ICSE 2025 EXAMINATION

Sample Question Paper - 15

Mathematics

Time: 2 ½ hours **Total Marks: 80**

General Instructions:

1. *Answers to this Paper must be written on the paper provided separately.*
2. *You will not be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.*
3. *The time given at the head of this Paper is the time allowed for writing the answers.*
4. *Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
5. *The intended marks for questions or parts of questions are given in brackets [ ]*

Section A

(Attempt all questions from this section.)

Question 1

Choose the correct answers to the questions from the given options. [15]

i) If $A = \begin{bmatrix} 5 & 3 \\ -1 & 2 \end{bmatrix}$, find (A – 2I).

(a) $\begin{bmatrix} 3 & 3 \\ -1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & 3 \\ -1 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 3 \\ -1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}$

ii) Find the value of m if $\frac{2}{3}$ is a solution of the equation $3x^2 + mx + 2 = 0$.

(a) $-2\sqrt{6}$

(b) -5

(c) $-2\sqrt{3}$

(d) -6

iii) A dealer A in Bihar buys an article for Rs. 8000 from a dealer B in Punjab. The dealer A sells this article for Rs. 9000 to a dealer C in the same state. If the rate of GST at each stage is 18%, find the price (including GST) paid by the dealer A.

(a) Rs. 1440

(b) Rs. 9440

(c) Rs. 6560

(d) Rs. 6000

iv) The roots of the equation $3x^2 - 4\sqrt{3}x + 4 = 0$ are

(a) $\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(b) $-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

(d) $-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

v) Which term of the A.P. 1, 4, 7, 10, ... is 58?

(a) 18

(b) 19

(c) 20

(d) 21

vi) If $\frac{x^2 + y^2}{x^2 - y^2} = \frac{17}{8}$ then $x : y =$

(a) 5 : 1

(b) 1 : 5

(c) 3 : 5

(d) 5 : 3

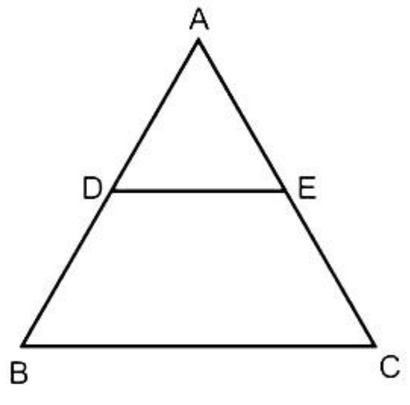
vii) In the given figure, DE || BC and D divides AB in the ratio 1 : 2. Find $\frac{AE}{EC}$.

(a) $\frac{1}{4}$

(b) $\frac{1}{8}$

(c) $\frac{1}{2}$

(d) $\frac{1}{6}$



viii) How many balls each of radius 1 cm can be made by melting a bigger ball whose diameter is 8 cm?

(a) 8
(b) 12
(c) 64
(d) 28

ix) A single letter is selected randomly from the word 'probability'. Find the probability that it is a vowel.

(a) 0
(b) 1
(c) $\frac{6}{11}$
(d) $\frac{4}{11}$

x) The locus of a moving point M equidistant from two points R and S is

(a) the angles bisector of angle R
(b) the angle bisector of angle S
(c) the perpendicular bisector of RS
(d) the segment RS

xi) Find the modal class for the following distribution:

Class interval	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	7	10	4	8	11	5

(a) 30–40
(b) 40–50
(c) 50–60
(d) 60–70

xii) The lengths of a vertical pole and its shadow are in the ratio $\sqrt{3}:1$. The angle of elevation of the sun is

(a) 90^o
(b) 30^o
(c) 45^o
(d) 60^o

xiii) **Statement 1:** When shares are at par, Market value = Nominal value

Statement 2: When shares are at a discount, Market value > Nominal value

Which of the following is valid?

(a) Both the statements are true.

(b) Both the statements are false.

(c) Statement 1 is true, and Statement 2 is false.

(d) Statement 1 is false, and Statement 2 is true.

xiv) Which of the following point lies on the line represented by equation $3x + 5y = 2$?

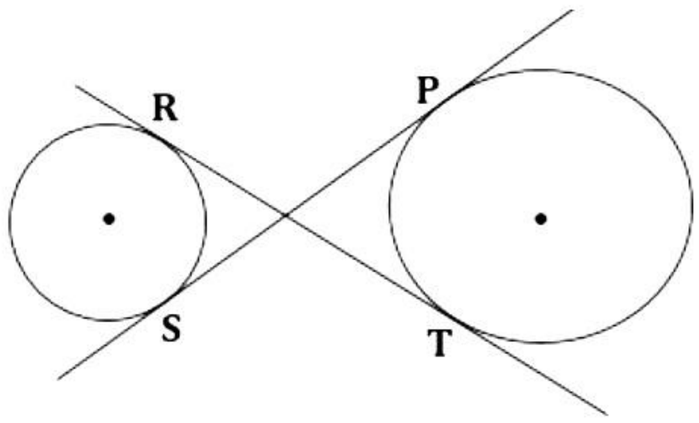
(a) $(-4, -2)$

(b) $(4, 2)$

(c) $(-4, 2)$

(d) $(4, -2)$

xv) **Assertion (A):** Tangent RT > Tangent PS



Reason (R): Two transverse common tangents are always equal.

(a) A is true, R is false

(b) A is false, R is true

(c) Both A and R are true, and R is the correct reason for A.

(d) Both A and R are true, and R is the incorrect reason for A.

Question 2

i) A right circular cone is 3.6 cm high and the radius of its base is 1.6 cm. It is melted and recast into a right circular cone with the radius of its base as 1.2 cm. Find its height. [4]

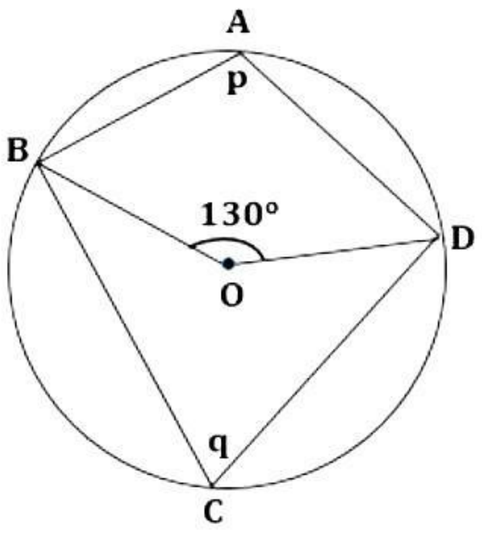
ii) Mr. Ratan gets Rs. 12132 as a maturity amount at the end of three years at the rate of 8% per annum in a recurring deposit account. Find his monthly instalment. [4]

iii) Prove that $\sin^2 A + \cos^2 A = 1$ in right-angled triangle ABC. Also prove that $1 + \tan^2 A = \sec^2 A$. [4]

Question 3

i) In a first round of cricket tournament, the ratio of the number of schools from state A to the number of schools from state B was 5 : 29. In the second round, 3 schools from state A and 20 schools from state B were disqualified. The ratio between the number of schools from state A to the number of schools from state B is now 37 : 212. Find the number of schools from state A and state B before second round. [4]

ii) In the figure, O is the centre of the circle. Find the values of p and q. [4]



iii) The marks obtained by 100 students in a Mathematics test are given below: [5]

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	7	12	17	23	14	9	6	5	4

Draw an ogive for the given distribution on a graph sheet.
Use a scale of 2 cm = 10 units on both axes.
Use the ogive to estimate the

a) Median
b) Lower quartile
c) Number of students who obtained more than 85% marks in the test.
d) Number of students who did not pass in the test if the pass percentage was 35. [5]

Section B

(Attempt any four questions from this Section.)

Question 4

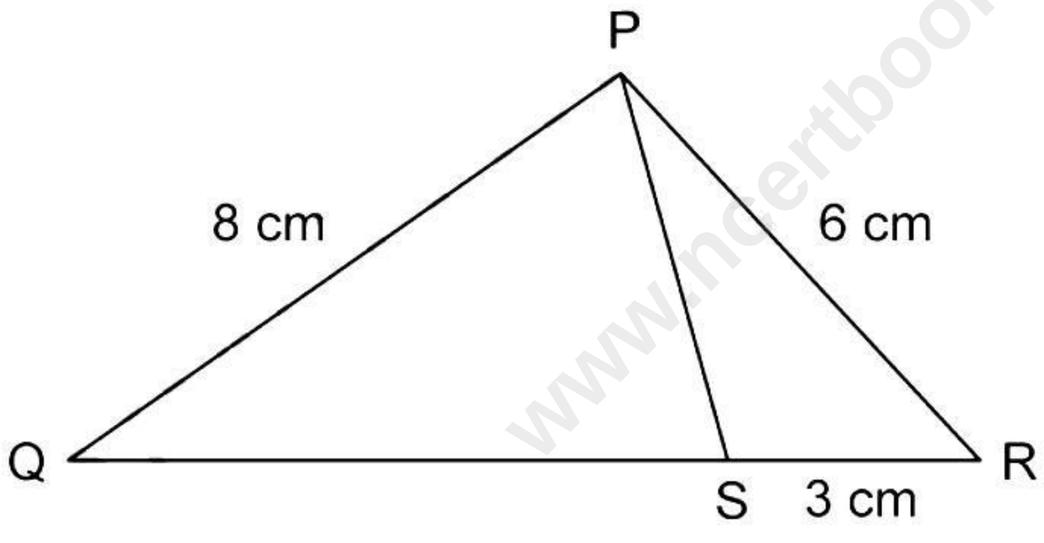
i) If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ and $A^2 - 5B^2 = 5C$. Find the matrix C where C is a 2 by 2 matrix. [3]

ii) One root of the quadratic equation $8x^2 + mx + 15 = 0$ is $\frac{3}{4}$. Find the value of m. Also, find the other root of the equation. [3]

iii) PQR is a triangle. S is a point on the side QR of ΔPQR such that $\angle PSR = \angle QPR$. Given QP = 8 cm, PR = 6 cm and SR = 3 cm. [4]

A. Prove that $\Delta PQR \sim \Delta SPR$.

B. Find the lengths of QR and PS.



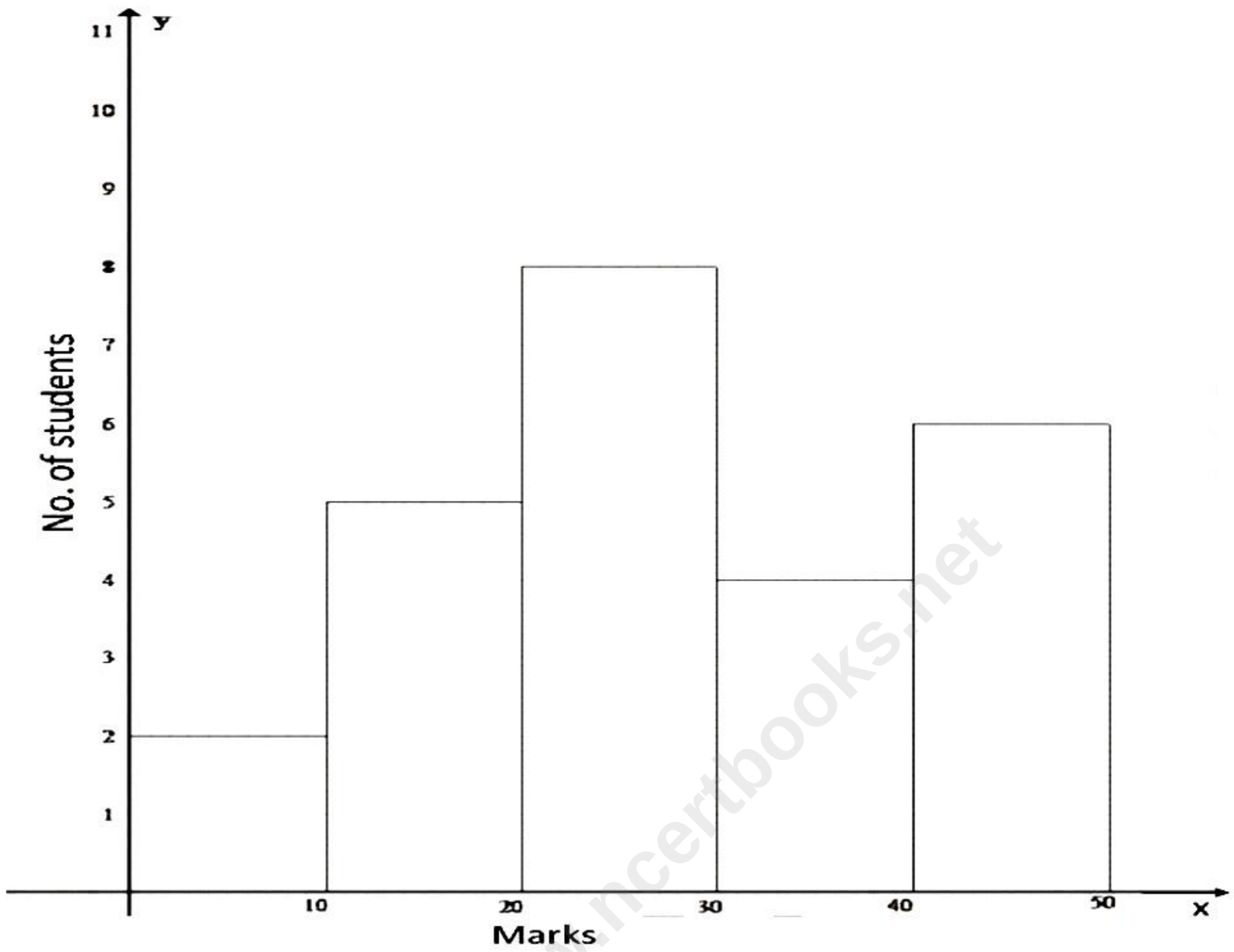
Question 5

i) The histogram below represents the scores obtained by 25 students in a Mathematics mental test. Use the data to: [3]

A. Frame a frequency distribution table.

B. Calculate mean.

C. Determine the Modal class.



ii) A computer mechanic in Delhi charges repairing cost from five different persons A, B, C, D and E with certain discounts. The repairing costs and the corresponding discounts are as given below: [3]

Name of the person	A	B	C	D	E
Repairing cost (in Rs.)	5500	6250	4800	7200	3500
Discount %	30	40	30	20	40

If the rate of GST is 18%, find the total money (including GST) received by the mechanic.

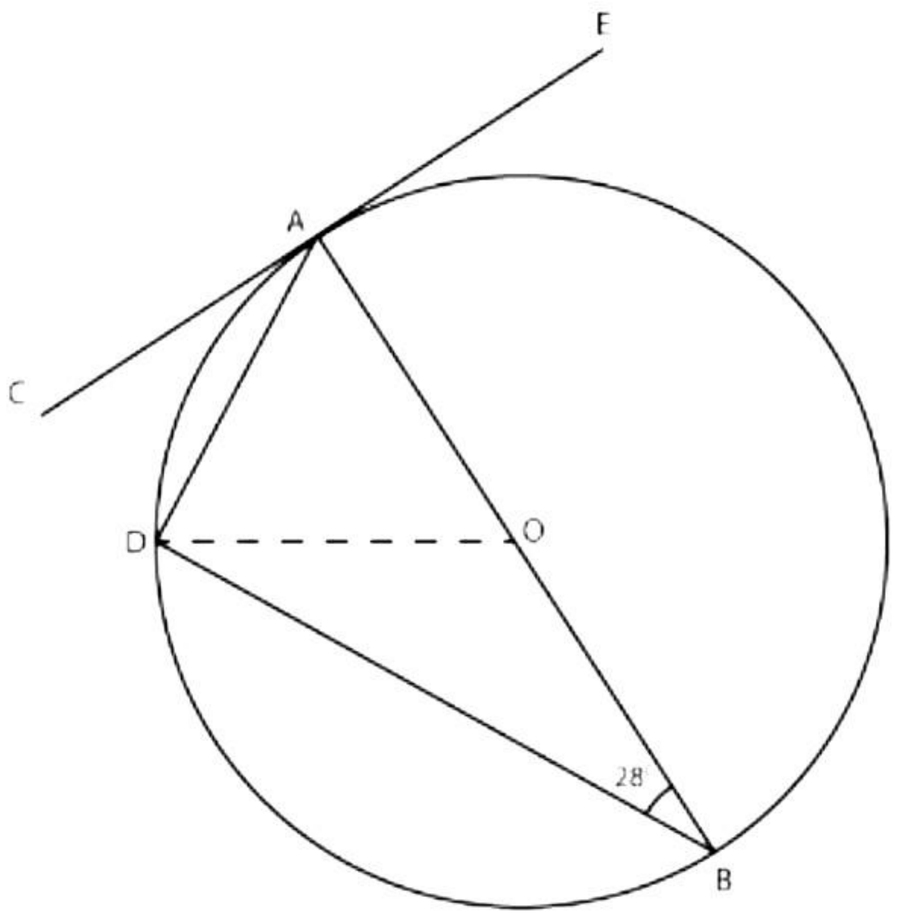
iii) In the given figure, O is the centre of the circle. CE is a tangent to the circle at A. If $\angle ABD = 28^{o}$, then find [4]

A. $\angle BDA$

B. $\angle BAD$

C. $\angle CAD$

D. $\angle ODB$



Question 6

i) How many terms of the GP $\frac{2}{9}, -\frac{1}{3}, \frac{1}{2}, \ldots$ must be added to get the sum equal to $-\frac{133}{144}$?

[3]

ii) The weight of 50 workers is given below: [3]

Weight in kg	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100	100 – 110	110 – 120
No. of workers	4	7	11	14	6	5	3

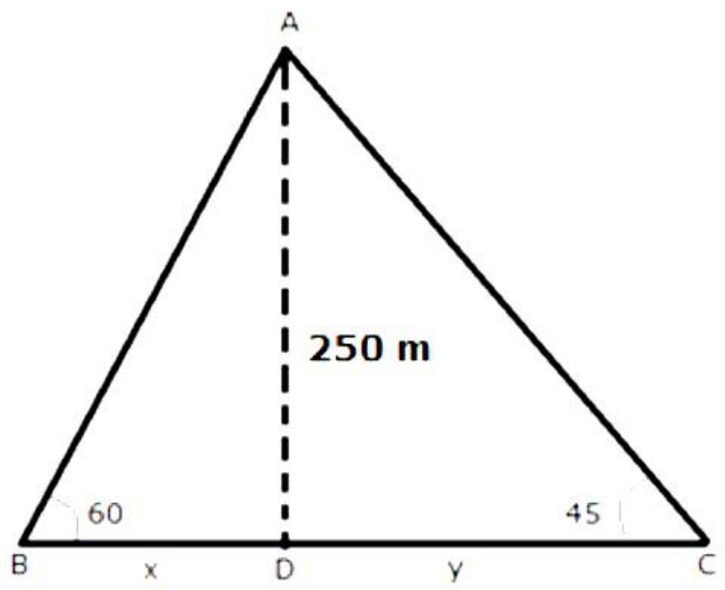
Draw an ogive of the given distribution using a graph sheet. Take 2 cm = 10 kg on one axis and 2 cm = 5 workers along the other axis. Use the ogive drawn to estimate the upper and lower quartiles.

iii) A metal pipe has a bore (inner diameter) of 5 cm. The pipe is 5 mm thick all round. Find the weight, in kilogram, of 2 metres of the pipe if 1 cm^3 of the metal weights 7.7 g. [4]

Question 7

i) P(7, 6), Q(−5, −4) and R(3, −10) are the vertices of a triangle PQR. Find [5]
(a) the slope of the altitude of PQ.
(b) the slope of the median PM.
(c) the slope of the line parallel to PR.
(d) the slope of the line parallel to QR.

ii) An aeroplane, at an altitude of 250 m, observes the angles of depression of two boats on the opposite banks of a river to be 45° and 60° respectively. Find the width of the river. Write the answer correct to the nearest whole number. [5]



Question 8

i) Solve the following inequation and represent the solution set on a number line.

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \leq 7\frac{1}{2},\ x \in I$$ [3]

ii) Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle. [3]

iii) If P(−b, 9a − 2) divides the line segment joining the points A(−3, 3a + 1) and B(5, 8a) in the ratio 3:1, find the values of a and b. [4]

Question 9

i) A school has 630 students. The ratio of the number of boys to the number of girls is 3:2. This ratio changes to 7:5 after the admission of 90 new students. Find the number of newly admitted boys. [3]

ii) Rs. 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received Rs. 100 more. Find the original number of children. [3]

iii) Using ruler and compass only, construct a ΔABC such that BC = 5 cm, AB = 6.5 cm and ∠ABC = 120°. [4]

(a) Construct a circumcircle of ΔABC.

(b) Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC.

Question 10

i) What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor? [3]

ii) Sixteen cards are labelled as a, b, c, … , m, n, o, p. They are put in a box and shuffled. A boy is asked to draw a card from the box. What is the probability that the card drawn is [3]

a. a vowel?
b. a consonant?
c. none of the letters of the word median?

iii) Use graph paper for this question. [4]
(Take 2 cm = 1 unit along both x-axis and y-axis.)
Plot the points O(0, 0), A(−4, 4), B(−3, 0) and C(0, −3).
Reflect points A and B on the y-axis and name them A' and B' respectively. Write down their co-ordinates.

Solution

Section A

Question 1

i) Correct Option: (a)

Explanation:

Given:

$$A=\begin{bmatrix}5 & 3\\ -1 & 2\end{bmatrix},$$

$$A-2I=\begin{bmatrix}5 & 3\\ -1 & 2\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}5-2 & 3\\ -1 & 2-2\end{bmatrix}=\begin{bmatrix}3 & 3\\ -1 & 0\end{bmatrix}$$

ii) Correct option: (b)

Explanation:

As $\frac{2}{3}$ satisfies the equation $3x^2 + mx + 2 = 0$

$$\Rightarrow 3\left(\frac{2}{3}\right)^2+m\left(\frac{2}{3}\right)+2=0$$

$$\Rightarrow \frac{4}{3}+2+\frac{2m}{3}=0$$

$$\Rightarrow \frac{2m}{3}=-\frac{10}{3}$$

$$\Rightarrow m=-5$$

iii) Correct Option: (b)

Explanation:

For dealer A,

C.P. = Rs. 8000, GST = 18% of 8000 = Rs. 1440

The price (including GST) paid by the dealer A

= C.P. + GST = 8000 + 1440 = Rs. 9440

iv) Correct Option: (a)

Explanation:

$3x^2 - 4\sqrt{3}x + 4 = 0$

$\Rightarrow 3x^2 - 2\sqrt{3}x - 2\sqrt{3}x + 4 = 0$

$\Rightarrow \sqrt{3}x\left(\sqrt{3}x - 2\right) - 2\left(\sqrt{3}x - 2\right) = 0$

$\Rightarrow \left(\sqrt{3}x - 2\right)\left(\sqrt{3}x - 2\right) = 0$

$\Rightarrow x = \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

v) Correct option: (c)

Explanation:

Here, a = 1, d = 3 and let the n^{th} term be 58.

$t_n = a + (n - 1)d$

$\Rightarrow 58 = 1 + (n - 1) \times 3$

$\Rightarrow (n - 1) \times 3 = 57$

$\Rightarrow n - 1 = 19$

$\Rightarrow n = 20$

vi) Correct Option: (d)

Explanation:

$\frac{x^2 + y^2}{x^2 - y^2} = \frac{17}{8}$

Applying componendo-dividendo,

$\frac{x^2 + y^2 + x^2 - y^2}{x^2 + y^2 - x^2 + y^2} = \frac{17 + 8}{17 - 8}$

$\Rightarrow \frac{2x^2}{2y^2} = \frac{25}{9}$

$\Rightarrow \frac{x^2}{y^2} = \frac{25}{9}$

$\Rightarrow \frac{x}{y} = \frac{5}{3}$

$\Rightarrow x : y = 5 : 3$

vii) Correct option: (c)

Explanation:

A line drawn parallel to one side of a triangle divides the other two sides proportionally.

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ ....(1)

Also, given that D divides AB in the ratio 1 : 2.

i.e. $\frac{AD}{DB}=\frac{1}{2}$ ....(2)

From (1) and (2)

$\therefore \frac{AE}{EC}=\frac{1}{2}$

viii) Correct Option: (c)

Explanation:

Diameter of bigger ball = 8 cm

⇒ Radius of bigger ball = 4 cm

Then, volume of bigger ball

$=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\times4\times4\times4=\frac{256\pi}{3}cm^3$

Radius of each small ball = 1 cm

Then, volume of each small ball

$=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\times1\times1\times1=\frac{4\pi}{3}cm^3$

Therefore, number of balls $=\frac{\frac{256\pi}{3}}{\frac{4\pi}{3}}=\frac{256\pi}{3}\times\frac{3}{4\pi}=64$

ix) Correct Option: (d)

Explanation:

Possible outcomes = S = {p, r, o, b, a, b, i, l, i, t, y}

n(S) = 11

Event of selection of vowels = E = {o, a, i, i}

n(E) = 4

Probability of selection of a vowel = $P(S)=\frac{n(E)}{n(S)}=\frac{4}{11}$

x) Correct Option: (c)

Explanation:

The locus of a moving point M, equidistant from two points R and S is the perpendicular bisector of RS.

xi) Correct option: (d)

Explanation:

The class with the highest frequency is called the Modal class.

xii) Correct option: (d)

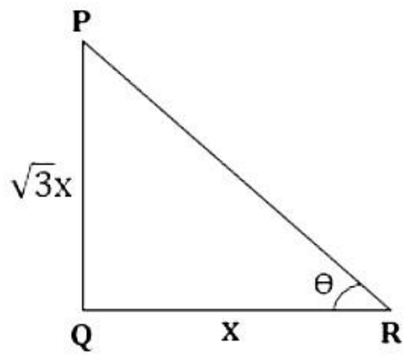
Explanation:

Let PQ be the pole and QR be its shadow.

$$\tan\theta = \frac{PQ}{QR} = \frac{\sqrt{3}x}{x} = \sqrt{3}$$

$$\Rightarrow \tan\theta = \tan 60^\circ$$

$$\Rightarrow \theta = 60^\circ$$



xiii) Correct Option: (c)

Explanation:

The statement given in Statement 1 is correct. Hence, Statement 1 is true.

When shares are at a discount, Market value < Nominal value.

Hence, Statement 2 is false.

xiv) Correct Option: (d)

Explanation:

Any point, which satisfies the equation of a line, lies on that line.

Given equation is $3x + 5y = 2$

Substituting $(4, -2)$ in given equation,

L.H.S. $= 3(4) + 5(-2) = 12 - 10 = 2 =$ R.H.S.

xv) Correct Option: (b)

Explanation:

The statement given in reason is correct and hence, reason is true.

RT and PS are two common transverse tangents.

$\Rightarrow$ Tangent RT = Tangent PS.

Hence, assertion is false.

Question 2

i) For first cone, height, $h_1 = 3.6$ cm and radius, $r_1 = 1.6$ cm

Then, Volume of the cone $= \frac{1}{3}\pi r_1^2 h_1$

$$= \frac{1}{3}\pi \times (1.6)^2 \times 3.6 \text{ cm}^3$$

For second cone, height $= h_2$, $r_2 = 1.2$ cm

Then, Volume of the cone $= \frac{1}{3}\pi r_2^2 h_2$

$$= \frac{1}{3}\pi \times (1.2)^2 \times h_2$$

According to the question,

Volume of the first cone = Volume of the second cone

$$\frac{1}{3}\pi\times(1.6)^2\times3.6=\frac{1}{3}\pi\times(1.2)^2\times h_2$$

$$\Rightarrow h_2=\frac{1.6\times1.6\times3.6}{1.2\times1.2}=6.4\text{ cm}$$

Therefore, the height of the second cone is 6.4 cm.

ii) Let the monthly instalment = Rs. P

n = 3 years = 36 months

r = 8%

Maturity value = Total sum deposited + Interest on it

$$\Rightarrow \text{Interest}=P\times n+P\times\frac{n(n+1)}{2\times12}\times\frac{r}{100}$$

$$\Rightarrow \text{Interest}=P\times n\left[1+\frac{(n+1)}{2\times12}\times\frac{r}{100}\right]$$

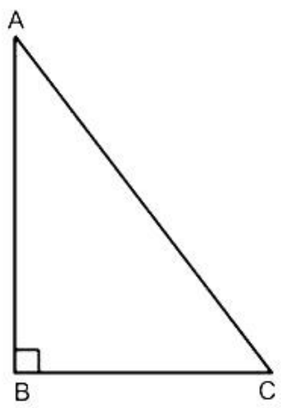
$$\Rightarrow 12132=P\times36\left[1+\frac{37}{2\times12}\times\frac{8}{100}\right]$$

$$\Rightarrow 337=P\times\frac{337}{300}$$

$$\Rightarrow P=300$$

Therefore, the monthly instalment by Mr. Ratan is Rs. 300.

iii) Let us take a right-angled triangle ABC, right angled at B.



Here, $\sin A=\frac{\text{Opposite side}}{\text{Hypotenuse}}=\frac{BC}{AC}$ and $\cos A=\frac{\text{Adjacent side}}{\text{Hypotenuse}}=\frac{AB}{AC}$

Consider

$$\sin^2 A+\cos^2 A$$

$$=\frac{BC^2}{AC^2}+\frac{AB^2}{AC^2}$$

$$=\frac{BC^2+AB^2}{AC^2}$$

$$=\frac{AC^2}{AC^2}=1$$

Hence, $\sin^2 A+\cos^2 A=1$

Also, to prove $1+\tan^2 A=\sec^2 A$

Consider

$$\text{L. H. S.} = 1 + \tan^2 A$$

$$= 1 + \left(\frac{\sin A}{\cos A}\right)^2$$

$$= 1 + \left(\frac{\text{Opposite side}}{\text{Adjacent side}}\right)^2$$

$$= 1 + \left(\frac{BC}{AB}\right)^2$$

$$= 1 + \frac{BC^2}{AB^2}$$

$$= \frac{AB^2 + BC^2}{AB^2}$$

$$= \frac{AC^2}{AB^2}$$

$$= \left(\frac{AC}{AB}\right)^2$$

$$= \left(\frac{1}{\cos A}\right)^2$$

$$= \sec^2 A$$

Hence, $1 + \tan^2 A = \sec^2 A$.

Question 3

i) In the first round,

Number of schools from state A = 5x

Number of schools from state B = 29x

In the second round,

Number of schools from state A = 5x – 3

Number of schools from state B = 29x – 20

Hence,

$$\frac{5x-3}{29x-20} = \frac{37}{212}$$

$$\Rightarrow 212(5x-3) = 37(29x-20)$$

$$\Rightarrow 1060x - 636 = 1073x - 740$$

$$\Rightarrow 1073x - 1060x = 740 - 636$$

$$\Rightarrow 13x = 104$$

$$\Rightarrow x = 8$$

$$\Rightarrow 5x = 5 \times 8 = 40 \text{ and } 29x = 29 \times 8 = 232$$

Therefore, the number of schools from state A and state B before second round are 40 and 232 respectively.

ii) Angle at the centre is double the angle at the circumference, subtended by the same arc.

$$\therefore \angle BCD = \frac{1}{2}\angle BOD$$

$$\Rightarrow q = \frac{1}{2} \times 130° = 65°$$

Now, ABCD is a cyclic quadrilateral.

And, opposite angles of a cyclic quadrilateral are supplementary.

Then, $p + q = 180°$

$\Rightarrow p + 65^o = 180°$

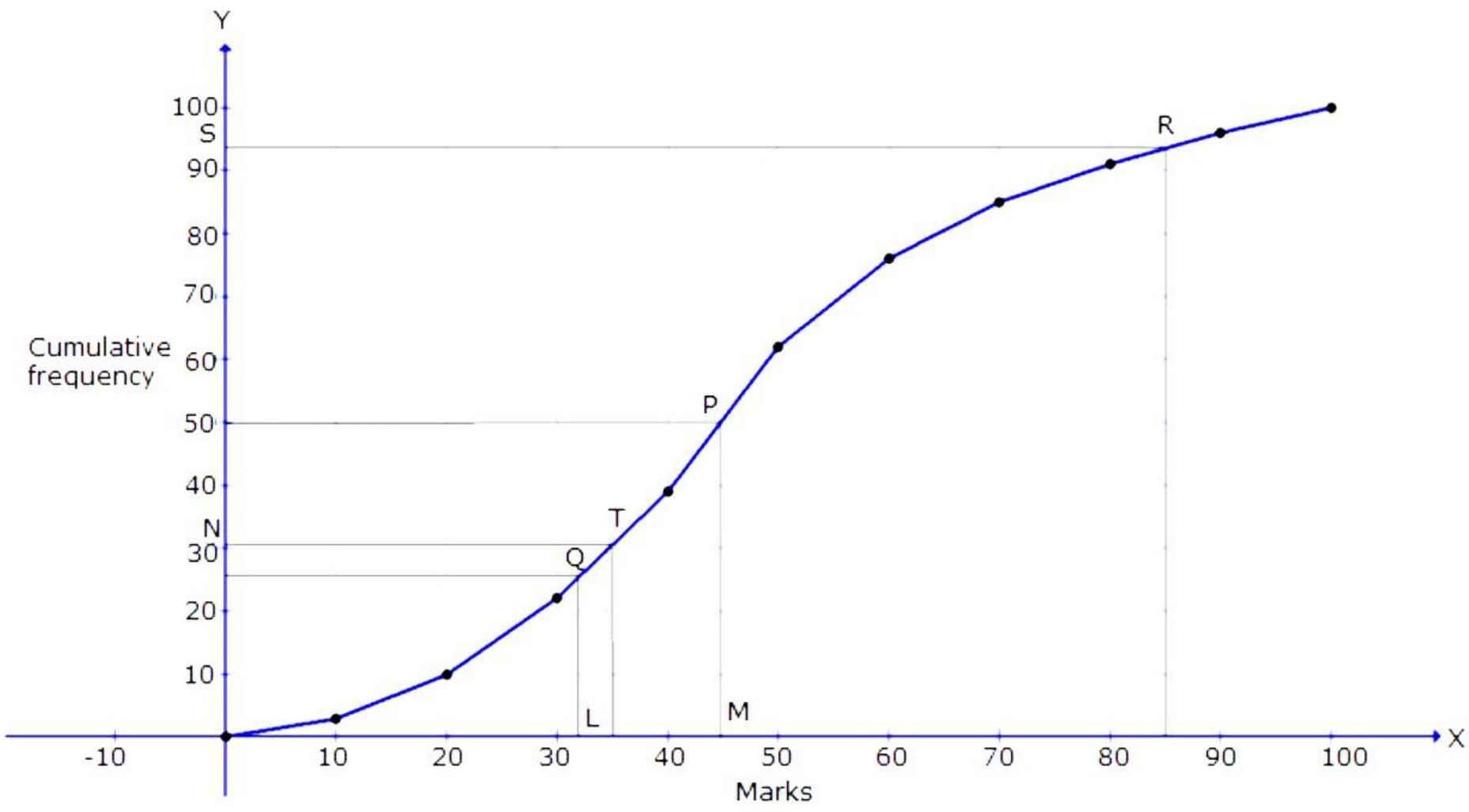
$\Rightarrow p = 115^o$

Therefore, $p = 115°$ and $q = 65°$.

iii) Draw the cumulative frequency table.

Marks	Number of Students (Frequency)	Cumulative Frequency
0-10	3	3
10-20	7	10
20-30	12	22
30-40	17	39
40-50	23	62
50-60	14	76
60-70	9	85
70-80	6	91
80-90	5	96
90-100	4	100

Plotting the points (10, 3), (20, 10), (30, 22), (40, 39), (50, 62), (60, 76), (70, 85), (80, 91), (90, 96) and (100, 100)



Scale: On x-axis, 1 unit = 10 marks, On y-axis, 1 unit = 10 students

a) Median = $\left(\frac{N}{2}\right)^{th}$ term $= \left(\frac{100}{2}\right)^{th}$ term $= 50^{th}$ term

Through mark 50 on the y-axis, draw a horizontal line which meets the curve at point P. Through point P, on the curve, draw a vertical line which meets x-axis at point M. The value of point M on x-axis is the median.
Thus, median = 45

b) Lower quartile = $\left(\frac{N}{4}\right)^{th}$ term $= \left(\frac{100}{4}\right)^{th}$ term $= 25^{th}$ term

Through mark 25 on the y-axis, draw a horizontal line which meets the curve at point Q. Through point Q, on the curve, draw a vertical line which meets x-axis at point L. The value of point L on x-axis is the median.
Thus, lower quartile = 32

c) Through 85 on the x-axis, draw a vertical line which meets the curve at point R. Through point R, on the curve, draw a horizontal line which meets y-axis at point S.
The point where this line touches the y-axis is the number of students who obtained less than 85% marks = 93
Thus, number of students who obtained more than 85% marks =100 – 93 = 7

d) Through 35 on the x-axis, draw a vertical line which meets the curve at point T. Through point T, on the curve, draw a horizontal line which meets y-axis at point N.
The point where this line touches the y-axis is the number of students who obtained less than 35% marks = 31

Section B

Question 4

i)

$$A^2 = A \times A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1\times 1 + 3\times 3 & 1\times 3 + 3\times 4 \\ 3\times 1 + 4\times 3 & 3\times 3 + 4\times 4 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -2\times -2 + 1\times -3 & -2\times 1 + 1\times 2 \\ -3\times -2 + 2\times -3 & -3\times 1 + 2\times 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Given: $A^2 - 5B^2 = 5C$

$$\Rightarrow \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 5C$$

$$\Rightarrow \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 5C$$

$$\Rightarrow \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = 5C$$

$$\Rightarrow 5\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = 5C$$

$$\Rightarrow C = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

ii) Given quadratic equation is $8x^2 + mx + 15 = 0$

One of the roots of the given equation is $\frac{3}{4}$, so it satisfies the given equation.

$$\Rightarrow 8\left(\frac{3}{4}\right)^2 + m\left(\frac{3}{4}\right) + 15 = 0$$

$$\Rightarrow \frac{9}{2} + 15 + m\left(\frac{3}{4}\right) = 0$$

$$\Rightarrow m\left(\frac{3}{4}\right) = -\frac{39}{2}$$

$$\Rightarrow m = -26$$

So, the given equation becomes $8x^2 - 26x + 15 = 0$

$$\Rightarrow 8x^2 - 20x - 6x + 15 = 0$$

$$\Rightarrow 4x(2x-5) - 3(2x-5) = 0$$

$$\Rightarrow (4x-3)(2x-5) = 0$$

$$\Rightarrow x = \frac{3}{4}, \frac{5}{2}$$

Hence, the other root is $\frac{5}{2}$.

iii)

A. In ΔPQR and ΔSPR,

$\angle QPR = \angle PSR$ ... (given)

$\angle PRQ = \angle SRP$ ... (common angle)

$\Rightarrow \Delta PQR \sim \Delta SPR$ ... (AA Test)

B. Since $\Delta PQR \sim \Delta SPR$ [from (i)]

$$\Rightarrow \frac{PQ}{SP} = \frac{QR}{PR} = \frac{PR}{SR} \quad \text{.... (a)}$$

Now, $\frac{QR}{PR} = \frac{PR}{SR}$

$$\Rightarrow \frac{QR}{6} = \frac{6}{3}$$

$$\Rightarrow QR = \frac{6 \times 6}{3} = 12 \text{ cm}$$

Also,

$$\frac{PQ}{SP} = \frac{PR}{SR} \quad \text{... from (a)}$$

$$\Rightarrow \frac{8}{SP} = \frac{6}{3}$$

$$\Rightarrow SP = \frac{8 \times 3}{6} = 4\,\text{cm}$$

Question 5

i)

A. The frequency distribution table is as follows:

Class interval	Frequency
0 – 10	2
10 – 20	5
20 – 30	8
30 – 40	4
40 – 50	6

B.

Class interval	Frequency (f)	Mean value (x)	fx
0 – 10	2	5	10
10 – 20	5	15	75
20 – 30	8	25	200
30 – 40	4	35	140
40 – 50	6	45	270
	$\Sigma f = 25$		$\Sigma f = 695$

$$\therefore \text{Mean} = \frac{\sum fx}{\sum f} = \frac{695}{25} = 27.8$$

C. Here the maximum frequency is 8 which is corresponding to class 20 – 30.
Hence, the modal class is 20 – 30.

ii)

Name of the person	Repairing cost (in Rs.)	Discount %	Discount	Selling price
A	5500	30	1650	3850
B	6250	40	2500	3750
C	4800	30	1440	3360
D	7200	20	1440	5760
E	3500	40	1400	2100
Total				18,820

$$\text{GST} = \frac{18}{100} \times 18820 = \text{Rs. } 3387.60$$

Therefore, total money (including GST) received by the mechanic
= Rs. (18,820 + 3387.60)
= Rs. 22,207.60

iii)

(a) O is the centre of the circle and AB is its diameter.
Now, $\angle BDA$ is an angle made by the semicircle.
$\Rightarrow \angle BDA = 90^o$

(b) In ΔBAD,
$\angle ABD + \angle BAD + \angle BDA = 180^o$
$\Rightarrow 28^o + \angle BAD + 90^o = 180^o$
$\Rightarrow \angle BAD = 62^o$

(c) $\angle CAD$ and $\angle ABD$ are the angles by the same chord but in different segments.
Then, by Alternate segment theorem, we have
$\angle CAD = \angle ABD = 28^o$

(d) In ΔODB,
$OD = OB$ ... (radii of the same circle)
$\Rightarrow \angle OBD = \angle ODB$ ... (Opposite angles of equal sides are also equal)
$\Rightarrow \angle ODB = 28^o$

Question 6

i) Here, $a = \frac{2}{9}$, $r = \frac{-1/3}{2/9} = -\frac{3}{2}$ and $S_n = -\frac{133}{144}$

Clearly, $|r| = \left|-\frac{3}{2}\right| = \frac{3}{2} \Rightarrow |r| > 1$

Therefore,

$$\therefore S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$\Rightarrow -\frac{133}{144} = \frac{\frac{2}{9}\left(\left(\frac{-3}{2}\right)^n - 1\right)}{-\frac{3}{2} - 1}$$

$$\Rightarrow -\frac{133}{144} = \frac{\frac{2}{9}\left(\left(\frac{-3}{2}\right)^n - 1\right)}{-\frac{5}{2}}$$

$$\Rightarrow -\frac{133}{144} \times \left(-\frac{5}{2}\right) \times \frac{9}{2} = \left(\frac{-3}{2}\right)^n - 1$$

$$\Rightarrow \frac{665}{64} = \left(\frac{-3}{2}\right)^n - 1$$

$$\Rightarrow \left(\frac{-3}{2}\right)^n = \frac{665}{64} + 1 = \frac{729}{64}$$

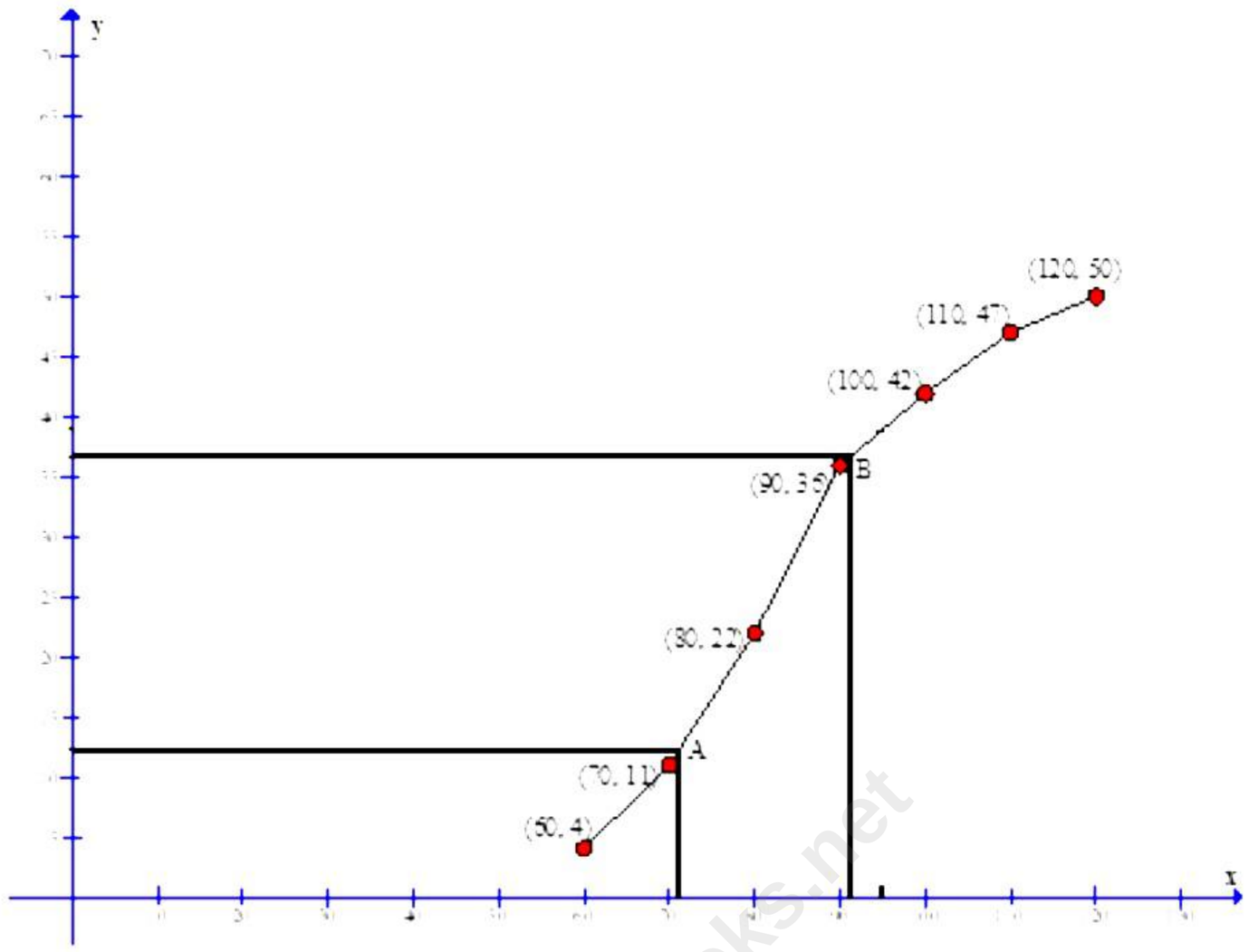
$$\Rightarrow \left(\frac{-3}{2}\right)^n = \left(-\frac{3}{2}\right)^6$$

$$\Rightarrow n = 6$$

ii) The cumulative frequency table of the given distribution table is as follows:

Weight in kg	Number of workers (f)	Cumulative frequency
50 – 60	4	4
60 – 70	7	11
70 – 80	11	22
80 – 90	14	36
90 – 100	6	42
100 – 110	5	47
110 – 120	3	50

Plot the points (60, 4), (70, 11), (80, 22), (90, 36), (100, 42), (110, 47) and (120, 50) on a graph paper and join them to get an ogive.



Number of workers, $N = 50$

Upper quartile $(Q_3) = \left(\frac{3 \times N}{4}\right)^{th}$ term $= \left(\frac{3 \times 50}{4}\right)^{th}$ term $= (37.5)^{th}$ term $= 92$

Lower quartile $(Q_1) = \left(\frac{N}{4}\right)^{th}$ term $= \left(\frac{50}{4}\right)^{th}$ term $= (12.5)^{th}$ term $= 71$

iii)

Inner radius of the pipe $= r = \frac{5}{2}$ cm $= 2.5$ cm

External radius of the pipe $= R =$ Inner radius of the pipe + Thickness of the pipe

$= 2.5$ cm $+ 0.5$ cm

$= 3$ cm

Length of the pipe $= h = 2$ m $= 200$ cm

Volume of the pipe = External Volume – Internal Volume

$$= \pi R^2 h - \pi r^2 h$$
$$= \pi\left(R^2 - r^2\right)h$$
$$= \pi(R - r)(R + r)h$$
$$= \frac{22}{7}(3 - 2.5)(3 + 2.5) \times 200$$
$$= \frac{22}{7} \times 0.5 \times 5.5 \times 200$$
$$= 1728.6 \text{ cm}^3$$

Since 1cm^3 of the metal weights 7.7 g,

$\therefore$ Weight of the pipe $= (1728.6 \times 7.7)\text{ g} = \left(\frac{1728.6 \times 7.7}{1000}\right)\text{kg} = 13.31\text{ kg}$

Question 7

i) Given, P(7, 6), Q(−5, −4) and R(3, −10) are the vertices of a ΔPQR.

(a) Slope of PQ $= \frac{-4-6}{-5-7} = \frac{-10}{-12} = \frac{5}{6}$

Therefore, slope of the altitude of PQ $= \frac{-1}{\text{slope of PQ}} = \frac{-1}{5/6} = -\frac{6}{5}$

(b) Since M is the mid-point of QR,

Coordinates of M are $\left(\frac{-5+3}{2}, \frac{-4-10}{2}\right) = \left(\frac{-2}{2}, \frac{-14}{2}\right) = (-1, -7)$

Therefore, the slope of the median PM $= \frac{-7-6}{-1-7} = \frac{-13}{-8} = \frac{13}{8}$

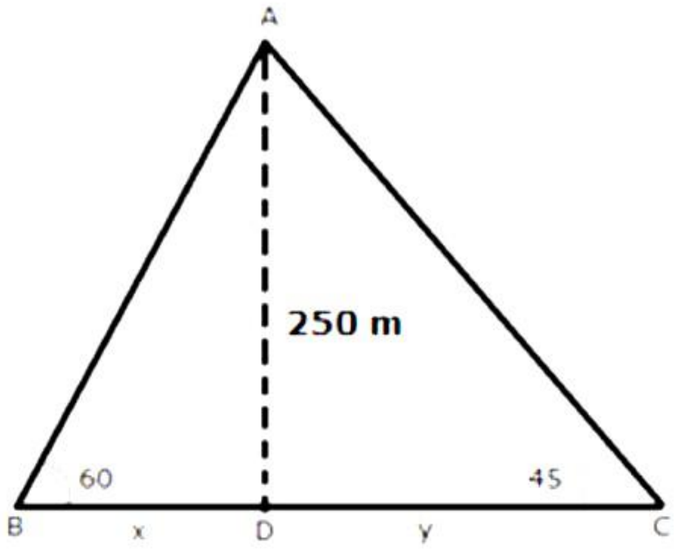
(c) Slope of PR $= \frac{-10-6}{3-7} = \frac{-16}{-4} = 4$

Therefore, the slope of the line parallel to PR = Slope of PR = 4

(d) Slope of QR $= \frac{-10-(-4)}{3-(-5)} = \frac{-6}{8} = -\frac{3}{4}$

Therefore, the slope of the line parallel to QR = Slope of QR $= -\frac{3}{4}$

ii)



Let A be the position of the airplane and let BC be the river.

Let D be the point on BC just below the airplane.

B and C be two boats on the opposite banks of the river with angles of depression 60° and 45° from A.

In ΔADC,

$$\tan 45^\circ = \frac{AD}{DC}$$

$$\Rightarrow 1 = \frac{250}{y}$$

$$\Rightarrow y = 250 \text{ m} = DC$$

In ΔADB,

$$\tan 60^\circ = \frac{AD}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{250}{x}$$

$$\Rightarrow x = \frac{250}{\sqrt{3}} = \frac{250\sqrt{3}}{3} = \frac{250 \times 1.732}{3} = 144.3 \text{ m} = BD$$

$$\therefore BC = BD + DC = 144.3 + 250 = 394.3 \approx 394 \text{ m}$$

Thus, the width of the river is 394 m.

Question 8

i) $-8\frac{1}{2} < -\frac{1}{2} - 4x \leq 7\frac{1}{2},\ x \in I$

Consider

$$-8\frac{1}{2} < -\frac{1}{2} - 4x$$

$$\Rightarrow \frac{-15}{2} < -\frac{1}{2} - 4x$$

$$\Rightarrow \frac{-15}{2} + \frac{1}{2} < -4x$$

$$\Rightarrow \frac{-14}{2} < -4x$$

$\Rightarrow -7 < -4x$

$\Rightarrow 7 > 4x$

$\Rightarrow x < \frac{7}{4}$

Consider

$-\frac{1}{2} - 4x \leq 7\frac{1}{2}$

$\Rightarrow -\frac{1}{2} - 4x \leq \frac{15}{2}$

$\Rightarrow -4x \leq \frac{15}{2} + \frac{1}{2}$

$\Rightarrow -4x \leq 8$

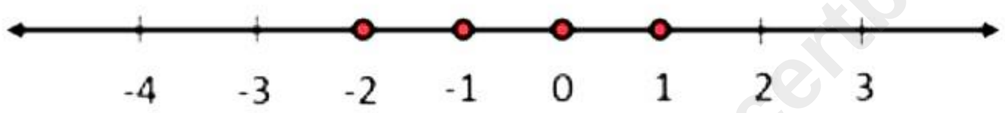
$\Rightarrow x \geq -2$

So,

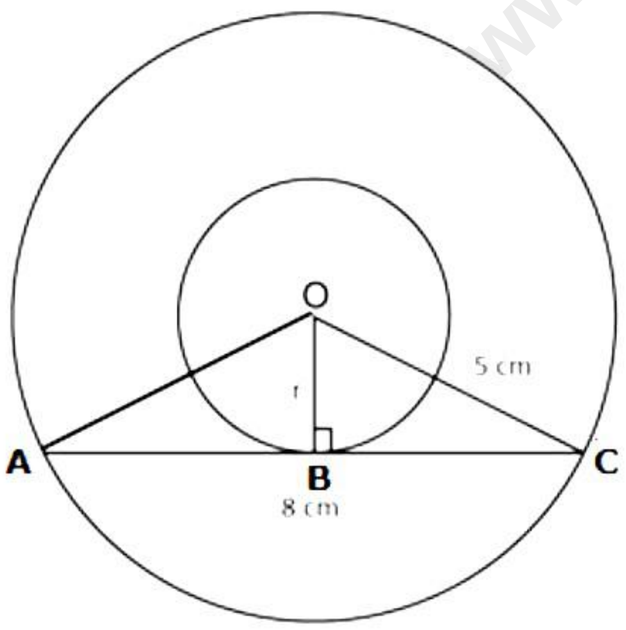
$\frac{7}{4} > x \geq -2$

As, $x \in I$

$x = \{-2, -1, 0, 1\}$



ii)



Let O be the centre of concentric circles.

Since AC is a tangent to the inner circle.

$\angle OBC = 90°$ ....(tangent is perpendicular to the radius of a circle)

AC is a chord of the outer circle.

We know that, the perpendicular drawn from the centre to a chord of a circle, bisects the chord.

So, $AC = 2BC \Rightarrow BC = 4$ cm

In ΔOBC, by Pythagoras theorem,

$OC^2 = OB^2 + BC^2$

$\Rightarrow 5^2 = r^2 + 4^2$

$\Rightarrow r^2 = 5^2 - 4^2 = 25 - 16 = 9$ cm

$\Rightarrow r = 3$ cm

Hence, the radius of the smaller circle is 3 cm.

iii) Take $(x_1, y_1) = (-3, 3a + 1)$, $(x_2, y_2) = (5, 8a)$ and $(x, y) = (-b, 9a - 2)$

Here, $m_1 = 3$ and $m_2 = 1$

$\therefore$ Coordinate of $P(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$\Rightarrow x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$ and $y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$\Rightarrow -b = \frac{3 \times 5 + 1 \times (-3)}{3 + 1}$ and $9a - 2 = \frac{3 \times 8a + 1(3a + 1)}{3 + 1}$

$\Rightarrow -b = \frac{15 - 3}{4}$ and $9a - 2 = \frac{24a + 3a + 1}{4}$

$\Rightarrow -4b = 12$ and $36a - 8 = 27a + 1$

$\Rightarrow b = -3$ and $9a = 9$

$\therefore a = 1$ and $b = -3$

Question 9

i) Let the number of boys be 3x.

Then, the number of girls = 2x

$\therefore 3x + 2x = 630$

$\Rightarrow 5x = 630$

$\Rightarrow x = 126$

Then, number of boys = $3x = 3 \times 126 = 378$

And, number of girls = $2x = 2 \times 126 = 252$

After admission of 90 new students, we have

Total number of students = $630 + 90 = 720$

Now, let the number of boys be 7x.

Then, number of girls = 5x

$7x + 5x = 720$

$\Rightarrow 12x = 720$

$\Rightarrow x = 60$

Number of boys = $7x = 7 \times 60 = 420$

Therefore, number of newly admitted boys = $420 - 378 = 42$

ii)

Let the original number of children be x.

It is given that Rs. 7500 is divided among x children.

$\Rightarrow$ Money received by each child $= \text{Rs.}\dfrac{7500}{x}$

If there were 20 less children, then money received by each child $= \text{Rs.}\ \dfrac{7500}{x-20}$

From the given information, we have

$$\frac{7500}{x-20}-\frac{7500}{x}=100$$

$$\Rightarrow \frac{75}{x-20}-\frac{75}{x}=1$$

$$\Rightarrow \frac{75x-75x+1500}{x^2-20x}=1$$

$$\Rightarrow 1500=x^2-20x$$

$$\Rightarrow x^2-20x-1500=0$$

$$\Rightarrow x^2-50x+30x-1500=0$$

$$\Rightarrow x(x-50)+30(x-50)=0$$

$$\Rightarrow (x-50)(x+30)=0$$

$$\Rightarrow x=50 \text{ or } x=-30$$

Since number of children cannot be negative, we reject $x=-30$.

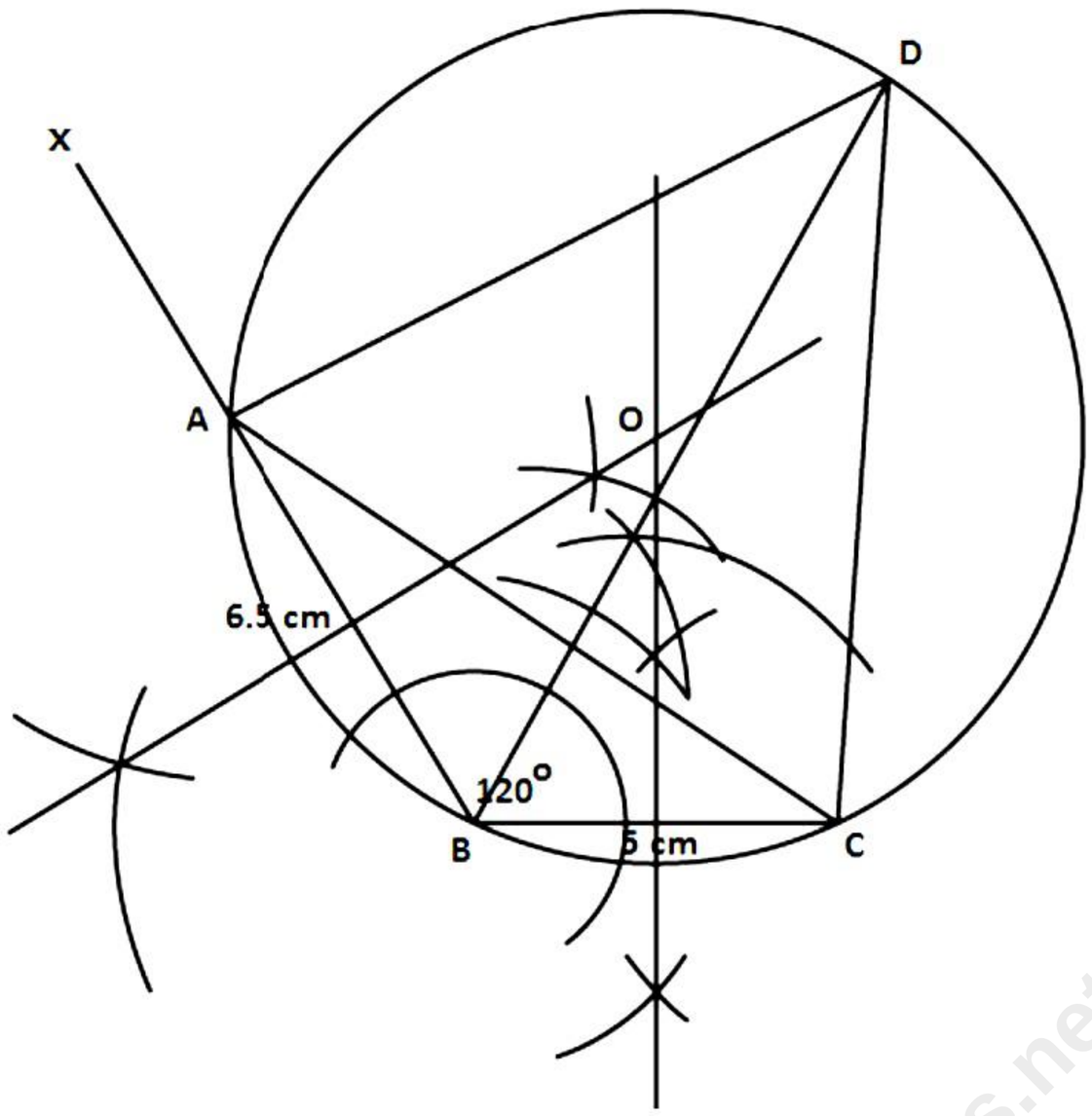
$\Rightarrow x=50$

Thus, the original number of children $=50$

iii)

Steps of construction:

1) Draw a line segment BC of length 5 cm.
2) At B, draw a ray BX making an angle of 120° with BC.
3) With B as centre and radius 6.5 cm, draw an arc to cut the ray BX at A. Join AC. ΔABC will be obtained.
4) Draw the perpendicular bisectors of AB and BC to meet at point O.
5) With O as centre and radius OA, draw a circle. The circle will circumscribe ΔABC.
6) Draw the angle bisector of $\angle ABC$ and let it meet circle at point D.
7) Join AD and DC to obtain the required cyclic quadrilateral ABCD such that point D is equidistant from AB and BC.



Solution 10

i) Here, $f(x) = 16x^3 - 8x^2 + 4x + 7$

Let the number subtracted be k from the given polynomial f(x).

Given that $2x + 1$ is a factor of f(x).

$$\therefore f\left(-\frac{1}{2}\right)=0$$

$$\Rightarrow 16\left(-\frac{1}{2}\right)^3-8\left(-\frac{1}{2}\right)^2+4\left(-\frac{1}{2}\right)+7-k=0$$

$$\Rightarrow 16\times\left(-\frac{1}{8}\right)-8\times\frac{1}{4}-2+7-k=0$$

$$\Rightarrow -2-2+5-k=0$$

$$\Rightarrow 1-k=0$$

$$\Rightarrow k=1$$

Therefore 1 must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor.

ii) Here, total number of all possible outcomes = 16

(a) a, e, i and o are the vowels.

Number of favourable outcomes = 4

$$\therefore \text{Required Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of all possible outcomes}} = \frac{4}{16} = \frac{1}{4}$$

(b) Number of consonants = 16 – 4 (vowels) = 12

$\therefore$ Number of favourable outcomes = 12

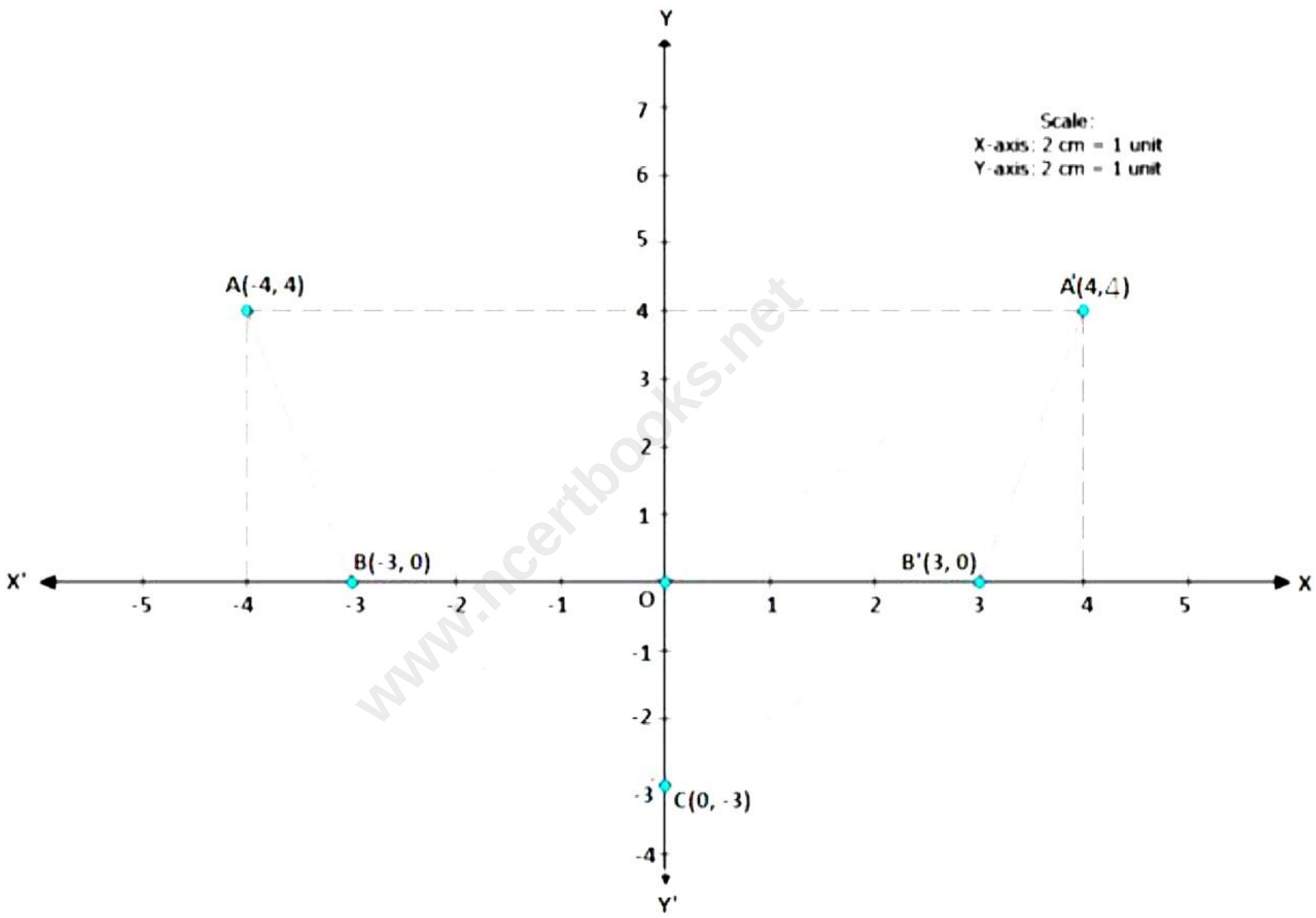
$$\therefore \text{ Required Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of all possible outcomes}} = \frac{12}{16} = \frac{3}{4}$$

(c) Median contains 6 letters.

$\therefore$ Number of favourable outcomes $= 16 - 6 = 10$

$$\therefore \text{ Required Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of all possible outcomes}} = \frac{10}{16} = \frac{5}{8}$$

iii)



Hence, A' $= (4, 4)$ and B' $= (3, 0)$