ICSE 2025 EXAMINATION

Sample Question Paper - 13

Mathematics

Time: 2 ½ hours. **Total Marks: 80**

General Instructions:

1. *Answers to this Paper must be written on the paper provided separately.*
2. *You will not be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.*
3. *The time given at the head of this Paper is the time allowed for writing the answers.*
4. *Attempt **all** questions from **Section A** and **any four** questions from **Section B**.*
5. *The intended marks for questions or parts of questions are given in brackets [ ]*

Section A

(Attempt all questions from this section.)

Question 1

Choose the correct answers to the questions from the given options. [15]

i) Given matrix $A = \begin{bmatrix} 4\sin 30° & \cos 0° \\ \cos 0° & 4\sin 30° \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. If AX = B, then the order of matrix X is

(a) 1×1
(b) 2×2
(c) 1×2
(d) 2×1

ii) The value of 'x' which satisfies the equation $(x + 5)(x - 5) = 24$ will be

(a) 5
(b) 6
(c) 7
(d) 8

iii) For a transaction within Delhi, MRP = Rs. 12,000, Discount % = 30%, GST = 18%, then CGST =?

(a) Rs. 756
(b) Rs. 765
(c) Rs. 786
(d) Rs. 768

iv) The roots of a quadratic equation are real and equal if its

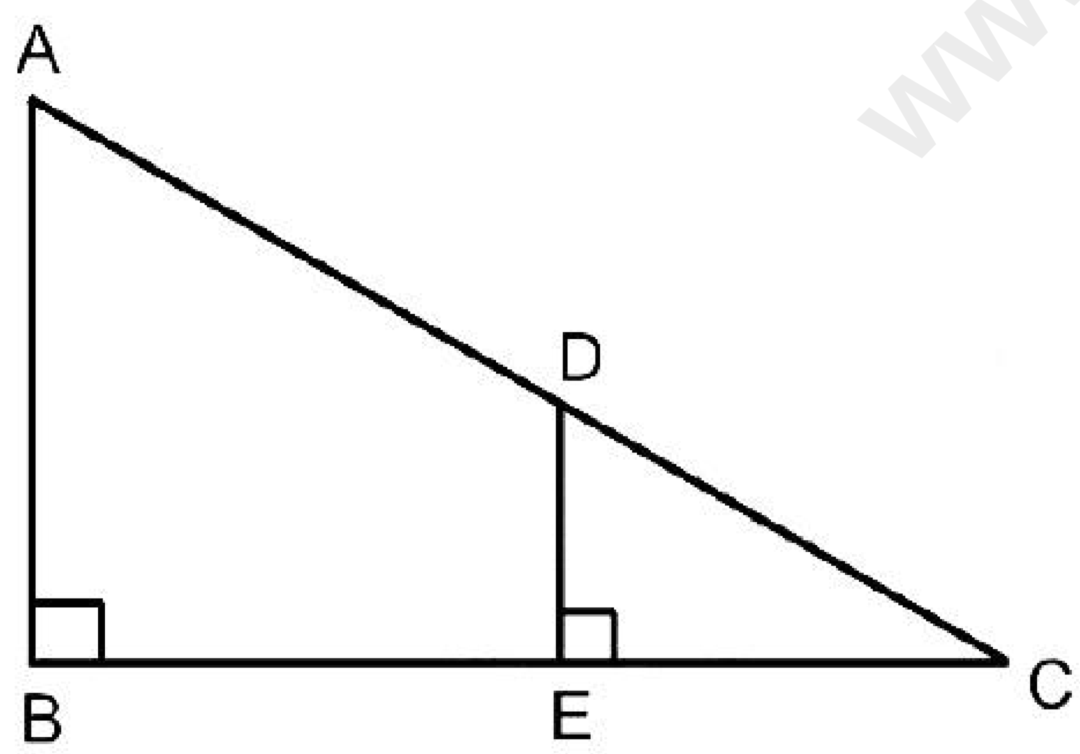
(a) discriminant < 0
(b) discriminant > 0
(c) discriminant = 0
(d) discriminant = 1

v) Find the 5^{th} term of the AP 30, 28, 26, 24, …
(a) 25
(b) 23
(c) 26
(d) 22

vi) For an A.P., $t_n = a + (n - 1)d$
Statement 1: If $d > 0$, the A.P. is decreasing.
Statement 2: If $d < 0$, the A.P. is increasing.
Which of the following is valid?
(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement 1 is true, and Statement 2 is false.
(d) Statement 1 is false, and Statement 2 is true.

vii) If a : b = c : d, then 4a + 3b : 4a – 3b =
(a) 3c + 4d : 3c – 4d
(b) 3c – 4d : 3c + 4d
(c) 4c – 3d : 4c + 3d
(d) 4c + 3d : 4c – 3d

viii) In the given figure, AB and DE are perpendiculars to BC. If AB = 5 cm, DE = 4 cm and AC = 13 cm, then $\Delta ABC \sim \Delta DEC$ by______



(a) SSS test
(b) AA test
(c) SAS test
(d) RHS test

ix) The height of a circular cylinder is 20 cm and the radius of its base is 7 cm, then its volume will be (Use $\pi = 22/7$)
(a) 3080 cm^3
(b) 3800 cm^3
(c) 3880 cm^3
(d) 3380 cm^3

x) A die has 6 faces marked by the given numbers as shown below:

1	2	3	-1	-2	-3

The die is thrown once. So the probability of getting a positive integer is

(a) 1/3
(b) 2/3
(c) 1/2
(d) 1

xi) The locus of a point equidistant from ____________________ is the bisector of the angle between the lines.

(a) two coincident lines
(b) two parallel lines
(c) two intersecting lines
(d) two perpendicular lines

xii) The angle between a tangent and a chord through the point of contact is ____________ an angle in the alternate segment.

(a) less than
(b) equal to
(c) greater than
(d) less than or equal to

xiii) The height of a pole is $\frac{1}{\sqrt{3}}$ times the length of its shadow. The angle of elevation of the sun is

(a) 60^o
(b) 45^o
(c) 30^0
(d) 90^o

xiv) The money required to buy 220, Rs. 30 shares at a discount of Rs. 5 is

(a) Rs. 5,000
(b) Rs. 5,500
(c) Rs. 4,000
(d) Rs. 4,500

xv) **Assertion (A):** If N = 100, assumed mean = 700 and $\sum f_i d_i = -780$, then the mean = 707.8.

Reason (R): Mean by assumed mean method = Assumed mean + $\frac{\sum f_i d_i}{N}$

(a) A is true, R is false.
(b) A is false, R is true.

(c) Both A and R are true, and R is the correct reason for A.

(d) both A and R are true, and R is the incorrect reason for A.

Question 2

i) A conical tent is to accommodate 77 persons. Each person must have 16 m^3 of air to breath. Given the radius of the tent as 7 m, find the height of the tent and also its curved surface area. [4]

ii) Mohan has a recurring deposit account in a bank for 2 years at 6% p.a. simple interest. If he gets Rs. 1,200 as interest at the time of maturity, find [4]

(a) the monthly instalment

(b) the amount of maturity.

iii) Prove that $\frac{\cot A - 1}{2 - \sec^2 A} = \frac{\cot A}{1 + \tan A}$ [4]

Question 3

i) If $\frac{x^2 + y^2}{x^2 - y^2} = 2\frac{1}{8}$, find [4]

A. $\frac{x}{y}$

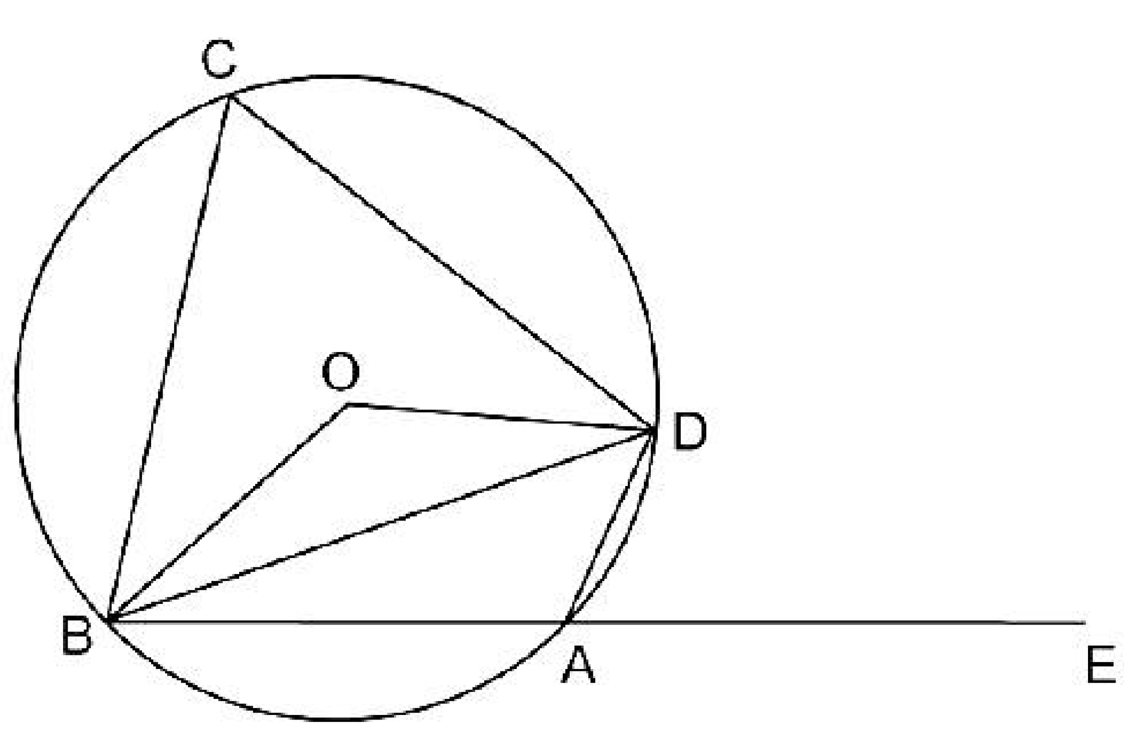
B. $\frac{x^3 + y^3}{x^3 - y^3}$

ii) In the figure given, O is the centre of the circle. $\angle DAE = 70°$. Find giving suitable reasons, the measure of [4]

A. $\angle BCD$

B. $\angle BOD$

C. $\angle OBD$



iii) The table shows the distribution of the scores obtained by 160 shooters in a shooting competition. Use a graph sheet and draw an ogive for the distribution. (Take 2 cm = 10 scores on the X-axis and 2 cm = 20 shooters on the Y-axis.)

Scores	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of shooters	9	13	20	26	30	22	15	10	8	7

Use your graph to estimate the following: [5]

(a) The median

(b) The interquartile range.

(c) The number of shooters who obtained a score of more than 85%.

Section B

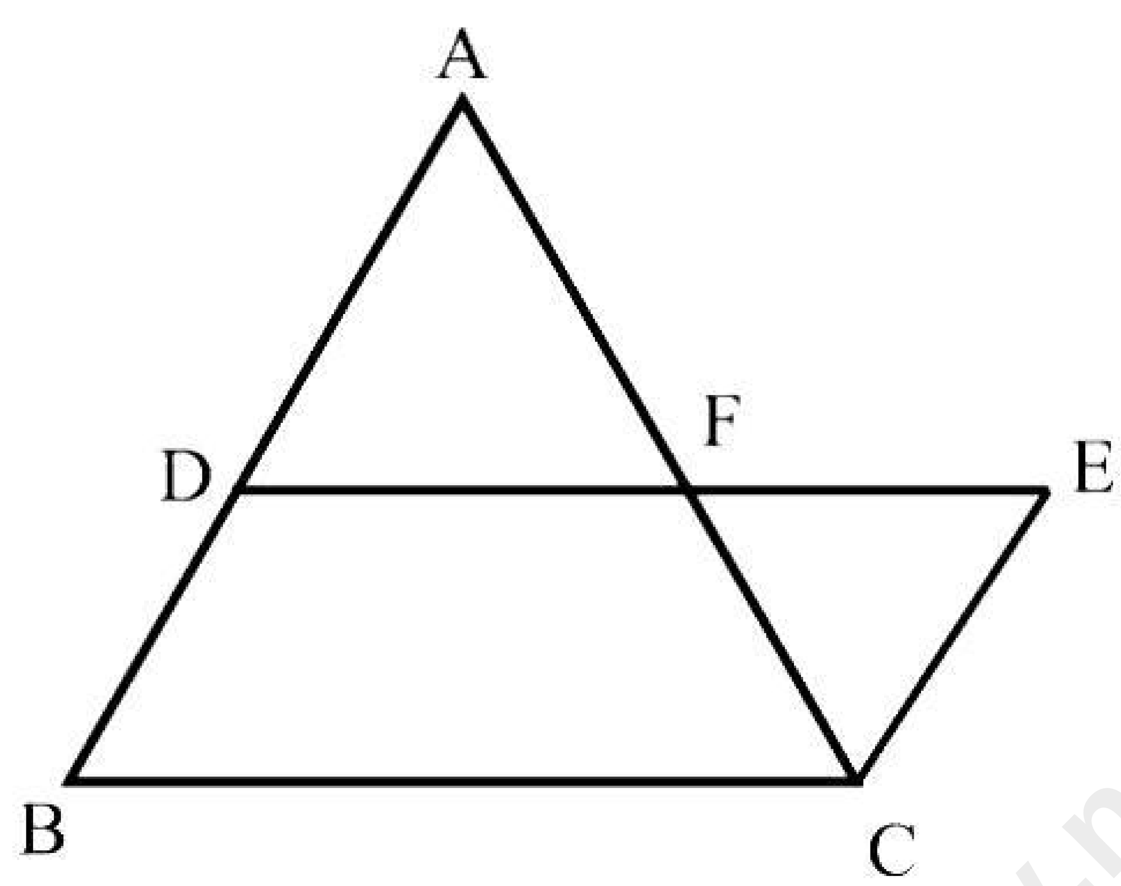
(Attempt any four questions from this Section.)

Question 4

i) If $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ and I is the identity matrix of the same order, then find AB + BI. [3]

ii) Solve: $\frac{x}{x-2} - \frac{x-2}{x} = 1\frac{1}{2}$ [3]

iii) In the given figure, ABC and CEF are two triangles where BA is parallel to CE and AF : AC = 5 : 8. [4]



a) Prove that ΔADF ~ ΔCEF
b) Find AD, if CE = 6 cm
c) Find FD, if FE = 9 cm

Question 5

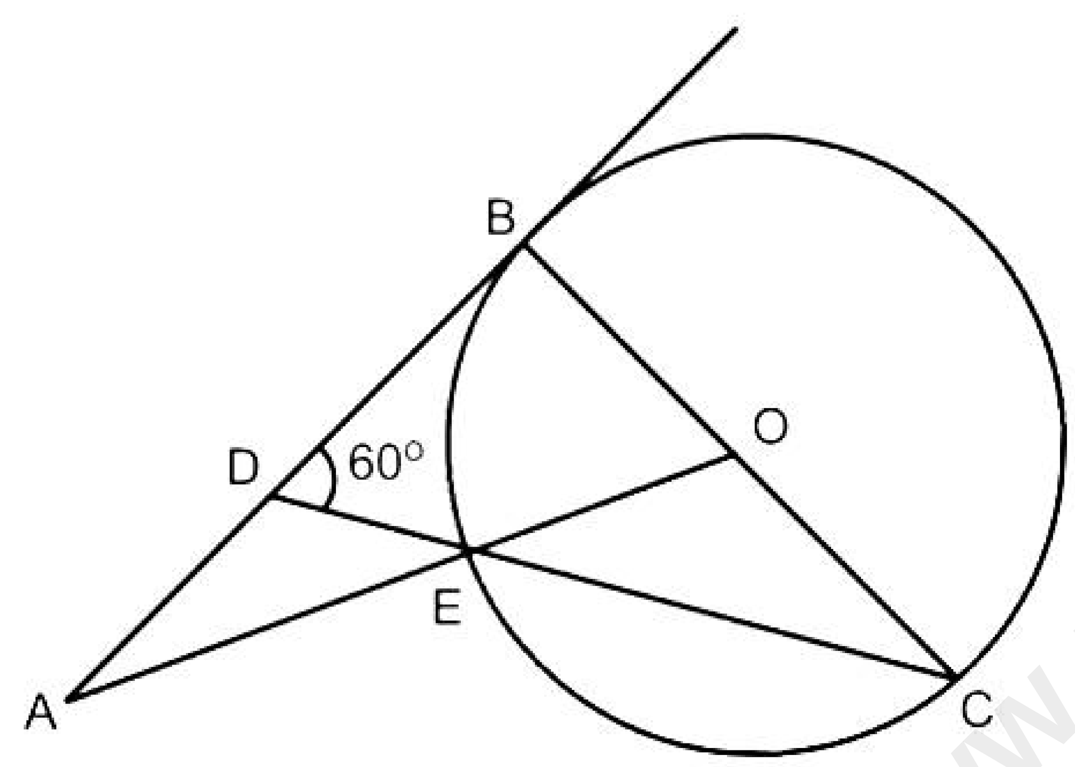
i) Calculate the mean marks of the following distribution using the step-deviation method. [3]

Class interval	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55
Frequency	8	15	25	17	14	11

ii) Consultancy services, worth Rs. 50,000, are transferred from Delhi to Calcutta at the rate of GST 18% and then from Calcutta to Nainital (with profit = Rs. 20,000) at the same rate of GST. Find the output tax at [3]

a) Delhi
b) Calcutta
c) Nainital

iii) In the given figure, O is the centre of the circle. AB is a tangent to it at point B. $\angle BDC = 60°$. Find $\angle BAO$. [4]



Question 6

i) The first and the last term of a G.P. are 3 and 96 respectively. If the common ratio is 2,
Find [3]
(a) The number of terms in the G.P.
(b) Sum of the n terms.

ii) Draw an ogive for the following frequency distribution using a graph paper which shows the marks obtained in Mathematics by 90 students. [3]

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of students	10	8	25	14	12	7	9	5

Use the ogive to estimate the median.

iii) From a solid cylinder whose height is 8 cm and radius is 6 cm, a conical cavity of height 8 cm and base radius of 6 cm is hollowed out. Find the volume of the remaining solid. (use $\pi = 3.14$) [4]

Question 7

i) The equation of the line is $4x - 5y = 9$. Find the [5]

a) Slope of the line.

b) Equation of the line perpendicular to the given line and passing through the intersection of the lines $x + y = 1$ and $2x + y = 2$.

ii) Two pillars of equal heights stand on either side of a roadway which is 180 m wide. At a point in the roadway between the pillars, the elevation at the top of the pillars are 30° and 60°. Find the height of the pillars and the position of the point. [5]

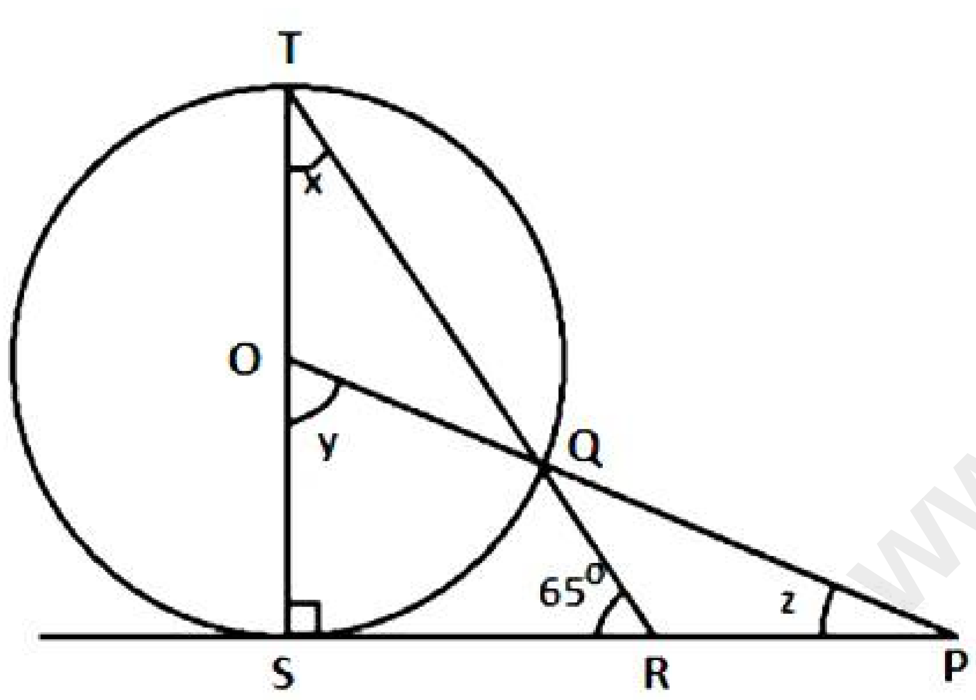
Question 8

i) Solve the following inequation and write the solution set: [3]

$13x - 5 < 15x + 4 < 7x + 12, x \in R$

Represent the solution on a real number line.

ii) In the figure given below, O is the centre of the circle and SP is a tangent. If $\angle SRT = 65^\circ$, find the values of x, y and z. [3]



iii) The mid-point of the line segment joining $(2a, 4)$ and $(-2, 2b)$ is $(1, 2a + 1)$. Find the values of a and b. [4]

Question 9

i) If $a : b = 9 : 5$, find $\frac{10a + 9b}{10a - 9b}$

[3]

ii) A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number. [3]

iii) Construct triangle ABC with AB = 5.5 cm, AC = 6 cm and $\angle BAC = 105^\circ$. Hence, [4]

(i) Construct the locus of points equidistant from BA and BC.

(ii) Construct the locus of points equidistant from B and C.

(iii) Mark the point which satisfies the above two loci as P. Measure and write the length of PC.

Question 10

i) (x – 1) is a factor of the expression $x^3 + ax^2 + bx + 9$. When this expression is divided by (x + 1), the remainder is 16. Find the values of a and b. [3]

ii) A bag contains 5 white balls, 6 red balls and 9 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is: [3]

(i) a white ball

(ii) a white or a red ball

(iii) Neither a green ball nor a white ball.

iii) Use a graph paper for this question. [4]

a) Plot point A (–3, –2). It is reflected about the y-axis to get the image A'. Find the co-ordinates of A'.

b) Plot point B(0, 3). It is reflected about the x-axis to get the image B'. Find the co-ordinates of B'.

c) Join points ABA'B'A. Find the distance between A' and B'.

Solution

Section A

Solution 1

i) Correct option: (d)

Explanation:

$$\text{Given, } A=\begin{bmatrix} 4\sin 30^\circ & \cos 0^\circ \\ \cos 0^\circ & 4\sin 30^\circ \end{bmatrix} \text{ and } B=\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Let the order of matrix X = m × n

Order of matrix A = 2 × 2

Order of matrix B = 2 × 1

Now, AX = B

$$\Rightarrow A_{2\times 2}\cdot X_{m\times n} = B_{2\times 1}$$

Thus, order of matrix X = m × n = 2 × 1

ii) Correct option: (c)

Explanation:

Given: (x + 5)(x – 5) = 24

$\Rightarrow x^2 - 5^2 = 24$

$\Rightarrow x^2 - 25 = 24$

$\Rightarrow x^2 = 49$

$\Rightarrow x = 7$

iii) Correct option: (a)

Explanation:

MRP = Rs. 12,000, Discount % = 30%, GST = 18%

Discount = 30% of 12,000 = $\frac{30}{100} \times 12000 = \text{Rs. } 3600$

Selling price (discounted value) = Rs. (12000 – 3600) = Rs. 8400

CGST = 9% of 8400 = Rs. 756

iv) Correct option: (c)

Explanation:

The roots of a quadratic equation are real and equal if its discriminant = 0.

v) Correct option: (d)

Explanation:

30, 28, 26, 24,...

$\Rightarrow a = 30, d = 28 - 30 = -2$

5^{th} term $= 24 + (-2) = 22$

vi) Correct option: (b)

For an A.P., $t_n = a + (n - 1)d$

Statement 1: If $d > 0$, the A.P. is increasing.

Statement 2: If $d < 0$, the A.P. is decreasing.

vii) Correct option: (d)

Explanation:

$a : b = c : d$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$

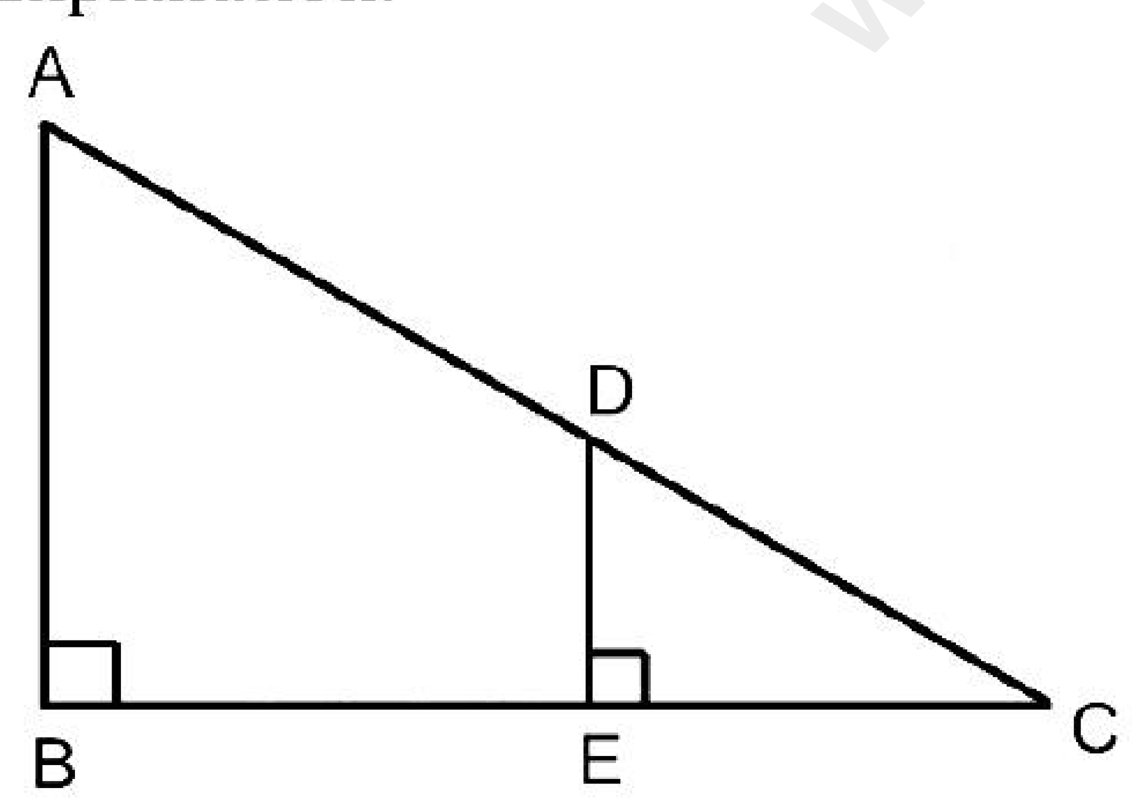
$\Rightarrow \frac{4a}{3b} = \frac{4c}{3d}$ $\quad \left(\text{Multiplying each side by } \frac{4}{3}\right)$

$\Rightarrow \frac{4a+3b}{4a-3b} = \frac{4c+3d}{4c-3d}$ $\quad (\text{By componendo-dividendo})$

$\Rightarrow 4a + 3b : 4a - 3b = 4c + 3d : 4c - 3d$

viii) Correct option: (b)

Explanation:



In ΔABC and ΔDEC,

$\angle ABC = \angle DEC = 90^\circ$

$\angle C = \angle C$ (Common)

$\therefore \Delta ABC \sim \Delta DEC$ (By AA similarity)

ix) Correct option: (a)

Explanation:

For a circular cylinder,

Height $= h = 20$ cm

Radius of the base $= r = 7$ cm

Volume of a cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 20$$

$$= 3080 \text{ cm}^3$$

x) Correct option: (c)

Explanation:

Given that the die has following numbers on the 6 faces:

1	2	3	-1	-2	-3

When the die is rolled, total number of outcomes = 6

There are 3 positive integers.

So, number of favorable outcomes = 3

$\Rightarrow$ Required probability $= \frac{3}{6} = \frac{1}{2}$

xi) Correct option: (c)

Explanation:

The locus of a point equidistant from two intersecting lines is the bisector of the angle between the lines.

xii) Correct option: (b)

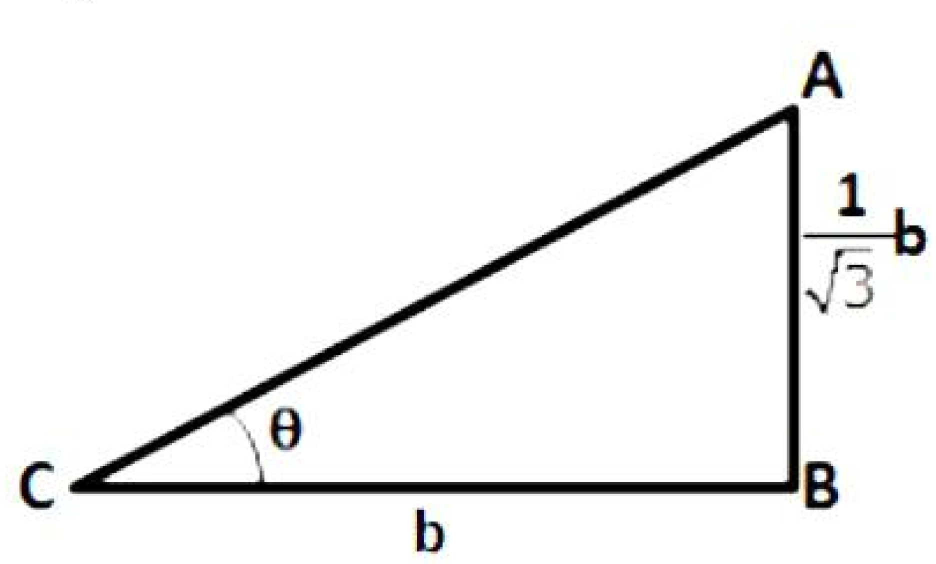
Explanation:

The angle between a tangent and a chord through the point of contact is **equal to** an angle in the alternate segment.

xiii)

Correct option: (c)

Explanation:



Let the length of shadow of a pole = b units

Then, height of the pole = $\frac{1}{\sqrt{3}}$ b units

If θ is the angle of elevation, then

$$\tan\theta = \frac{\frac{1}{\sqrt{3}}b}{b} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$\Rightarrow \theta = 30^\circ$

xiv) Correct option: (b)

Explanation:

Total shares = 220

Nominal value = Rs. 30

Market value = Rs. 30 – Rs. 5 = Rs. 25

Money required to buy 220 shares = Rs. 25 × 220 = Rs. 5,500

xv) Correct option: (b)

Explanation:

The statement given in reason is correct and hence reason is true.

Given: N = 100, assumed mean = 700 and $\sum f_i d_i = -780$

Mean = Assumed mean + $\frac{\sum f_i d_i}{N}$

$$= 700 + \frac{-780}{100}$$

$$= 700 - 7.8$$

$$= 692.2$$

Hence, the assertion is false.

Question 2

i) According to the condition in the question,

$$77 \times 16 = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow 77 \times 16 = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h$$

$$\Rightarrow h = \frac{77 \times 16 \times 3 \times 7}{22 \times 7 \times 7}$$

$$\Rightarrow h = 24 \text{ m}$$

We know that,

$l^2 = r^2 + h^2$

$\Rightarrow l^2 = (7)^2 + (24)^2$

$\Rightarrow l^2 = 49 + 576$

$\Rightarrow l^2 = 625$

$\Rightarrow l = 25$ m

$\therefore$ Curved Surface Area $= \pi rl = \dfrac{22}{7} \times 7 \times 25 = 550$ m^2

Therefore, the height of the tent is 24 m and its curved surface area is 550 m^2.

ii) Interest, I = Rs. 1,200
Time, n = 2 years = 2 × 12 = 24 months
Rate, r = 6%
(a) To find: Monthly instalment, P
Now,

$$I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow 1{,}200 = P \times \frac{24 \times 25}{24} \times \frac{6}{100}$$

$$\Rightarrow 1{,}200 = P \times \frac{3}{2}$$

$$\Rightarrow P = \frac{1{,}200 \times 2}{3}$$

$$\Rightarrow P = \text{Rs. } 800$$

So, the monthly instalment is Rs. 800.

(b) Total sum deposited = P × n = Rs. 800 × 24 = Rs. 19,200
$\therefore$ Amount of maturity = Total sum deposited + Interest on it
= Rs. (19,200 + 1,200)
= Rs. 20,400

iii) $$\text{L.H.S.} = \frac{\cot A - 1}{2 - \sec^2 A}$$

$$= \frac{\frac{1}{\tan A} - 1}{2 - \left(1 + \tan^2 A\right)}$$

$$= \frac{1 - \tan A}{\tan A\left(1 - \tan^2 A\right)}$$

$$= \frac{(1 - \tan A)}{\tan A\,(1 - \tan A)(1 + \tan A)}$$

$$= \frac{1}{\tan A(1 + \tan A)}$$

$$= \frac{1}{\tan A} \times \frac{1}{(1 + \tan A)}$$

$= \frac{\cot A}{1+\tan A}$

$= \text{R.H.S.}$

Hence, L.H.S. = R.H.S.

Question 3

i)

A. $\frac{x^2+y^2}{x^2-y^2} = 2\frac{1}{8}$

$\Rightarrow \frac{x^2+y^2}{x^2-y^2} = \frac{17}{8}$

Using componendo and dividendo,

$\Rightarrow \frac{x^2+y^2+x^2-y^2}{x^2+y^2-\left(x^2-y^2\right)} = \frac{17+8}{17-8}$

$\Rightarrow \frac{2x^2}{2y^2} = \frac{25}{9}$

$\Rightarrow \frac{x^2}{y^2} = \frac{25}{9}$

$\Rightarrow \frac{x}{y} = \frac{5}{3}$

B. Now, $\frac{x}{y} = \frac{5}{3}$

$\Rightarrow \frac{x^3}{y^3} = \frac{5^3}{3^3}$

$\Rightarrow \frac{x^3}{y^3} = \frac{125}{27}$

Using componendo and dividendo,

$\Rightarrow \frac{x^3+y^3}{x^3-y^3} = \frac{125+27}{125-27}$

$\Rightarrow \frac{x^3+y^3}{x^3-y^3} = \frac{152}{98}$

$\Rightarrow \frac{x^3+y^3}{x^3-y^3} = \frac{76}{49}$

ii)

A. $\angle$DAE and $\angle$DAB form a linear pair.

$\Rightarrow \angle DAE + \angle DAB = 180°$

$\Rightarrow 70^{o} + \angle DAB = 180°$

$\therefore \angle DAB = 110°$

Also,

$\angle BCD + \angle DAB = 180°$ ... (Opp. angles of cyclic quadrilateral BADC)

$\Rightarrow \angle BCD + 110° = 180°$

$\therefore \angle BCD = 70°$

B. $\angle BCD = ½ \angle BOD$ ... (angles subtended by an arc at the centre and on the circle)

$\Rightarrow 70° = ½ \angle BOD$

$\therefore \angle BOD = 140°$

C. In ΔBOD,

$OB = OD$ ... (Radii of the same circle)

So, $\angle OBD = \angle ODB$ ... (Isosceles triangle theorem)

Now, $\angle OBD + \angle ODB + \angle BOD = 180°$ ... (Sum of angles of triangle)

$\Rightarrow 2\angle OBD + 140° = 180°$

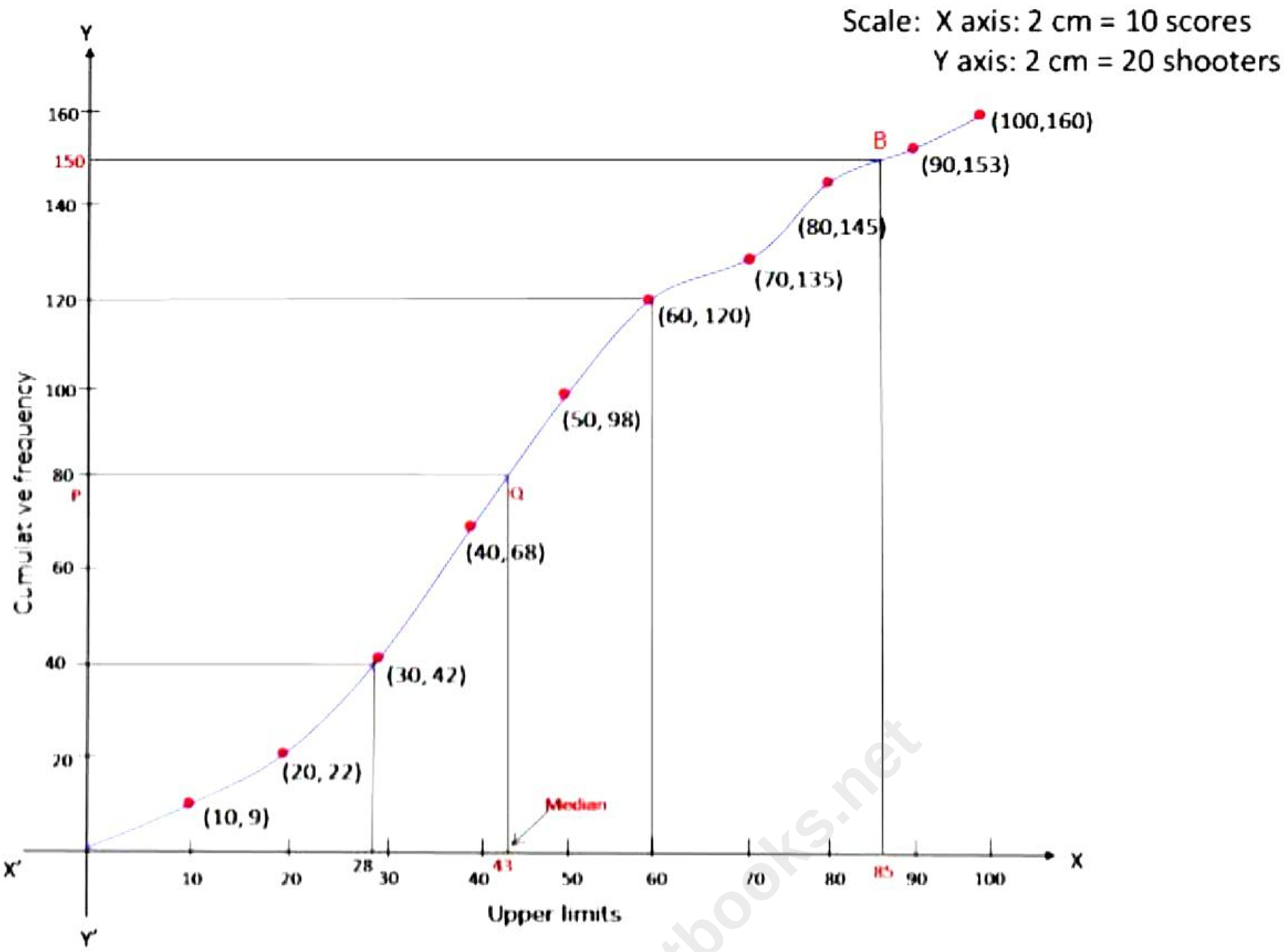
$\Rightarrow 2\angle OBD = 40°$

$\Rightarrow \angle OBD = 20°$

iii)

Scores	f	c.f.
0 – 10	9	9
10 – 20	13	22
20 – 30	20	42
30 – 40	26	68
40 – 50	30	98
50 – 60	22	120
60 – 70	15	135
70 – 80	10	145
80 – 90	8	153
90 – 100	7	160
	n = 160	

The ogive is shown below:



(a) Median $= \left(\frac{n}{2}\right)^{th}$ term $= \left(\frac{160}{2}\right)^{th}$ term $= 80^{th}$ term

Through mark 80 on y-axis, draw a horizontal line which meets the ogive drawn at point Q.

Through Q, draw a vertical line which meets the x-axis at the mark of 43.

$\Rightarrow$ Median $= 43$

(b) Since the number of terms $= 160$

Lower quartile $(Q_1) = \left(\frac{160}{4}\right)^{th}$ term $= 40^{th}$ term $= 28$

Upper quartile $(Q_3) = \left(\frac{3 \times 160}{4}\right)^{th}$ term $= 120^{th}$ term $= 60$

$\therefore$ Inter-quartile range $= Q_3 - Q_1 = 60 - 28 = 32$

(c) 85% scores = 85% of 100 = 85

Through mark of 85 on x-axis, draw a vertical line that meets the ogive drawn at point B.

Through the point B, draw a horizontal line that meets the y-axis at the mark of 150.

⇒ Number of shooters who obtained more than 85% score = 160 – 150 = 10

Section B

Solution 4

i)

Given,

$$\Rightarrow A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 8-1 & -4+3 \\ 20-3 & -10+9 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix}$$

$$\Rightarrow BI = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\Rightarrow AB + BI = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 11 & -3 \\ 16 & 2 \end{bmatrix}$$

ii)

$$\frac{x}{x-2} - \frac{x-2}{x} = 1\frac{1}{2}$$

$$\Rightarrow \frac{x^2-(x-2)^2}{x(x-2)} = \frac{3}{2}$$

$$\Rightarrow 2\left[x^2-(x-2)^2\right] = 3x(x-2)$$

$$\Rightarrow 2\left[x^2-x^2+4x-4\right] = 3x^2-6x$$

$$\Rightarrow 2(4x-4) = 3x^2-6x$$

$$\Rightarrow 8x-8 = 3x^2-6x$$

$$\Rightarrow 3x^2-6x-8x+8 = 0$$

$$\Rightarrow 3x^2-14x+8 = 0$$

$$\Rightarrow 3x^2-12x-2x+8 = 0$$

$$\Rightarrow 3x(x-4)-2(x-4) = 0$$

$$\Rightarrow (x-4)(3x-2) = 0$$

$$\Rightarrow x = 4 \text{ or } x = \frac{2}{3}$$

iii)

a) In ΔADF and ΔCEF,

$\angle DFA = \angle EFC$ (vertically opposite angles)

$\angle DAF = \angle ECF$ (alternate angles)

$\therefore \Delta ADF \sim \Delta CEF$ (By AA similarity)

b)

Since, $\frac{AF}{AC} = \frac{5}{8}$

$\therefore \frac{AC}{AF} = \frac{8}{5}$

$\therefore \frac{AF + FC}{AF} = \frac{5+3}{5}$

$\therefore 1 + \frac{FC}{AF} = 1 + \frac{3}{5}$

$\therefore \frac{FC}{AF} = \frac{3}{5}$

$\therefore \frac{FA}{FC} = \frac{5}{3}$ ... (i)

$\because \Delta ADF \sim \Delta CEF$

$\therefore \frac{FA}{FC} = \frac{AD}{CE} = \frac{FD}{FE}$

$\therefore \frac{FA}{FC} = \frac{AD}{CE}$

$\therefore \frac{5}{3} = \frac{AD}{6}$

$\therefore AD = 10$ cm

c)

Since $\Delta ADF \sim \Delta CEF$

$\frac{FA}{FC} = \frac{AD}{CE} = \frac{FD}{FE}$

$\therefore \frac{FA}{FC} = \frac{FD}{FE}$

$\therefore \frac{5}{3} = \frac{FD}{9}$

$\therefore FD = 15$ cm

Solution 5

i)

Let us prepare the frequency table using the direct method as follows:

C.I.	f	Class mark (x)	Assumed mean A = 37.5 d = x – A	$t = \frac{x-A}{i} = \frac{x-37.5}{5}$	ft
25–30	8	27.5	−10	−2	−16
30–35	15	32.5	−5	−1	−15
35–40	25	37.5	0	0	0
40–45	17	42.5	5	1	17
45–50	14	47.5	10	2	28
50–55	11	52.5	15	3	33
	$n = \sum f = 90$				$\sum ft = 47$

$\text{Mean } = A + \frac{\sum ft}{n} \times i$

$\Rightarrow \text{Mean } = 37.5 + \frac{47}{90} \times 5 = 37.5 + 2.61 = 40.11$

ii)

a) Output tax in Delhi (interstate):
IGST = 18% of 50,000 = Rs. 9000
Output tax in Delhi = Rs. 9000

b) Output tax in Calcutta:
C.P. in Calcutta = Rs. 50,000
Profit = Rs. 20,000
S.P. in Calcutta = 50,000 + 20,000 = Rs. 70,000
IGST = 18% of 70,000 = Rs. 12,600
Output tax in Calcutta = Rs. 12,600

c) Since, the dealer in Nainital does not sell the product.
Output GST (tax on sale) = Rs. 0

iii)

In ΔBDC,

$\angle DBC = 90°$ (tangent ⊥ radius)

$\angle BDC = 60°$ ...(1)(given)

$\angle DCB + \angle DBC + \angle BDC = 180°$ (angle sum property in the triangle)

$\therefore \angle DCB + 90° + 60° = 180°$

$\therefore \angle DCB = 30°$

In ΔOEC,

$OE = OC$ (radii of the same circle)

$\therefore \angle OEC = \angle OCE$ (isosceles triangle property)

$\therefore \angle OEC = \angle BCD = 30°$

And, $\angle DEA = \angle OEC = 30°$ (vertically opposite angles)

Now, $\angle ADE + \angle BDE = 180°$ (straight line property)

$\therefore \angle ADE + 60° = 180°$

$\therefore \angle ADE = 120°$

In ΔAED,

$\angle DAE + \angle ADE + \angle DEA = 180°$ (angle sum property in the triangle)

$\therefore \angle DAE + 120° + 30° = 180°$

$\therefore \angle DAE = 30°$

$\therefore \angle BAO = \angle DAE = 30°$

Solution 6

i)

(a) First term, a = 3

Last term, l = 96

Common ratio, r = 2

Let the number of terms in the G.P. = n

$\Rightarrow ar^{n-1} = 96$

$\Rightarrow 3 \times (2)^{n-1} = 96$

$\Rightarrow (2)^{n-1} = 32$

$\Rightarrow (2)^{n-1} = 2^5$

$\Rightarrow n - 1 = 5$

$\Rightarrow n = 6$

(b) Sum of n terms = S_n

$r = 2 \Rightarrow |r| > 1$

$$\therefore S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{3(2^6 - 1)}{2 - 1}$$

$= 3(64 - 1)$

$= 189$

ii)

Let us first prepare the cumulative frequency table as follows:

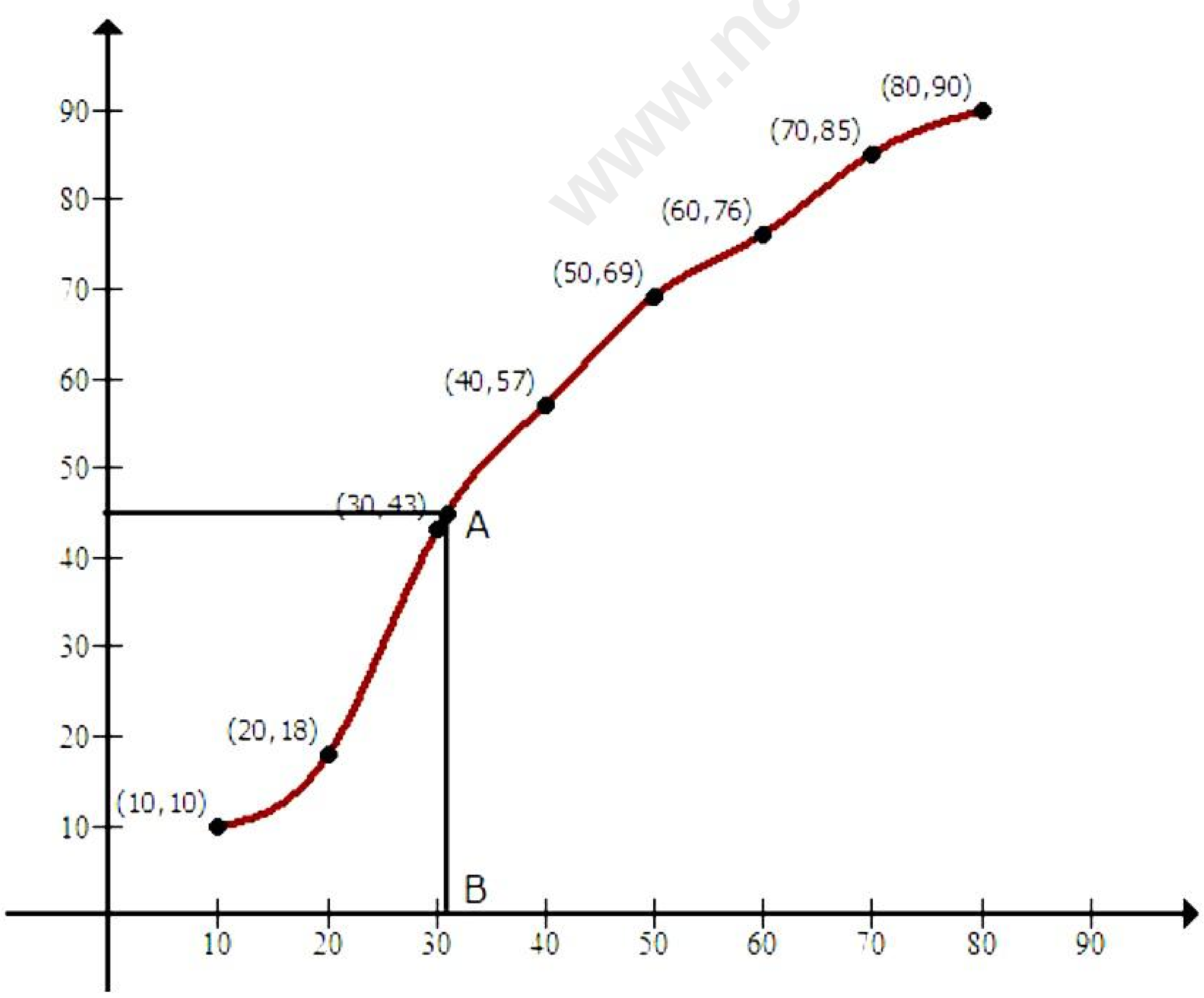
Class Interval	Frequency	Cumulative frequency (c.f.)
0–10	10	10
10–20	8	18
20–30	25	43
30–40	14	57
40–50	12	69
50–60	7	76
60–70	9	85
70–80	5	90
	$n = \sum f = 90$	

On a graph paper, mark class intervals along the x-axis and cumulative frequencies along the y-axis.

On this graph, mark points (10, 10), (20, 18), (30, 43), (40, 57), (50, 69), (60, 76), (70, 85) and (80, 90).

Then draw a free-hand curve passing through the points marked, starting from the upper limit of the first class and terminating at the upper limit of the last class.

The curve so obtained is the ogive curve.



Total no. of terms = n = 90

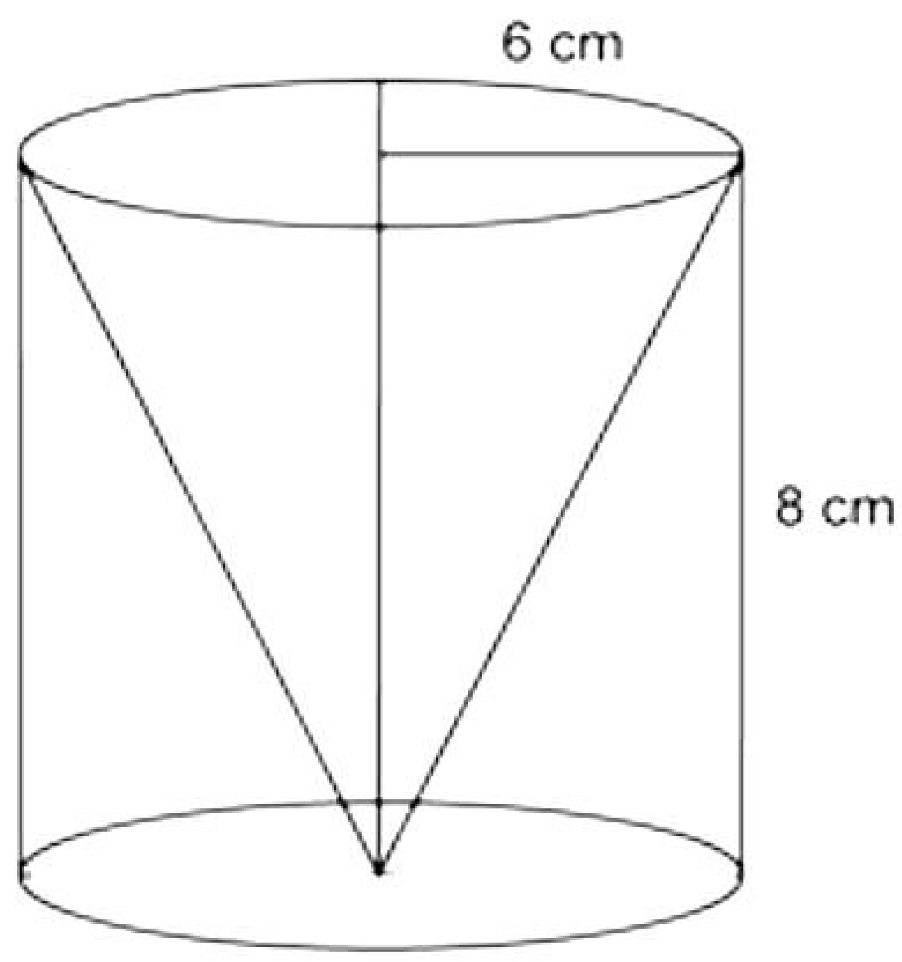
$$\therefore \text{ Median} = \left(\frac{n}{2}\right)^{th} \text{ term} = \left(\frac{90}{2}\right)^{th} \text{ term} = 45^{th} \text{ term}$$

Now, draw a horizontal line from 45 marked on the y-axis which meets the curve at point A and then draw a vertical line through A which meets the x-axis at point B.

The value of B on the x-axis is the median = 31.

iii)

For the conical cavity, radius r = 6 cm and height = 8 cm



$\Rightarrow$ Volume of the remaining solid = Volume of the cylinder – Volume of the cone

$= \pi r^2 h - \frac{1}{3}\pi r^2 h$

$= 3.14 \times 6^2 \times 8\left(1 - \frac{1}{3}\right)$

$= 602.88 \text{ cm}^3$

Therefore, the volume of the remaining solid is 602.88 cm^3.

Solution 7

i)

a) Given equation of the line is 4x – 5y = 9

$\therefore$ 5y = 4x – 9

$\therefore y = \frac{4x}{5} - \frac{9}{5}$

Comparing it with y = mx + c, we get $m = \frac{4}{5}$, which is the required slope.

b) To find the point of intersection of the two lines

x + y = 1 ... (i)

2x + y = 2 ... (ii)

Subtracting (i) from (ii), we get, x = 1

Putting it in (i), we get, y = 0

Thus, the intersection point of the given two lines is (1, 0).

Now, the equation of the required line is passing through (1, 0) and perpendicular to the line 4x – 5y = 9.

Hence, the slope of the required line = $m_1 = \frac{-1}{\frac{4}{5}} = \frac{-5}{4}$, and the equation is given by

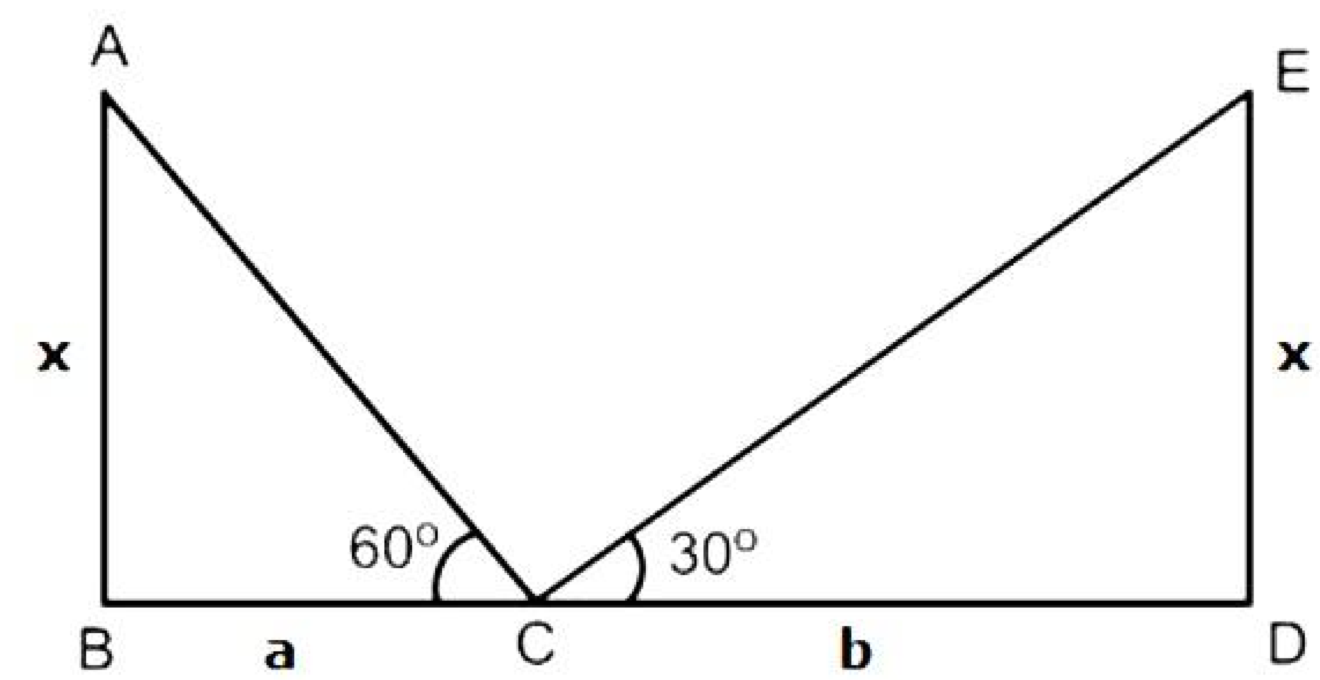
$y - y_1 = m_1(x - x_1)$

$\therefore y - 0 = \frac{-5}{4}(x - 1)$

$\therefore 4y = -5x + 5$
$\therefore 5x + 4y = 5$

ii)

Let AB and ED be two pillars standing on either side of the road.
Let AB = x = ED
BD is the width of the road and BD = BC + CD
It is given that BD = 180 m
$\Rightarrow a + b = 180$ m



In triangle ABC,

$$\tan 60^\circ = \frac{x}{a}$$

$$\Rightarrow \sqrt{3} = \frac{x}{a}$$

$$\Rightarrow x = a\sqrt{3} \quad ...(i)$$

In triangle CDE,

$$\tan 30^\circ = \frac{x}{b}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{b}$$

$$\Rightarrow x = \frac{b}{\sqrt{3}} \quad ...(ii)$$

From (i) and (ii), we have

$$a\sqrt{3} = \frac{b}{\sqrt{3}}$$

$$\Rightarrow b = 3a$$

$$a + b = 180$$

$$a + 3a = 180$$

$$\Rightarrow 4a = 180$$

$$\Rightarrow a = 45 \text{ m}$$

$$\Rightarrow b = 3a = 3 \times 45 = 135 \text{ m}$$

$$\therefore x = a\sqrt{3} \text{ m} = 45\sqrt{3} \text{ m}$$

Hence, the height of each pillar is $45\sqrt{3}$ m.
And, the position of the point from pillar AB is 45 m and from pillar ED is 135 m.

Solution 8

i)

$13x - 5 < 15x + 4 < 7x + 12, x \in R$
Consider,

$13x - 5 < 15x + 4$ $\quad$ $15x + 4 < 7x + 12$

$-5 - 4 < 15x - 13x$ $\quad$ $15x - 7x < 12 - 4$

$-9 < 2x$ $\quad$ $8x < 8$

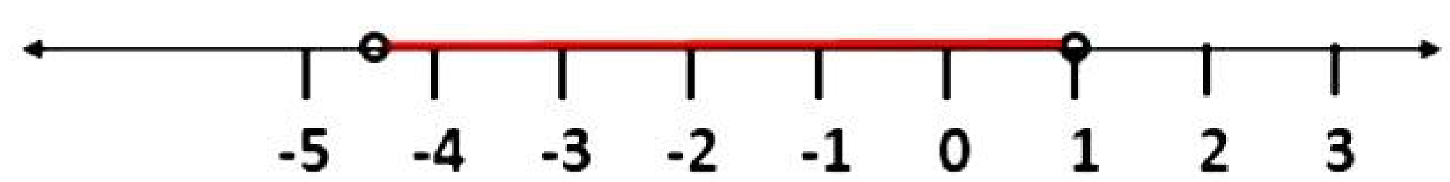
$-\frac{9}{2} < x$ $\quad$ $x < 1$

$\therefore -\frac{9}{2} < x < 1$

i.e. $-4.5 < x < 1$

$\therefore$ Solution set $= \{x : -4.5 < x < 1,\ x \in R\}$

The solution on the number line is as follows:



ii)

In the given figure, TS $\perp$ SP,

$\angle TSR = \angle OSP = 90^\circ$

In ΔTSR,

$\angle TSR + \angle TRS + \angle RTS = 180^\circ$

$\Rightarrow 90^\circ + 65^\circ + x = 180^\circ$

$\Rightarrow x = 180^\circ - 90^\circ - 65^\circ$

$\Rightarrow x = 25^\circ$

Now, $y = 2x$ [Angle subtended at the centre is double that of the angle subtended by the arc at the same centre]

$\Rightarrow y = 2 \times 25^\circ = 50^\circ$

In ΔOSP,

$\angle OSP + \angle SPO + \angle POS = 180^\circ$

$\Rightarrow 90^\circ + z + 50^\circ = 180^\circ$

$\Rightarrow z = 180^\circ - 140^\circ$

$\therefore z = 40^\circ$

iii)

The mid-point of the line segment joining (2a, 4) and (−2, 2b) is (1, 2a + 1).

$\Rightarrow \left(\frac{2a-2}{2}, \frac{4+2b}{2}\right) = (1, 2a+1)$

$\Rightarrow \frac{2a-2}{2} = 1, \frac{4+2b}{2} = 2a+1$

$\Rightarrow a = 2$

Substituting $a = 2$ in $\frac{4+2b}{2} = 2a+1$,

$\frac{4+2b}{2} = 2 \times 2 + 1$

$\Rightarrow 4 + 2b = 10$

$\Rightarrow 2b = 6$

$\Rightarrow b = 3$

Solution 9

i)

$$\frac{a}{b} = \frac{9}{5}$$

$$\frac{10a+9b}{10a-9b} = \frac{10\times\frac{a}{b}+9}{10\times\frac{a}{b}-9}$$

$$= \frac{10\times\frac{9}{5}+9}{10\times\frac{9}{5}-9}$$

$$= \frac{27}{9}$$

$$= 3$$

ii)

Let the digit at the tens place be 'a' and at units place be 'b'.

The two-digit so formed will be 10a + b.

According to first condition, product of its digits is 6.

$\Rightarrow a \times b = 6$

$\Rightarrow b = \frac{6}{a}$ ....(1)

According to second condition,

$10a + b + 9 = 10b + a$

$\Rightarrow 9a - 9b = -9$

$\Rightarrow a - b = -1$

$\Rightarrow a - \frac{6}{a} = -1$ [From (i)]

$\Rightarrow a^2 - 6 = -a$

$\Rightarrow a^2 + a - 6 = 0$

$\Rightarrow a^2 + 3a - 2a - 6 = 0$

$\Rightarrow a(a+3) - 2(a+3) = 0$

$\Rightarrow (a+3)(a-2) = 0$

$\Rightarrow a = -3$ or $a = 2$

Since a digit cannot be negative, a = 2.

$\Rightarrow b = \frac{6}{a} = \frac{6}{2} = 3$

Thus, the required two-digit number = 10a + b = 10(2) + 3 = 23

iii)

1. Draw a line segment AB of length 5.5 cm.
2. Construct $\angle BAX = 105°$ using a protractor.
3. Draw an arc AC with radius AC = 6 cm on AX with centre at A.
4. Join BC.

Thus ΔABC is the required triangle.

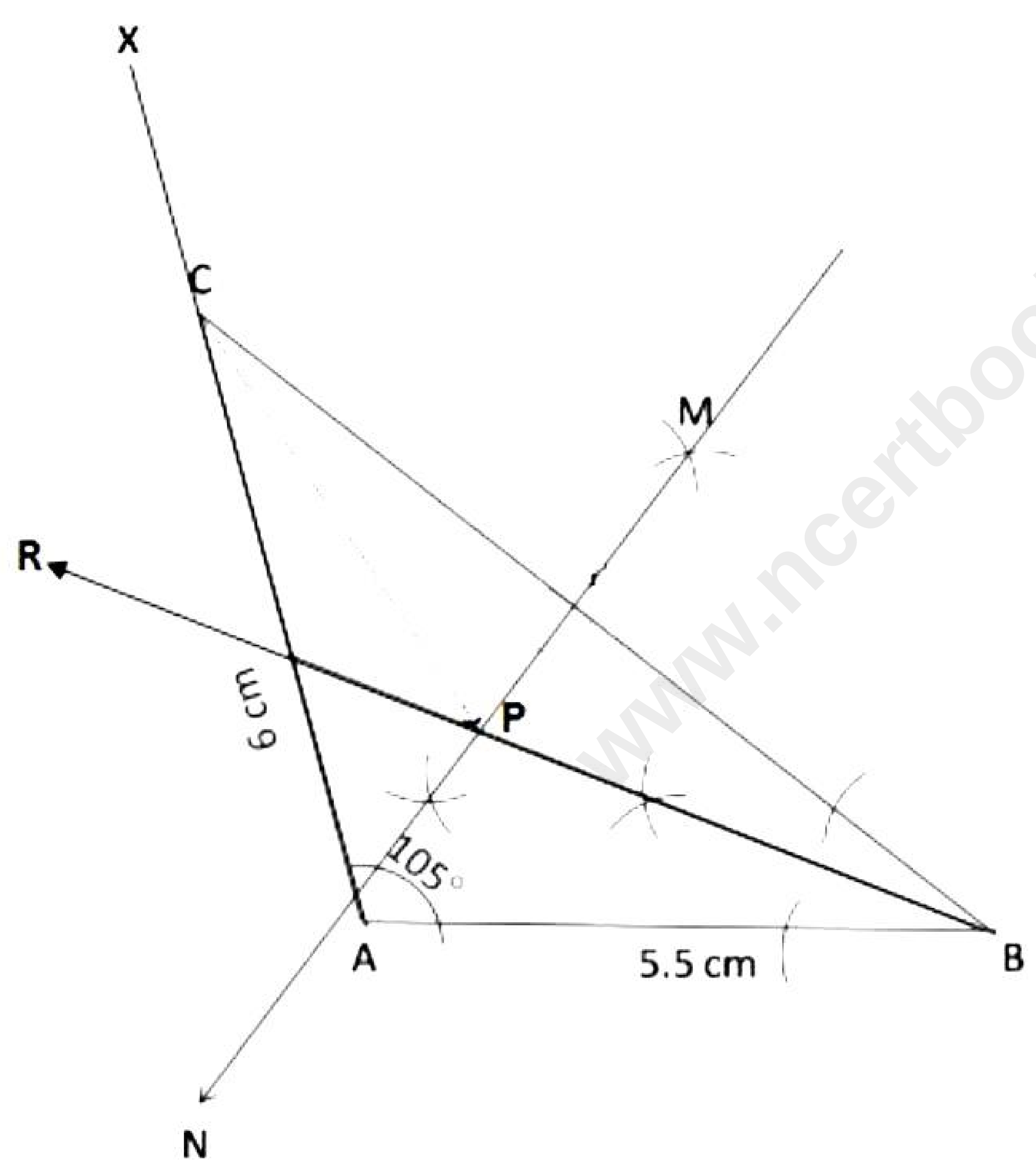
(i) Draw BR, the bisector of $\angle ABC$, which is the locus of points equidistant from BA and BC.

(ii) Draw MN, the perpendicular bisector of BC, which is the locus of points equidistant from B and C.

(iii) The angle bisector of $\angle ABC$ and perpendicular bisector of BC meet at point P.

Thus, P satisfies the above two loci.

Length of PC = 4.8 cm



Solution 10

i)

Let $p(x) = x^3 + ax^2 + bx + 9$ be the given polynomial.

$(x - 1)$ is a factor of $p(x)$.

$\Rightarrow p(1) = 0$

$\Rightarrow 1 + a + b + 9 = 0$

$\Rightarrow a + b = -10$ ... (i)

It is given that p(x) leaves the remainder 16 when it is divided by x + 1.

Therefore,

$\Rightarrow p(-1) = 16$

$\Rightarrow (-1)^3 + a(-1)^2 + b(-1) + 9 = 16$

$\Rightarrow -1 + a - b + 9 = 16$

$\Rightarrow$ a – b = 8 … (ii)

Adding (i) and (ii), we get

$2a = -2 \Rightarrow a = -1$

Then, $b = -10 + 1 = -9$

Hence, $a = -1$ and $b = -9$.

ii)

Total number of balls in a bag = 5 + 6 + 9 = 20

(i) Number of white balls = 5 = Number of favourable cases

$$\therefore P(\text{White ball}) = \frac{\text{Number of favourable cases}}{\text{Total number of balls}} = \frac{5}{20} = \frac{1}{4}$$

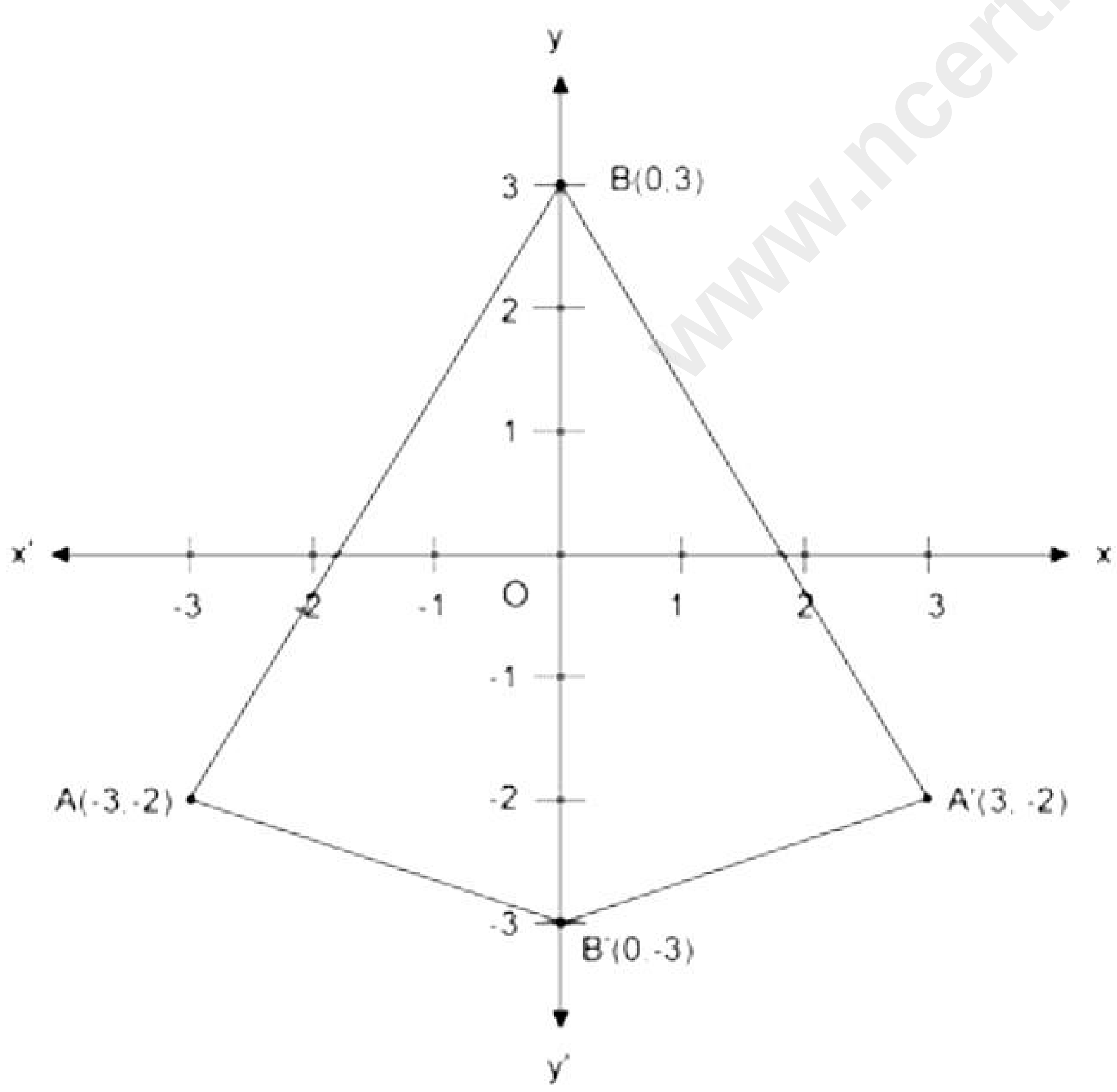
(ii) Number of white balls = 5, Number of red balls = 6

Number of favourable cases = 5 + 6 = 11

$$\therefore P(\text{White ball or Red ball}) = \frac{\text{Number of favourable cases}}{\text{Total number of balls}} = \frac{11}{20}$$

(iii) $P(\text{Neither green ball nor white ball}) = P(\text{Red ball}) = \frac{\text{Number of Red balls}}{\text{Total number of balls}} = \frac{6}{20} = \frac{3}{10}$

iii)



a) Point A(−3, −2) is reflected about the y-axis to get the image A'.
Hence, the co-ordinates of A' are (3, −2).

b) Point B(0, 3) is reflected about the x-axis to get the image B'.
Hence, the co-ordinates of B' are (0, −3).

c) Distance between A'(3, -2) and B'(0, -3) is

$$A'B' = \sqrt{(3-0)^2 + (-2+3)^2} = \sqrt{9+1} = \sqrt{10} \text{ units}$$