2 + c^2$ [3]

(c) A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height 'h'. At a point on the plane, the angle of elevation of the bottom of the flagstaff is α and that of the top of the flagstaff is β. Prove that the height of the tower is $\frac{h \tan\alpha}{\tan\beta - \tan\alpha}$. [4]

Q. 9.

(a) D is the midpoint of side BC of a ΔABC. AD is bisected at the point E and BE produced cuts AC at the point X. Prove that BE : EX = 3 : 1. [3]

(b) Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, satisfies the equation $A^3 - 4A^2 + A = 0$ [3]

(c) A man invests Rs. 3465 in buying shares of nominal value of Rs. 45 and sells them at a 10% premium. The dividend on the shares is 14% per annum.

i. Calculate the number of shares he buys

ii. Calculate the dividend he receives. [4]

Q. 10.

(a) The marks of 200 students in an exam were recorded as follows:

Marks%	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of students	7	11	20	46	57	37	15	7

Draw the cumulative frequency table and hence, draw the Ogive and use it to find

i. the median

ii. the number of students who score more than 40% marks [6]

(b) A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km per hour more, it would have taken 30 minute less for the journey. Find the original speed of the train. [4]

Q. 11.

(a) Construct a circle inscribed inside a given regular hexagon of side 3 cm. [3]

(b) A copper wire, 4 mm in diameter, is evenly wound about a cylinder whose length is 24 cm and diameter 20 cm so as to cover the whole surface. Find the length and weight of the wire assuming the specific gravity to be 8.8 gm/cm^3. [3]

(c) If $x = \dfrac{\sqrt{a+3b} + \sqrt{a - 3b}}{\sqrt{a+3b} - \sqrt{a - 3b}}$, Prove that: $3b^2 - 2ax + 3b = 0$ [4]

Solution

SECTION – A (40 Marks)

Q.1.

(a) $2x - 9 < 7$ $\qquad$ $3x + 9 \le 25$,

$2x < 16$ $\qquad$ $3x \le 16$

$x < 8$ $\qquad$ $x \le \frac{16}{3}$

Required solution is $x \le \frac{16}{3}$

The solution is represented on the graph as:

(b) Let the listed price of the jacket be Rs. x

Discounted price = $\text{L.P.}\left(1 - \frac{D}{100}\right) = x\left(1 - \frac{20}{100}\right) = \text{Rs. } \frac{4x}{5}$

Sales tax = $\frac{4x}{5} \times \frac{5}{100} = \text{Rs. } \frac{x}{25}$

Cost price of the jacket = $\frac{4x}{5} + \frac{x}{25} = \frac{21x}{25}$

According to the problem,

$\frac{21x}{25} = 462$

$\Rightarrow x = 550$

Hence, the list price of the jacket = Rs. 550

(c) Total price for one month = $\frac{150 \times 8(8 + 1)}{2} = 150 \times 4 \times 9 = \text{Rs. } 5400$

Interest on Rs. 5400 for one month = $\frac{5400 \times 8 \times 1}{12 \times 100} = \text{Rs. } 36$

Maturity value = Rs. $(150 \times 8 + 36)$ = Rs. $(1200 + 36)$ = Rs. 1236

Q.2.

(a) Let a − d, a and a + d be three terms in A.P.

According to the question,

$a - d + a + a + d = 3$

$3a = 3$ or $a = 1$

$(a - d)(a)(a + d) = -8$

$a(a^2 - d^2) = -8$

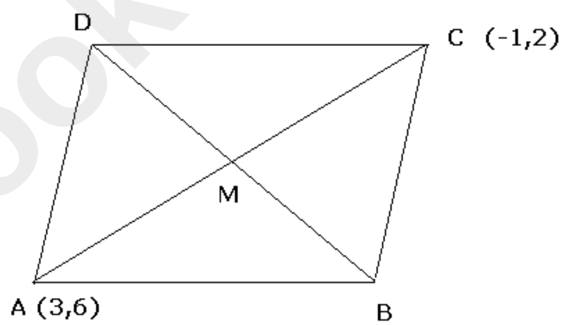
Putting the value of a = 1, we get

$1 - d^2 = -8$

$d^2 = 9$ or $d = \pm 3$

Thus, the required three terms are −2, 1, 4 or 4, 1, −2.

(b) BD will be the perpendicular bisector of AC



Midpoint of AC = $\left(\frac{3-1}{2}, \frac{6+2}{2}\right) = (1, 4)$

Slope of AC = $\frac{y_2 - y_2}{x_2 - x_1} = \frac{2 - 6}{-1 - 3} = \frac{-4}{-4} = 1$

So, slope of BD = −1

BD is passing through the midpoint of AC.

Equation of BD, $y - y_1 = m(x - x_1)$

$\Rightarrow y - 4 = -1(x - 1)$

$\Rightarrow y - 4 = -x + 1$

$\Rightarrow x + y - 5 = 0$

(c) $A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}$

$$A^2 = A \times A = \begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} (2\times 2)+(-2\times -3) & (2\times -2)+(-2\times 4) \\ (-3\times 2)+(4\times -3) & (-3\times -2)+(4\times 4) \end{bmatrix}$$

$$= \begin{bmatrix} 4+6 & -4-8 \\ -6-12 & 6+16 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -12 \\ -18 & 22 \end{bmatrix}$$

$$-A^2 = (-1)\times\begin{bmatrix} 10 & -12 \\ -18 & 22 \end{bmatrix} = \begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix}$$

$$6A = 6\begin{bmatrix} 2 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix}$$

$$-A^2 + 6A = \begin{bmatrix} -10 & 12 \\ 18 & -22 \end{bmatrix} + \begin{bmatrix} 12 & -12 \\ -18 & 24 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Q.3

(a)

$$\text{L.H.S.} = \frac{\cos(90^\circ - A)}{\sin A} + \frac{\tan B}{\cot(90^\circ - B)}$$

$$= \frac{\sin A}{\sin A} + \frac{\tan B}{\tan B} \quad [\because \cos(90^\circ - A) = \sin A \text{ and } \cot(90^\circ - B) = \tan B]$$

$= 1 + 1$

$= 2$

$=$ R.H.S

(b) Total number of cards = 18

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

i. Number of favourable outcomes = 7

(Prime numbers between 1 and 18 are 2, 3, 5, 7, 11, 13, and 17)

$$P(\text{a prime no.}) = \frac{7}{18}$$

ii. Factors of 18 are 1, 2, 3, 6, 9, and 18

Number of favourable outcomes = 6

$$P(\text{a factor of 18}) = \frac{6}{18} = \frac{1}{3}$$

iii. Numbers divisible by 2 and 3 are 6, 12 and 18

Number of favourable outcomes = 3

$$P(\text{a number divisible by 2 and 3}) = \frac{3}{18} = \frac{1}{6}$$

(c) Reflection on the x-axis is represented by

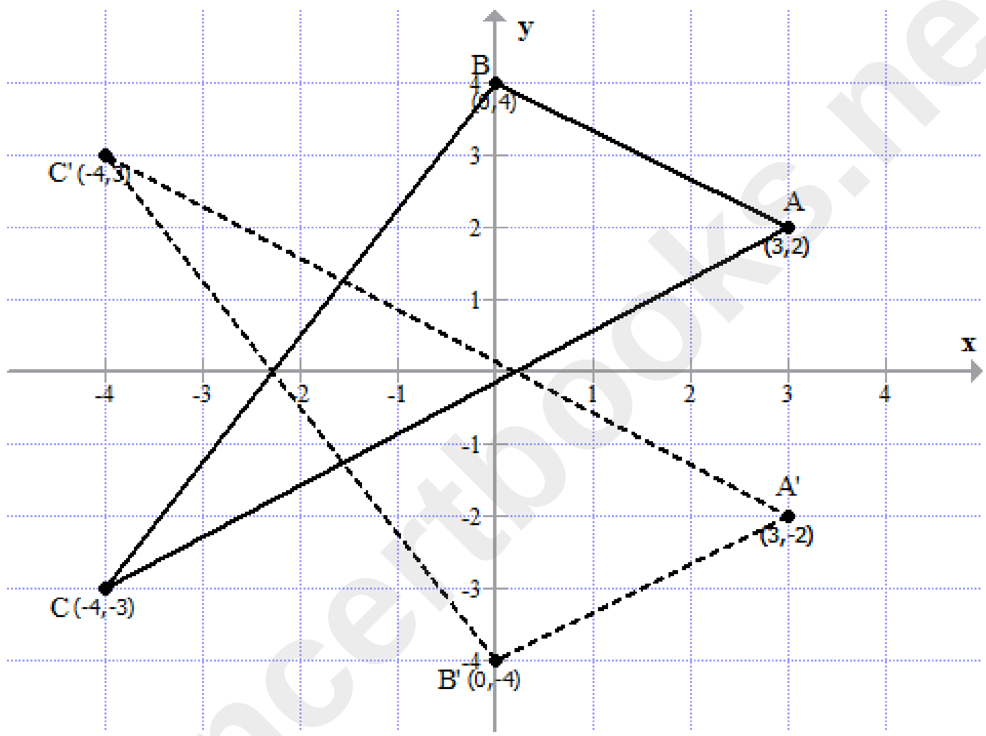
$r : (x, y) \to (x, -y)$

$\therefore$ A(3, 2) $\to$ A'(3, −2)

B(0, 4) $\to$ B'(0, −4)

By measuring the lengths of the sides of ΔABC and ΔA'B'C', we can say that

ΔABC $\cong$ ΔA'B'C [By SSS rule]



Q.4.

(a) (x − m) is a factor of $x^3 - (m^2 - 1)x + 2$

Then for x = m

$m^3 - (m^2 - 1)m + 2 = 0$

$\Rightarrow m^3 - m^3 + m + 2 = 0$

Or m = −2

Now, 3m + 6 = k

$\Rightarrow 3(-2) + 6 = k$

$\Rightarrow k = 0$

$\therefore$ m = −2 and k = 0

(b) $\angle BAC = \angle BDC$

$\therefore\ m\angle BAC = 35°$

In ΔAEB,

$m\angle EAB + m\angle AEB + m\angle EBA = 180°$

$35° + m\angle AEB + 50° = 180°$

$m\angle AEB = 180° - 35° - 50° = 95°$

$m\angle CEB + m\angle AEB = 180°$ [linear pair angles]

$m\angle CEB + 95° = 180°$

$m\angle CEB = 180° - 95° = 85°$

In ΔADB,

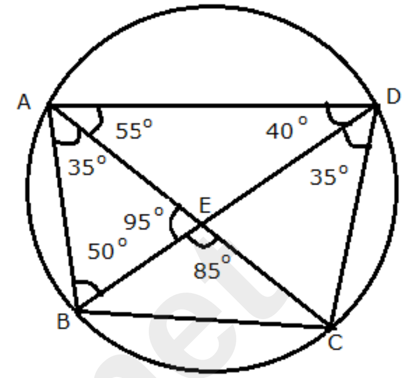
$m\angle ADB + m\angle DAB + m\angle ABD = 180°$

$m\angle ADB + (55° + 35°) + 50° = 180°$

$m\angle ADB + 140° = 180°$

$m\angle ADB = 180° - 140°$

$m\angle ADB = 40°$



(c) Drawing Table and computing values

Class	x	f	fx
0-20	10	5	50
20-40	30	p	30p
40-60	50	10	500
60-80	70	q	70q
80-100	90	7	630
100-120	110	8	880
		50	2060 + 30p + 70q

Now, $5 + p + 10 + q + 7 + 8 = 50$

$\Rightarrow p + q = 50 - 30$

$\Rightarrow p + q = 20$ ------ (1)

$\because \bar{x} = \dfrac{\sum fx}{\sum f}$

$\Rightarrow 62.8 = \dfrac{2060 + 30p + 70q}{50}$

$\Rightarrow 62.8 = \dfrac{10(206 + 3p + 7q)}{50}$

$\Rightarrow 314 = 206 + 3p + 7q$

$\Rightarrow 3p + 7q = 108$ ----- (2)

$\Rightarrow (1) \times 3 \Rightarrow 3p + 3q = 60$ ----- (3)

Subtracting (3) from (2)

$\Rightarrow 4q = 48 \Rightarrow q = 12$

$\Rightarrow p + 12 = 20 \Rightarrow p = 8$ [from (1)]

$\therefore\ p = 8$ and $q = 12$

SECTION – B (40 Marks)

Q. 5.

(a) $2x^2 + 2x - 3 = 0$

Comparing it with $ax^2 + bx + c = 0$,

we get $a = 2$, $b = 2$, $c = -3$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4 \times 2 \times (-3)}}{2 \times 2}$$

$$= \frac{-2 \pm \sqrt{4 + 24}}{4}$$

$$= \frac{-2 \pm \sqrt{28}}{4}$$

$$= \frac{-2 \pm 2\sqrt{7}}{4}$$

$$= \frac{-1 \pm \sqrt{7}}{2}$$

$$= \frac{-1 \pm 2.645}{2}$$

$$= \frac{-3.645}{2}, \frac{1.645}{2}$$

$$= -1.8225, 0.8225$$

$\therefore x = -1.8$ and 0.8

(b) Diameter of a cone, d = 12 cm

$\Rightarrow$ Radius, r = 6 cm

Volume of cone = 660 cm^3

$$V = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow 660 = \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times h$$

$$\Rightarrow h = \frac{660 \times 7}{22 \times 12} cm$$

$\therefore$ h = 17.5 cm

i. Hence, height of the cone = 17.5 cm

ii. $\ell^2 = r^2 + h^2$

$$\Rightarrow \ell^2 = (6)^2 + \left(\frac{35}{2}\right)^2$$

$$\Rightarrow \ell^2 = 36 + \frac{1225}{4}$$

$$\Rightarrow \ell^2 = \frac{144 + 1225}{4}$$

$$\Rightarrow \ell = \sqrt{\frac{1369}{4}}$$

$$\Rightarrow \ell = \frac{37}{2} \text{ cm}$$

$$\Rightarrow \ell = 18.5 \text{ cm}$$

$\therefore$ Slant height = 18.5 cm

iii. Total surface area = $\pi r(r + \ell)$

$$= \frac{22}{7} \times 6(6 + 18.5) cm^2$$

$$= \frac{22}{7} \times 6 \times 24.5 \ cm^2$$

$$= 462 \ cm^2$$

$\therefore$ Total surface area = 462 cm^2

(c) If a, b, c and d are in G.P., we have

$\frac{a}{b}=\frac{b}{c}, \frac{b}{c}=\frac{c}{d}$ and $\frac{a}{b}=\frac{c}{d}$

$\Rightarrow b^2 = ac,\ c^2 = bd$ and $ad = bc$

Now consider the given expression

$(a-b+c)(b+c+d)$

$= ab+ac+ad-b^2-bc-bd+bc+c^2+cd$

$= ab+ac+ad-ac-bc-bd+bc+bd+cd \left[\because b^2 = ac,\ c^2 = bd\right]$

$= ab+ad+cd$

$= ab+bc+cd \left[\because ad=bc\right]$

L.H.S = R.H.S

Hence proved.

Q. 6.

(a) $m\angle BAC = 90°$ (given)

And AC is the diameter

So, AB is the tangent at A.

$AB^2 = BD.BC = BD.(BD + DC)$

$= 9 \times (9 + 7)$

$= 9 \times 16 = 144$

$AB = \sqrt{144} = 12$ cm

(b) Amount of tax collected by the manufacturer

= 5% of Rs. 7,000 = Rs. 350

Since the wholesaler earns a profit of Rs. 1000, the value added by wholesaler

= Rs. 1,000

Amount of VAT to be paid by wholesaler = 5% of Rs. 1,000 = Rs. 50

As the trader earns a profit of Rs. 1,500, the value added by trader = Rs. 1,500

Amount of VAT to be paid by trader = 5% of 1,500 = Rs. 75

$\therefore$ The amount of tax (under VAT) received by the government

= Rs. 350 + Rs. 50 + Rs. 75

= Rs. 475

(c) L.H.S. = $\cos^6 A + \sin^6 A$

$= (\cos^2 A)^3 + (\sin^2 A)^3$

$= (\cos^2 A + \sin^2 A)^3 - 3\cos^2 A \sin^2 A(\cos^2 A + \sin^2 A)$ $[a^3 + b^3 = (a + b)^3 - 3ab\ (a + b)\}$

$= (1)^3 - 3\cos^2 A \sin^2 A\ (1)$ (since $\cos^2 A + \sin^2 A = 1$)

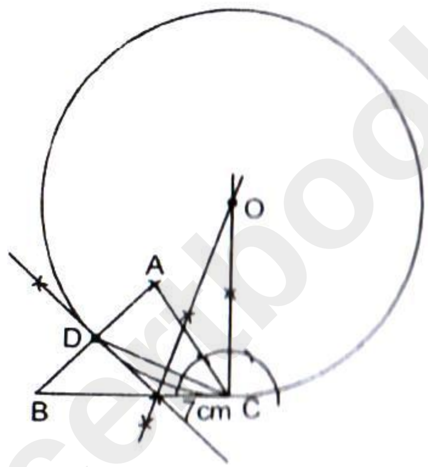
$= 1 - 3\cos^2 A \sin^2 A$

= R.H.S.

Q.7.

(a) Steps of construction

i. Draw BC = 7cm
ii. With B as centre and radius BA = 6 cm, draw an arc
iii. With C as centre and radius CA = 5 cm, draw another arc intersecting the arc of step (ii) at A.
iv. Join AB and AC
v. Draw perpendicular bisector of AB intersecting AB at D. D is the midpoint of AB
vi. Construct perpendicular to BC at C
vii. Draw perpendicular bisector of CD intersecting the perpendicular to BC at O.
viii. With O as centre and OC as radius draw a circle.



(b) Midpoint of line joining (2, 3) and (4, 5)

$$M \equiv \left(\frac{4+2}{2}, \frac{5+3}{2}\right) \Rightarrow M \equiv (3,\ 4)$$

Equation of the given line is $5x - 2y = 8$

$\therefore$ Slope of the given line is $\frac{5}{2}$

Therefore slope of the perpendicular to the line $5x - 2y = 8$ is $-\frac{2}{5}$

Equation of the perpendicular is $y = mx + k$

$\Rightarrow y = -\frac{2}{5}x + k$

$\Rightarrow 2x + 5y - k = 0$

It passes through M(3, 4).

$\therefore\ 2 \times 3 + 5 \times 4 - k = 0$

$\Rightarrow k = 26$

Hence, $2x + 5y - 26 = 0$ is the required equation.

(c)

Class	x_i	f_i	$f_i x_i$
0-20	$\frac{0+20}{2}=10$	17	170
20-40	30	f_1	$30f_2$
40-60	50	32	1600
60-80	70	f_2	$70f_2$
80-100	90	19	1710
		$\sum f_i = 120$	$\sum f_i x_i = 3480 + 30f_1 + 70f_2$

So, Mean $(\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 50 = \frac{3480 + 30f_1 + 70f_2}{120}$

$\Rightarrow 6000 = 3480 + 30f_1 + 70f_2$

$\Rightarrow 30f_1 + 70f_2 = 2520$

$\Rightarrow 3f_1 + 7f_2 = 252$ --- (1)

Also $120 = 17 + f_1 + 32 + f_2 + 19$

$\Rightarrow f_1 + f_2 + 68 = 120$

$\Rightarrow f_1 + f_2 = 52$ ---(2)

On multiplying equation (1) by 1 and equation (2) by 3, and then subtracting the two equations, we get

$(3f_1 + 7f_2 = 252) - (3f_1 + 3f_2 = 156)$

$\Rightarrow 4f_2 = 96$

$\Rightarrow f_2 = 24$

Put $f_2 = 24$ in equation (2), we get $f_1 + f_2 = 52$

$\Rightarrow f_1 + 24 = 52$

$\Rightarrow f_1 = 28$

Q.8.

(a) Since $(x - 2)$ is a factor of $x^3 + ax^2 + bx + 6$,

$\therefore$ For $x = 2$,

$f(2) = (2)^3 + a(2)^2 + b(2) + 6 = 0$

$8 + 4a + 2b + 6 = 0$

$4a + 2b + 14 = 0$

$2a + b + 7 = 0$

$\Rightarrow 2a + b = -7$ ----- (1)

Also, when given expression is divided by $(x - 3)$, the remainder is 3.

Thus for $x = 3$,

$(3)^3 + a(3)^2 + b(3) + 6 = 3$

$27 + 9a + 3b + 6 = 0$

$9a + 3b = -30$

$\Rightarrow 3a + b = -10$ ----- (2)

By solving equation (1) and (2), we get a = -3

By putting the value of 'a' in equation (1), we get

$2(-3) + b = -7$

$\Rightarrow -6 + b = -7$

$\Rightarrow b = -1$

$\therefore a = -3$ and $b = -1$

(b)

If a, b, and c are in continued proportion, then

Put $\frac{a}{b} = \frac{b}{c} = k$

So, $a = bk = ck^2; b = ck$

L.H.S. $= (a + b + c)(a - b + c)$

$= (ck^2 + ck + c)(ck^2 - ck + c)$

$= c(k^2 + k + 1).c(k^2 - k + 1)$

$= c^2(k^2 + k + 1)(k^2 - k + 1)$

$= c^2(k^4 + k^2 + 1)$

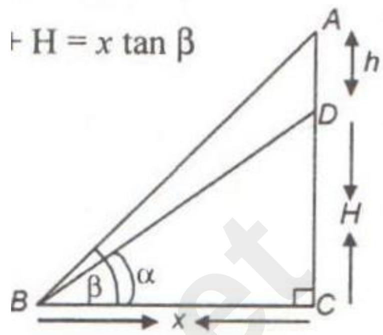
R.H.S. $= a^2 + b^2 + c^2$

$= c^2k^4 + c^2k^2 + c^2$

$= c^2(k^4 + k^2 + 1)$

L.H.S. = R.H.S.

(c) Let BC = x and DC = H



$\frac{H}{x} = \tan\alpha$

$\Rightarrow x = \frac{H}{\tan\alpha}$

$\frac{h + H}{x} = \tan\beta$

$\Rightarrow h + H = x\tan\beta$

$\Rightarrow h + H = \frac{H}{\tan\alpha}\tan\beta$ (Subs. value of x in the above eq.)

$\Rightarrow h\tan\alpha + H\tan\alpha = H\tan\beta$

$\Rightarrow H = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$

Q.9.

(a)

Construction: Draw DG || BX

In ΔBCX, D is the midpoint of BC and DG || BX

$\Rightarrow$ G is the midpoint of CX (converse of midpoint theorem)

$\Rightarrow DG = \frac{1}{2}BX$ (midpoint theorem)

$\Rightarrow BX = 2DG$ .... (1)

Similarly EX || DG in ΔADG, we get $EX = \frac{1}{2}DG$

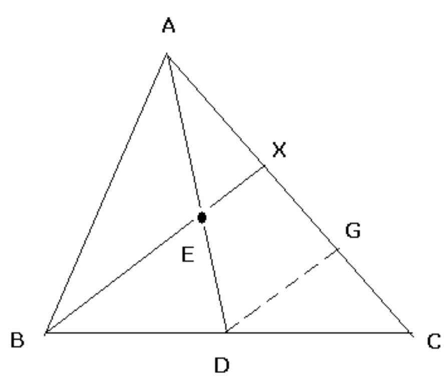
$\Rightarrow 2EX = DG$ .... (2)

Putting DG = 2EX from (2) in (1), we get $BX = 2 \times 2EX$

BX = 4EX or BE + EX = 4EX

$\Rightarrow$ BE = 3EX

$\Rightarrow$ BE : EX = 3 : 1



(b)

$$A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

Now, L.H.S. = $A^3 - 4A^2 + A = 0$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

= 0

= R.H.S.

(c)

i. Nominal value of one share = Rs. 45

Premium = 10%

$\therefore$ Market value of one share

$$= \text{Rs.}\left(45 + \frac{45 \times 10}{100}\right)$$

= Rs.(45 + 4.50) = Rs. 49.50

Investment = Rs. 3,465

$$\therefore \text{No. of shares } = \frac{3465}{49.50} = 70$$

ii. Nominal value of 70 shares = Rs. (70×45) = Rs. 3,150

Rate of dividend = 14%

$\therefore$ Dividend received

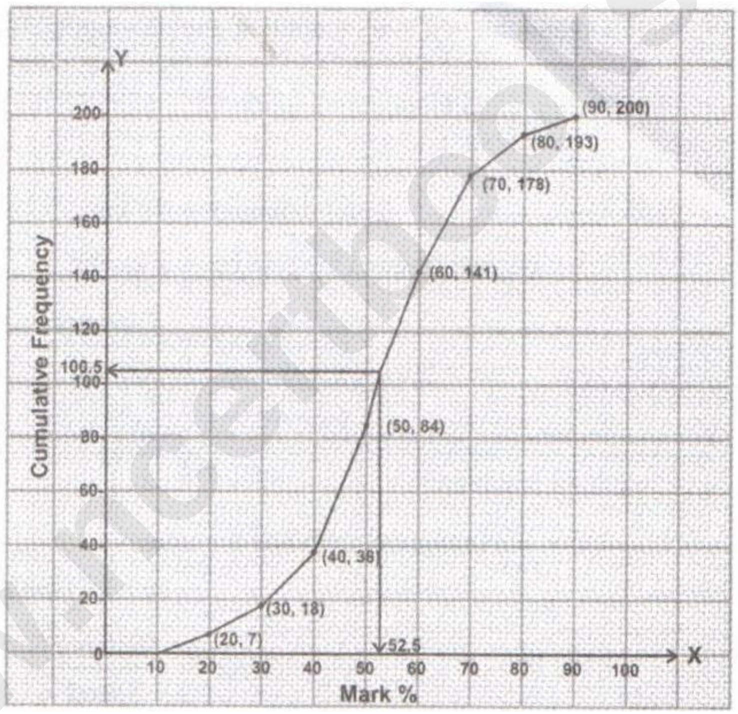
$$= \frac{14}{100} \times 3150$$

= Rs. 441

Q.10.

(a)

Marks %	No. of students	Cumulative (frequency)
10-20	7	7
20-30	11	18
30-40	20	38
40-50	46	84
50-60	57	141
60-70	37	178
70-80	15	193
80-90	7	200



i. Median = $\dfrac{\frac{n}{2}+\left(\frac{n}{2}+1\right)}{2}$ th observation

$= \dfrac{\frac{200}{2}+\left(\frac{200}{2}+1\right)}{2}$ th observation

$= \dfrac{101+100}{2}$ th observation

= 100.5th observation

= 52.5%

ii. Number of students getting more than 40% marks = 200 − 38 = 162.

(b) Total distance = 90 km

Let original speed of the train be = x km/hr

$\therefore$ Time = $\dfrac{\text{Distance}}{\text{speed}} = \dfrac{90}{x}$ hr

Now, the increased speed of the train = (x + 15) km/hr

$\therefore$ Time = $\dfrac{\text{Distance}}{\text{speed}} = \dfrac{90}{x + 15}$ hr

According to the problem

$$\frac{90}{x} - \frac{90}{x + 15} = \frac{1}{2}$$

$$\Rightarrow \frac{90(x + 15) - 90x}{x(x + 15)} = \frac{1}{2}$$

$$\Rightarrow \frac{90x + 1350 - 90x}{x^2 + 15x} = \frac{1}{2}$$

$$\Rightarrow 2 \times 1350 = x^2 + 15x$$

$$\Rightarrow x^2 + 15x - 2700 = 0$$

$$\Rightarrow x^2 + 60x - 45x - 2700 = 0$$

$$\Rightarrow x(x + 60) - 45(x + 60) = 0$$

$$\Rightarrow (x + 60)(x - 45) = 0$$

Either $x + 60 = 0$ or $x - 45 = 0$

$\Rightarrow x = -60$ or $x = 45$

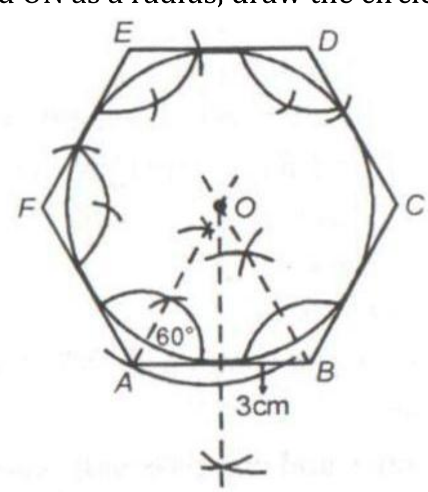
Rejecting $x = -60$ [as speed cannot be negative]

Hence, original speed of the train = 45 km/hr.

Q.11.

(a) Steps of Construction:

i. Draw a hexagon of side 3 cm.
ii. Draw bisectors of any two angles, let these meet at O.
iii. From O, draw $\perp$ on any side, call it ON.
iv. Take O as a centre and ON as a radius, draw the circle.



(b) For wire d = 4 mm = 0.4 cm

Number of round to cover 24 cm = $\frac{24}{0.4} = 60$

For cylinder D = 20 cm

Length of the wire in one round = Circumference of base of cylinder

$= \pi(20)$

$= 20\pi$ cm

Radius of the wire = $\frac{0.4}{2} = 0.2$ cm

Volume of the wire = $\pi(0.2)^2 \times 1200\pi = 48\pi^2 \text{ cm}^3$

Weight of the wire = $48\pi^2 \times 8.88 = 426.24\, \pi^2$ gm

(c) $\frac{x}{1} = \frac{\sqrt{a+3b} + \sqrt{a-3b}}{\sqrt{a+3b} - \sqrt{a-3b}}$,

Using componendo and dividendo

$$\frac{x+1}{x-1} = \frac{\sqrt{a+3b} + \sqrt{a-3b} + (\sqrt{a+3b} - \sqrt{a-3b})}{\sqrt{a+3b} + \sqrt{a-3b} - (\sqrt{a+3b} - \sqrt{a-3b})}$$

$$\Rightarrow \frac{x+1}{x-1} = \frac{2\sqrt{a+3b}}{2\sqrt{a-3b}}$$

$$\Rightarrow \frac{x+1}{x-1} = \sqrt{\frac{a+3b}{a-3b}}$$

On squaring both sides, we get

$$\frac{x^2+2x+1}{x^2-2x+1} = \frac{a+3b}{a-3b}$$

Again using componendo and dividendo

$$\frac{x^2+2x+1+(x^2-2x+1)}{x^2+2x+1-(x^2-2x+1)} = \frac{a+3b+(a-3b)}{a+3b-(a-3b)}$$

$$\frac{2x^2+2}{4x} = \frac{2a}{6b}$$

$$\Rightarrow \frac{x^2+1}{2x} = \frac{a}{3b}$$

$$\Rightarrow 3bx^2 + 3b = 2ax$$

$$\Rightarrow 3bx^2 - 2ax + 3b = 0$$