ICSE 2025 EXAMINATION

Sample Question Paper – 10

Mathematics

Time: 2 Hours **Max. Marks: 80**

General Instructions:

1. Answer to this Paper must be written on the paper provided separately
2. You will not be allowed to write during first 15 minutes.
3. This time is to be spent in reading the question paper.
4. The time given at the head of this Paper is the time allowed for writing the answers.
5. Attempt all questions from Section A and any four questions from Section B.
6. All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
7. Omission of essential working will result in loss of marks.
8. The intended marks for questions or parts of questions are given in brackets [ ].
9. Mathematical tables are provided.

SECTION-A

(Attempt all questions from this Section.)

QUESTION 1.
Choose the correct answers to the questions from the given options.

(Do not copy the questions, write the correct answer only.)

(i) Rahman invests ₹8400 on ₹100 shares at ₹70 if the company pays him 10% dividend then numbers of shares he buys is :

(a) 120 (b) 108
(c) 125 (d) 132

Answer: (a) 120

(ii) Varsha deposited ₹360 per month in a cumulative time deposit account with PNB for 2 years. If the rate of interest is 7% per annum, then the amount she get at the time of maturity is:

(a) ₹2,790 (b) ₹8,720
(c) ₹9,270 (d) ₹7,290

Answer: (c) ₹9,270

(iii) If $3(x-1) \geq 2(x-3)$ then

(a) $x > -3$

(b) $x < -3$

(c) $x \leq -3$

(d) $x \geq -3$

Answer: (c) $x \leq -3$

(iv) Statement 1: The roots of the quadratic equation $x^2 + 4x + 4 = 0$ are imaginary.
Statement 2 : If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

(a) Both the statement are true.
(b) Both the statement are false.
(c) Statement 1 is true and statement 2 is false.
(d) Statement 1 is false and statement 2 is true.

Answer: (d) Statement 1 is false and statement 2 is true.

(v) If $(x^2 + y^2) : (x^2 - y^2) = 17 : 8$, then $x : y$ is:

(a) 3 : 7

(b) 25 : 9

(c) 8 : 17

(d) 5 : 3

Answer: (d) 5 : 3

(vi) If $x - 2$ is a factor of $x^3 + 2x^2 - kx + 10$, then the value of k is:

(a) 23

(b) 5

(c) 13

(d) 16

Answer: (C) 13

(vii) If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 7 \end{bmatrix}$, then the values of x and y are :

(a) $x = 3,\ y = -8$

(b) $x = 4,\ y = -4$

(c) $x = 4,\ y = -8$

(d) $x = 2,\ y = -4$

Answer: (a) $x = 3,\ y = -8$

(viii) **Assertion :** $a_n - a_{n-1}$ is not independent of n then the given sequence is an AP.

Reason : Common difference $d = a_n - a_{n-1}$ is constant or independent of n.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Answer: (d) Assertion (A) is false but reason (R) is true.

(ix) If the point $P(2,1)$ lies on the line segment joining points $A(4,2)$ and $B(8,4)$, then

(a) $AP = \frac{1}{3}AB$ (b) $AP = PB$

(c) $PB = \frac{1}{3}AB$ (d) $AP = \frac{1}{2}AB$

Answer: (d) $AP = \frac{1}{2}AB$

(x) Which of the following is the slope of a line perpendicular to PQ if $P(5, -3)$ and $Q(7, 3)$

(a) $-\frac{3}{5}$ (b) $-\frac{5}{3}$

(c) $-\frac{1}{3}$ (d) $\frac{3}{2}$

Answer:

(xi) If triangle ABC is similar to triangle DEF such that 2AB = DE and BC = 8 cm then find EF.

(a) 8 cm (b) 10 cm

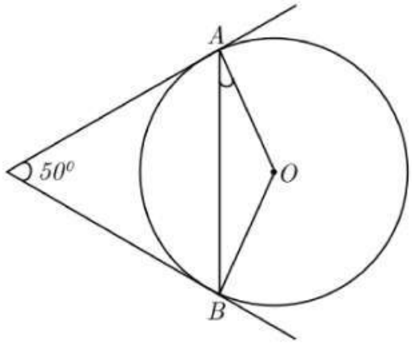
(c) 16 cm (d) 12 cm

Answer: (a) $-\frac{3}{5}$

(xii) In the given figure PA and PB are tangents to a circle with centre O. If ∠APB = 50 °, then ∠OAB is equal to

(a) 25° (b) 30°

(c) 40° (d) 50°



Answer: (a) 25°

(xiii) Volumes of two spheres are in the ratio 64 : 27. The the ratio of their surface areas is

(a) 3 : 4 (b) 4 : 3
(c) 9 : 16 (d) 16 : 9

Answer: (d) 16 : 9

(xiv) If sec 5A = cosec(A + 30°), where 5A is an acute angle, then the value of A is

(a) 15° (b) 5°
(c) 20° (d) 10°

Answer: (d) 10°

(xv) Which of the following cannot be the probability of an event?

(a) 1/3 (b) 0.1
(c) 3% (d) 17/16

Answer: (d) 17/16

QUESTION 2.

(i) The surface area of a solid metallic sphere is 2464 cm^2. It is melted and recast into solid right circular cones of radius 3.5 cm and height 7 cm. Calculate

(i) The radius of the sphere.

(ii) The number of cones recast. (take π = 22/7)

Answer:

i. Total surface area of the sphere = $4\pi r^2$, where r is the radius of the sphere.

Thus,

$4\pi r^2 = 2464 \text{ cm}^2$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 2464$

$\Rightarrow r^2 = 196$

$\Rightarrow r = 14$ cm

$\therefore R = 14$ cm

ii. Volume of sphere melted = $\frac{4}{3}\pi R^3$

$= \frac{4}{3} \times \pi \times 14 \times 14 \times 14$

Radius of each cone recasted = r = 3.5 cm

Height of each cone recasted = h = 7 cm

$\therefore$ Volume of each cone recasted = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 3.5 \times 3.5 \times 7$

$\therefore$ Number of cones recasted

$= \frac{\text{Volume of sphere melted}}{\text{Volume of each cone formed}}$

$= \frac{\frac{4}{3} \times \pi \times 14 \times 14 \times 14}{\frac{1}{3} \times \pi \times 3.5 \times 3.5 \times 7}$

$= \frac{4 \times 14 \times 14 \times 14}{3.5 \times 3.5 \times 7}$

$= 4 \times 4 \times 4 \times 2$

$= 128$

(ii) Sharukh opened a recurring deposit in a bank and deposited ₹800 per month for $1\frac{1}{2}$ years. If the received ₹15084 at the time of maturity, find the rate of interest per annum.

Answer:

Shahrukh deposited Rs. 800 per month for n = $1\frac{1}{2}$ years =18 months

∴ Total money deposited 18 x Rs. 800 Rs. 14400

Given that the maturity value = Rs 15084

Intrest paid by the banl

= Maturity Value - Total sum deposited

= 15084 - 14400

= Rs 684

Now

$$I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow 684 = 800 \times \frac{18(18+1)}{2 \times 12} \times \frac{r}{100}$$

$$\Rightarrow \frac{684 \times 2 \times 12 \times 100}{800 \times 18 \times 19} = r$$

=> r = 6%

(iii) Prove that $\frac{\tan^2\theta}{(\sec\theta - 1)^2} = \frac{1+\cos\theta}{1-\cos\theta}$.

Answer:

$$\frac{\tan^2\theta}{(\sec\theta - 1)^2} = \frac{\sec^2\theta - 1}{(\sec\theta - 1)^2}$$

$$= \frac{(\sec\theta - 1)(\sec\theta + 1)}{(\sec\theta - 1)^2}$$

$$= \frac{\sec\theta + 1}{\sec\theta - 1}$$

$$= \frac{\frac{1}{\cos\theta} + 1}{\frac{1}{\cos\theta} - 1}$$

$$= \frac{1+\cos\theta}{1-\cos\theta} \qquad \text{Hence proved}$$

QUESTION 3.

(i) A man invests ₹9,600 ₹100 shares at ₹80. if the company pays him 18% dividend, find

(i) the number of shares, he buys,
(ii) his total dividend.
(iii) his percentage return on the shares.

Answer:

1) Market value = Rs. 80

Sum invested = Rs. 9600

Number of shares = $\frac{9600}{80} = 120$

2) Nomial value (face value) = Number of shares c face value

= 120 xx Rs 100

= Rs 12000

Annual income = (Dividend)$\% \times$ $Nationalvalue$

$= \frac{18}{100} \times 12000$

= 2160

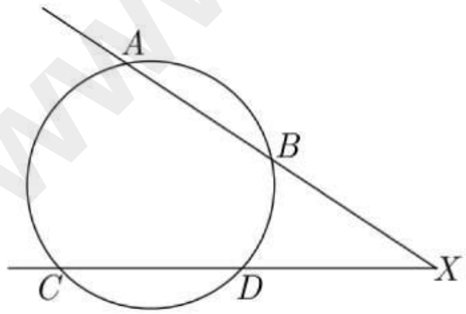
(iii) His percentage return on the shares.

$$\text{Return percentage} = \frac{\text{Total Dividend}}{\text{Total Investment}} \times 100$$

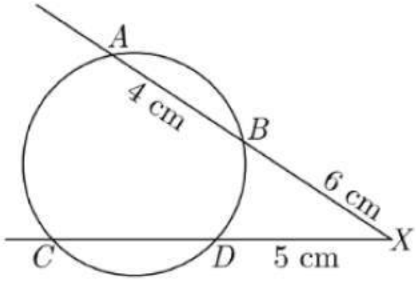
$$= \frac{2160}{9600} \times 100$$

$$= 22.5\%$$

(ii) In the given figure, chords AB and CD, when extended meet at X. Given AB = 4cm, BX = 6cm, XD = 5cm. Calculate the length of CD.



Answer:



We know that XB.XA = XD.XC

Or XB.(XB + BA) = XD.(XD + CD)

Or 6(6 + 4) = 5(5 + CD)

Or 60 = 5(5 + CD)

Or 5 + CD = $\frac{60}{5}$ = 12

Or CD = 12 – 5 = 7 cm.

(iii) The following tables shows the distribution of the heights of a group of factory workers.

Height (in cm)	Number of workers
150-155	6
155-160	12
160-165	18
165-170	20
170-175	13
175-180	8
180-185	6

(i) Determine the cumulative frequencies.

(ii) Draw the cumulative frequency curve on a graph paper.

Use 2 cm = 5 cm height on one axis and 2 cm = 10 workers on the other.

(iii) From your graph, write down the median height in cm.

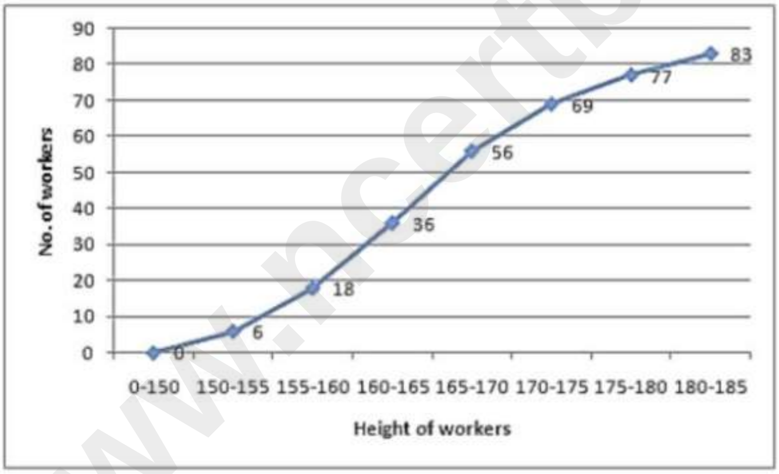
Answer:

(i)

Height (in cm)	No. of workers	c.f.
150-155	6	6
155-160	12	18
160-165	18	36
165-170	20	56
170-175	13	69
175-180	8	77
180-185	6	83

The Cumulative frequency are 6, 18, 36, 56, 69, 77, 83.

(ii)



(iii) Here $N = 83$ thus $\frac{N}{2} = \frac{83}{2}$. Now, we locate the point on the ogive, whose y-ordinate is $\frac{83}{2}$. The x co-ordinate corresponding to this ordinate is 166.3 cm. Hence, median height is 166.3 cm.

SECTION-B

(Attempt any four questions.)

QUESTION 4.

(i) Find the values of x, y, a and b if

$$\begin{bmatrix} x+y & a+b \\ a-b & x-3y \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 3 & -7 \end{bmatrix}$$

Answer:

From the given matrix equation, we have four equations:

$x + y = 5$

$a + b = -1$

$a - b = 3$

$x - 3y = -7$

Solving for a and b

From equations 2 and 3:

$a + b = -1$

$a - b = 3$

Adding and subtracting these equations:

$2a = 2$

$2b = -4$

Therefore:

$a = 1$

$b = -2$

Solving for x and y

From equations 1 and 4:

$x + y = 5$

$x - 3y = -7$

Solving these simultaneously:

$x = 2$

$y = 3$

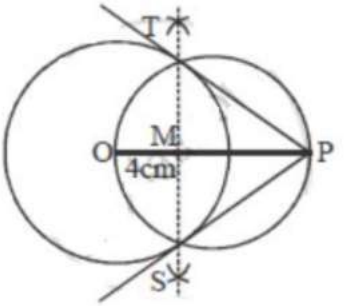
Therefore, the complete solution is:

$x = 2, y = 3, a = 1, b = -2$

(ii) Use ruler and compass only for answering this question.

Draw a circle of radius 4 cm. Mark the centre as O. Mark a point P outside the circle at a distance of 7 cm from the centre. Construct two tangents to the circle from the external point P. Measure and write down the length of any one tangent.

Answer:



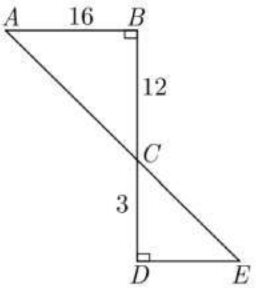
i. Draw a line segment OP = 7 cm
ii. With centre O and radius 4 cm, draw a circle.
iii. Draw the mid point of OP.
iv. With centre M and diameter OP, draw a circle which intersect the circle at T and S.
v. Joint PT and PS
PT and PS are the required tangent on measuring the length of PT = PS = 5.74 cm

(iii) In given figure AB and ED are perpendiculars to BD. AE meets BD at C. If $AB = 16$ cm, $BC = 12$ cm and $CD = 3$ cm :

(a) Show that $\Delta ABC \sim \Delta EDC$

(b) Find the lengths of DE and CE.

(c) Find area ΔABC : area ΔEDC.



Answer:

(a) Showing ΔABC ~ ΔEDC

To prove the triangles are similar, we can use the following facts:

- AB ⊥ BD and ED ⊥ BD (given)
- These perpendicular lines form corresponding angles
- Angle C is common to both triangles

Therefore, ΔABC ~ ΔEDC by AA (Angle-Angle) similarity criterion.

(b) Finding lengths of DE and CE

Using the similarity of triangles:
$\frac{BC}{DC} = \frac{AB}{DE} = \frac{AC}{EC}$

We know:

- AB = 16 cm
- BC = 12 cm
- CD = 3 cm

From the ratio of corresponding sides:
$\frac{BC}{DC} = \frac{12}{3} = 4$

Therefore:

- DE = AB ÷ 4 = 16 ÷ 4 = 4 cm
- CE can be found using Pythagorean theorem in ΔEDC:
  $CE = \sqrt{DE^2 + DC^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$ cm

(c) Finding area ratio

Area ratio of similar triangles is equal to the square of their similarity ratio:
$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta EDC} = \left(\frac{BC}{DC}\right)^2 = 4^2 = 16$

Therefore, Area of ΔABC : Area of ΔEDC = 16:1

QUESTION 5.

(i) Find the mean number of plants per house from the following data :

Number of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of houses	1	2	1	5	6	2	3

Answer:

We prepare following table to find mean.

Class	$x_i = \frac{l_1 + l_2}{2}$	f_i	$f_i x_i$
0-2	1	1	1
2-4	3	2	6
4-6	5	1	5
6-8	7	5	35
8-10	9	6	54
10-12	11	2	22
12-14	13	3	39
	Total	20	162

Mean $M = \frac{\sum f_i x_i}{\sum f_i}$

$$= \frac{162}{20} = 8.1$$

Mean number of plants per house is 8.1.

(ii) Riya bought the following articles from a departmental store :

S. No.	**Item**	**Price**	**Rate of GST**
1.	Fruit Juice	₹300	12%
2.	Coffee	₹1200	5%

Find the :

(a) Total GST paid.

(b) Total bill amount including GST.

Answer:

Step 1: Calculate GST for each item

The formula to calculate GST is:

$$\text{GST} = \text{Price} \times \frac{\text{Rate of GST}}{100}.$$

1. **GST on Fruit Juice:**

$$\text{GST on Fruit Juice} = 300 \times \frac{12}{100}.$$

Simplify:

$$\text{GST on Fruit Juice} = 300 \times 0.12 = 36\,₹.$$

2. **GST on Coffee:**

$$\text{GST on Coffee} = 1200 \times \frac{5}{100}.$$

Simplify:

$$\text{GST on Coffee} = 1200 \times 0.05 = 60\,₹.$$

Step 2: Find the Total GST Paid

The total GST paid is the sum of GST on both items:

$$\text{Total GST} = \text{GST on Fruit Juice} + \text{GST on Coffee}.$$

Substitute the values:

$$\text{Total GST} = 36 + 60 = 96\,₹.$$

Step 3: Find the Total Bill Amount Including GST

The total bill amount is:

$$\text{Total Bill} = \text{Price of Fruit Juice} + \text{Price of Coffee} + \text{Total GST}.$$

Substitute the values:

$$\text{Total Bill} = 300 + 1200 + 96.$$

Simplify:

$$\text{Total Bill} = 1596\,₹.$$

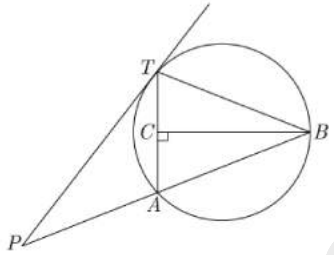
Final Answers:

(a) Total GST paid = ₹96.
(b) Total bill amount including GST = ₹1596.

(iii) In the given figure, PT is a tangent to the circle at T, chord BA is produced to meet the tangent at P. Perpendicular BC bisects the chord TA at C. If $PA = 9$ cm and $TB = 7$ cm, find the lengths of :

(a) AB

(b) PT



Answer:

We have redrawn the figure below.

(a) AB

In ΔABC and ΔTBC :

Since BC is perpendicular bisector of TA,

$$TC = CA$$

$$\angle TCB = \angle ACB = 90^\circ$$

Due to common angle,

$$BC = BC$$

By SAS similarity criterion we get

$$\Delta ABC \cong \Delta TBC$$

By CPCT we get

$$TB = AB$$

Then, $AB = TB$

$$= 7 \text{ cm}$$

(b) PT

Using properly $PA \times PB = PT^2$

$$9 \times (9 + 7) = PT^2$$

$$PT^2 = 9 \times 16$$

$$PT = 3 \times 4$$

$$= 12 \text{ cm}$$

QUESTION 6.

(i) The 5^{th}, 8^{th} and 11^{th} terms of a G.P. are p, q and s respectively. Show that $q^2 = ps$.

Answer:

Let a be the first term and r be the common ratio of given GP.

In given GP, we have

$$a^5 = p,\ a_8 = q \text{ and} a_{11} = s$$

Let a be the first term and r be the common ratio.

From $a_n = ar^{n-1}$ we can write

$$a_5 = ar^6 = p$$

$$a_8 = ar^7 = q$$

and $$a_{11} = ar^{10} = s$$

Now, $$q^2 = ps$$

$$ar^8 \times ar^8 = ar^5 \times ar^{11}$$

$$a^2 r^{16} = a^2 r^{16}$$

LHS = RHS Hence Proved

(ii) The digit of a positive number of three digits are in AP and their sum is 15. The number obtained by reversing the digits is 594 less then the original number. Find the number.

Answer:

Let digits of the number be (a – d), a and (a + d) respectively.

∴ The required number is 100 (a – d) + 10a + (a + d) .

Given : The sum of the digits = 15

⇨ (a-d) + a + (a+d) = 15

3a=15⇨ a = 5

Now, the number on reversing the digits is 100 (a + d) + 10a + (a – d) .

According to the question

100(a – d) + 10a + a + d = 100 (a + d) + 10a + (a – d) + 594

on solving we get, d = -3

The digits of the number are (5 – (–3)), 5, (5 + (–3) = 8, 5, 2

And the required number is 8 × 100 + 5 × 10 + 2 = 852

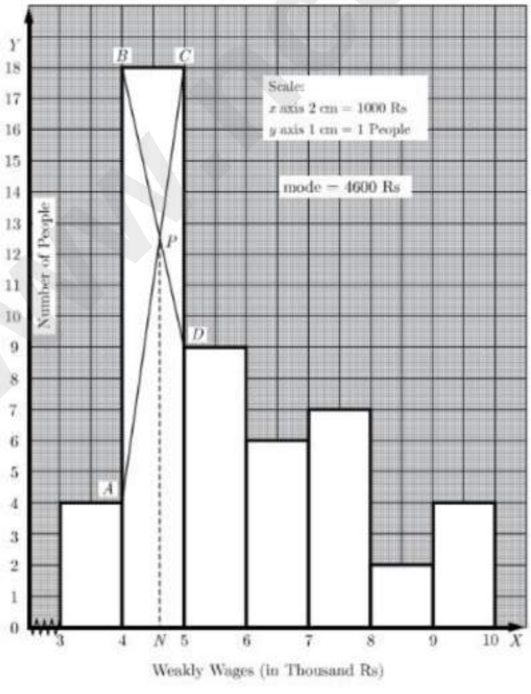
(iii) Draw a Histogram for the given data, using a graph paper.

Weekly wages (in ₹)	Number of people
3000-4000	4
4000-5000	18
5000-6000	9
6000-7000	6
7000-8000	7
8000-9000	2
9000-10000	4

Estimate the mode from the graph.

Answer:

The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 4600. Hence, the required mode is 4600.

QUESTION 7.

(i) M and N are two points on the X-axis and Y-axis, respectively. P(3, 2) divides the line segment MN in the ratio 2 : 3.

Find

(a) the coordinates of M and N.

(b) slope of the line MN.

Answer:

Let coordinates of M is (a, 0) and M is (0, b).

Point P divides MN in 2 : 3 ratio

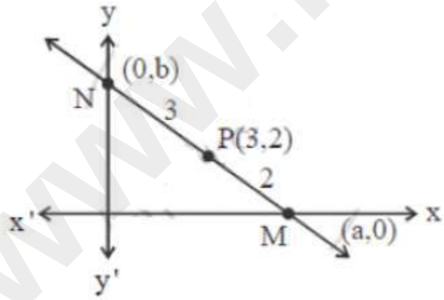
$$\therefore 3 = \frac{3a + 2 \times 0}{3 + 2} \text{ and } 2 = \frac{3 \times 0 + 2 \times b}{3 + 2}$$

3a = 15 and 10 = 2b

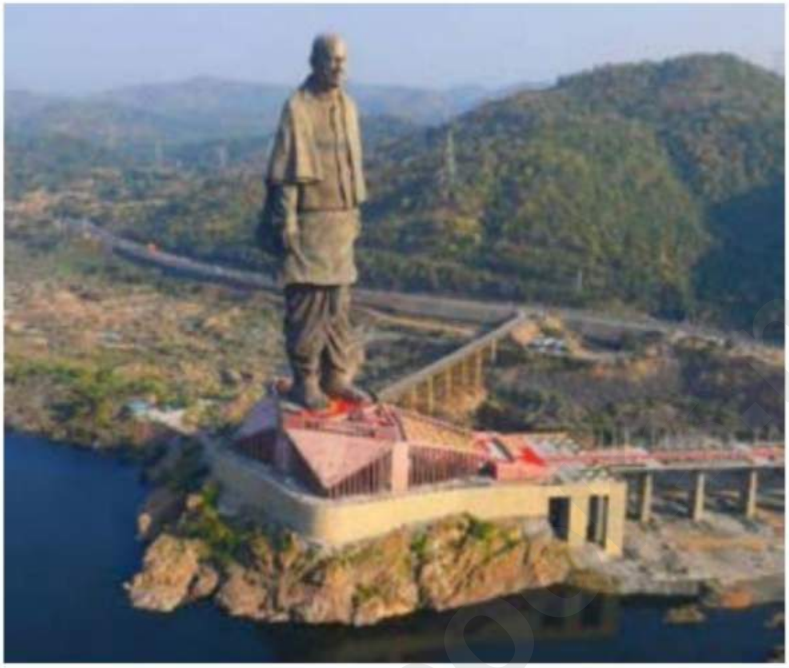
a = 5 and b = 5

(i) The coordinates of M is (5, 0) and N(0, 5)

(ii) Slope of the line MN $= \frac{0 - 5}{5 - 0} = -1$



(ii) Statue of Unity is a colossal statue of Indian statesman and independence activist Sardar Vallabh bhai Patel, who was the first Deputy Prime Minister and Home minister of independent India.



(a) For a person standing 240 m from the center of the base of the statue, the angle of elevation to the top of the statue is 45°. How tall is the statue?

(b) A cop in helicopter near the top of the statue, notices a car wreck some distance from the statue. If the angle of depression from the cop's eyes to the wreck is 60°, how far away is the accident from the centre of base of the statue?

Answer:

Part (a): Height of the Statue

When angle of elevation is 45° and distance is 240 m:

- Using tangent ratio: $\tan(45^\circ) = \frac{\text{height}}{\text{distance}}$
- $\text{height} = 240 \times \tan(45^\circ)$
- Since $\tan(45^\circ) = 1$, the height is 240 meters

Part (b): Distance of the Car Wreck

Given:

- Height = 240 m (from part a)
- Angle of depression = 60° (angle of elevation = 60°)
- Using tangent ratio: $\tan(60^\circ) = \frac{\text{height}}{\text{distance}}$
- Therefore: $\text{distance} = \frac{\text{height}}{\tan(60^\circ)} = \frac{240}{\tan(60^\circ)}$
- Distance ≈ 138.56 meters

The Statue of Unity is indeed a remarkable engineering achievement, standing against the backdrop of the Narmada River and surrounding hills

QUESTION 8.

(i) Solve the following in equation, and graph the solution on the number line:

$2x - 5 \leq 5x + 4 < 11,\ x \in R.$

Answer:

$$2x - 5 \leq 5x + 4 < 11,\ x \in \mathbb{R}.$$

This is a **double inequality**, so we split it into two parts:

1. $2x - 5 \leq 5x + 4,$
2. $5x + 4 < 11.$

Step 1: Solve $2x - 5 \leq 5x + 4$

$$2x - 5 \leq 5x + 4.$$

- Subtract $2x$ from both sides:

$$-5 \leq 3x + 4.$$

- Subtract 4 from both sides:

$$-5 - 4 \leq 3x.$$

Simplify:

$$-9 \leq 3x.$$

- Divide both sides by 3:

$$x \geq -3.$$

Step 2: Solve $5x + 4 < 11$

$$5x + 4 < 11.$$

- Subtract 4 from both sides:

$$5x < 7.$$

- Divide both sides by 5:

$$x < \frac{7}{5}.$$

Step 3: Combine the Results

From Step 1 and Step 2, the solution is:

$$-3 \leq x < \frac{7}{5}.$$

This is the range of x that satisfies the double inequality.

Step 4: Graph the Solution on the Number Line

To represent the solution $-3 \leq x < \frac{7}{5}$ on the number line:

1. Draw a solid circle at $x = -3$ to indicate **inclusive**.

Step 4: Graph the Solution on the Number Line

To represent the solution $-3 \leq x < \frac{7}{5}$ on the number line:

1. Draw a solid circle at $x = -3$ to indicate **inclusive**.

2. Draw an open circle at $x = \frac{7}{5}$ (or 1.4) to indicate **not inclusive**.
3. Shade the region between -3 and $\frac{7}{5}$.

Final Answer:

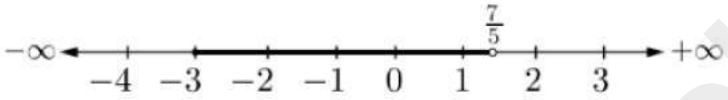
The solution is:

$$\boxed{-3 \leq x < \frac{7}{5}}.$$

or $-3 \leq x < \frac{7}{5}$ where $x \in R$

Hence solution set is $\{x : -3 \leq x < \frac{7}{5}, x \in R,\}$

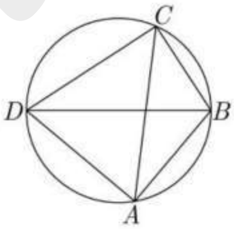
The graph of the solution set is shown by the thick portion of the number line. The solid circle at -3 indicates that the number -3 is included among the solutions and the open circle at $\frac{7}{5}$ indicates that the number $\frac{7}{5}$ is not included among the solutions.



(ii) In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of $\angle CAQ$ and $\angle PAC$. If $\angle BAQ = 30^\circ$, prove that

(a) BD is a diameter of the circle.

(b) ABC is an isosceles triangle.



Answer:

Part (a): Proving BD is a diameter

1. Given that PQ is tangent at A, we know that radius at point A is perpendicular to tangent PQ.
2. AB and AD are bisectors of ∠CAQ and ∠PAC respectively.
    - Given that ∠BAQ = 30°
    - Since AB bisects ∠CAQ, ∠CAB = ∠BAQ = 30°
    - Similarly, since AD bisects ∠PAC, ∠PAD = ∠DAC = 30°

3. In circle geometry:

- ∠BAD = 60° (sum of ∠BAQ + ∠PAD = 30° + 30°)
- ∠BCD = 120° (angles in same segment)
- ∠BDC = 30° (angles in a circle sum to 180°)

4. Therefore:

- ∠CBD = 30° (isosceles triangle property)
- ∠CBD + ∠CDB = 60° = 180° ÷ 2

Since the angle subtended by BD at the center is 180°, BD is a diameter.

Part (b): Proving ABC is isosceles

1. Since BD is a diameter (proved in part a):

- ∠BAD = 60° (proved earlier)
- ∠BCD = 120° (angle in semicircle)

2. In triangle ABC:

- ∠ABC = 60° (angle in alternate segment)
- ∠BCA = 60° (∠BCD - ∠BAD = 120° - 60°)
- ∠CAB = 60° (angles in triangle sum to 180°)

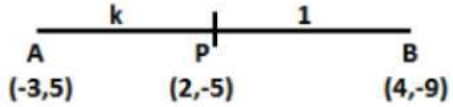
3. Since all angles in triangle ABC are 60°:

- ABC is equilateral
- Therefore, ABC is also isosceles

Thus, we have proved both parts (a) and (b).

(iii) In what ratio does the point P(2, –5) divide the line segment joining the points A(-3, 5) and B(4, –9)?

Answer: Let point P(2, –5) divide line segment joining points A(–3, 5) and B (4, –9) in the ratio k : 1.



Therefore, the coordinates of P is (2, –5) = $\left(\frac{4k-3}{k+1}, \frac{-9k+5}{k+1}\right)$.(by division formula)

By comparing x & y coordinates of P, we get $\frac{4k-3}{k+1}$ = 2 & $\frac{-9k+5}{k+1}$ = -5.

⇒ 4k – 3 = 2k + 2 & – 9k + 5 = – 5k – 5

⇒ 4k – 2k = 2 + 3 & – 5k + 9k = 5 + 5

⇒ 2k = 5 & 4k = 10

⇒ k = $\frac{5}{2}$

⇒ $\frac{k}{1} = \frac{5}{2}$

⇒ k : 1 = 5 : 2.

Hence, point P divide line segment AB in the ratio 5 : 2.

QUESTION 9.
(i) What number must be added to each of the numbers 4, 6, 8, 11 in order to get the four numbers in proportion?

Answer:

Let x be added to make them in proportion.

∴ (4 + x) : (6 + x) :: (8 + x) : (11 + x)

⇒ (4 + x)(11 + x) = (6 + x)(8 + x)

⇒ 44 + 4x + 11x + x^2 = 48 + 6x + 8x + x^2

⇒ 44 + 15x = 48 + 14x

⇒ 15x – 14x = 48 – 44

⇒ x = 4

(ii) A two digit number is four times the sum of the digits. It also equal to 3 times the product of digits. Find the number.

Answer:

Let the digits of the required number be x and y.

Now, the required number is $10x + y$.

According to the question,

$10x + y = 4(x + y)$

So,

$6x - 3y = 0$

$\Rightarrow 2x - y = 0$

$x = \frac{y}{2}$ .....(1)

Also,

$10x + y = 3xy$ .....(2)

From (1) and (2), we get

$$10\left(\frac{y}{2}\right) + y = 3\left(\frac{y}{2}\right)y$$

$$\Rightarrow 5y + y = \frac{3}{2}y^2$$

$$\Rightarrow 6y = \frac{3}{2}y^2$$

$$\Rightarrow y^2 - 4y = 0$$

$$\Rightarrow y(y-4) = 0$$

$$\Rightarrow y = 0, 4$$

So, $x = 0$ for $y = 0$ and $x = 2$ for $y = 4$.

Hence, the required number is 24.

(iii) Use ruler and compass only for this question.

(a) Construct ΔABC, where $AB = 3.5$ cm, $\angle ABC = 60^\circ$.

(b) Construct the locus of points inside the triangle, which are equidistant from BA and BC.

(c) Construct the locus of points inside the triangle which are equidistant from B and C.

(d) Mark the point P which is equidistant from AB, BC and also equidistant from B and C. Measure and record the length of PB.

Answer:

Steps of Construction :

Step 1 : Draw a line segment $BC = 6$ cm.

Step 2 : From the point B, draw $\angle XBC = 60^\circ$.

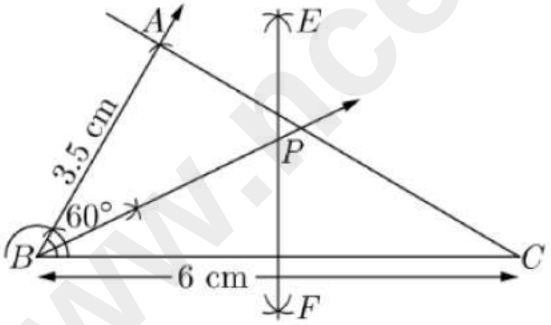
Step 3 : Taking radius 3.5 cm, with B as centre cut $BA = 3.5$ cm from BX.

Step 4 : Join A to C, which is the required ΔABC.

Step 5 : Draw angle bisector of $\angle ABC$, which is the locus of points inside the triangle, which are equidistant from BA and BC.

Step 6 : Draw perpendicular bisector EF of BC, which intersects the angle bisector at point P. Then, EF is the locus of points inside the triangle, which are equidistant from B and C.

Step 7 : The intersection point of angle bisector and EF is the required point P, which is equidistant from AB, BC, B and C. On measuring the length of PB is 3.5 cm.



QUESTION 10.

(i) Factorize completely using factor theorem:

$2x^3 - x^2 - 13x - 6$

Answer:

Step 1: Check for possible roots using the **Rational Root Theorem**

The **Rational Root Theorem** states that any rational root of the polynomial $P(x) = 2x^3 - x^2 - 13x - 6$ is of the form:

$$\text{Possible roots} = \pm\frac{\text{factors of constant term}}{\text{factors of leading coefficient}}$$

- Constant term = -6, factors are $\pm1, \pm2, \pm3, \pm6$.
- Leading coefficient = 2, factors are $\pm1, \pm2$.

Thus, the possible roots are:

$$\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}$$

Step 2: Test the roots using substitution

Let's test each possible root by substituting into $P(x) = 2x^3 - x^2 - 13x - 6$.

Testing $x = -2$:

Substitute $x = -2$ into $P(x)$:

$$P(-2) = 2(-2)^3 - (-2)^2 - 13(-2) - 6$$

$$P(-2) = 2(-8) - 4 + 26 - 6$$

$$P(-2) = -16 - 4 + 26 - 6 = 0$$

Since $P(-2) = 0$, $x = -2$ is a root of the polynomial.

Step 3: Factorize using $(x + 2)$

Now that $x = -2$ is a root, we can factorize $P(x)$ as:

$$P(x) = (x + 2)Q(x),$$

where $Q(x)$ is a quadratic polynomial obtained by dividing $P(x)$ by $(x + 2)$.

We perform polynomial division:

1. Divide the first term $2x^3$ by x:
   Result = $2x^2$.

2. Multiply $(x+2)$ by $2x^2$:
   $2x^2(x+2) = 2x^3 + 4x^2$.
3. Subtract $(2x^3 + 4x^2)$ from $2x^3 - x^2$:
   Result = $-5x^2$.
4. Divide $-5x^2$ by x:
   Result = $-5x$.
5. Multiply $(x+2)$ by $-5x$:
   $-5x(x+2) = -5x^2 - 10x$.
6. Subtract $(-5x^2 - 10x)$ from $-5x^2 - 13x$:
   Result = $-3x$.
7. Divide $-3x$ by x:
   Result = -3.
8. Multiply $(x+2)$ by -3:
   $-3(x+2) = -3x - 6$.
9. Subtract $(-3x - 6)$ from $-3x - 6$:
   Remainder = 0.

Thus, the quotient is $Q(x) = 2x^2 - 5x - 3$, and we have:

$$P(x) = (x+2)(2x^2 - 5x - 3).$$

Step 4: Factorize the quadratic $2x^2 - 5x - 3$

We now factorize $2x^2 - 5x - 3$ by splitting the middle term:

1. Find two numbers whose product is $2 \times (-3) = -6$ and whose sum is -5:
   These numbers are -6 and 1.
2. Split the middle term $-5x$ into $-6x$ and x:

$$2x^2 - 5x - 3 = 2x^2 - 6x + x - 3.$$

3. Group terms and factorize:

$$2x^2 - 6x + x - 3 = 2x(x-3) + 1(x-3).$$

4. Factor out the common term $(x-3)$:

$$2x^2 - 5x - 3 = (2x+1)(x-3).$$

Step 5: Write the complete factorization

Substitute back into the original polynomial:

$$P(x) = (x+2)(2x^2 - 5x - 3).$$

$$P(x) = (x+2)(2x+1)(x-3).$$

Final Answer:

The complete factorization of $2x^3 - x^2 - 13x - 6$ is:

$$\boxed{(x+2)(2x+1)(x-3)}.$$

(ii) A circular dartboard has a total radius of 8 inch, with circular bands that are 2 inch wide, as shown in figure. Abhinav is skilled enough to hit this board 100% of the time so he always score at least two points each time he throw a dart. Assume the probabilities are related to area, on the next dart that he throw.

(a) What is the probability that he score at least 4 ?

(b) What is the probability that he score at least 6 ?

(c) What is the probability that he hit bull's eye ?

(d) What is the probability that he score exactly 4 points ?

Answer:

(a) Probability that Abhinav scores at least 4 points

To score **at least 4 points**, Abhinav must hit:

- The **4-point region**,
- The **6-point region**, or
- The **bull's eye** (8 points).

The combined area for these regions is:

$$\text{Area (at least 4 points)} = \text{Area of Bull's Eye} + \text{Area of 4 points} + \text{Area of 6 points}.$$

$$\text{Area (at least 4 points)} = 4\pi + 12\pi + 20\pi = 36\pi.$$

The probability is:

$$P(\text{at least 4 points}) = \frac{\text{Area (at least 4 points)}}{\text{Total Area}} = \frac{36\pi}{64\pi} = \frac{36}{64} = \frac{9}{16}.$$

(b) The probability that you score at least 6,
Let E_2 be the event that she hit the circle in the radius of 4 inch to score 6. Favorable outcome will be area of this 4 inch circle.

Favorable outcome, $n(E_2) = \pi 4^2 \text{ in}^2$

Probability, $P(E_2) = \frac{n(E_2)}{n(S)}$

$$= \frac{\pi 4^2}{\pi 8^2} = \frac{1}{4}$$

(c) Probability that Abhinav hits the bull's eye

The bull's eye has an area of 4π. The probability is:

$$P(\text{bull's eye}) = \frac{\text{Area of Bull's Eye}}{\text{Total Area}} = \frac{4\pi}{64\pi} = \frac{4}{64} = \frac{1}{16}.$$

(d) The probability that you score exactly 4 points,
Let E_4 be the event that she hit the circular band in the radius of 4 inch to 6 inch. Favorable outcome will be area of this circular band.

Favorable outcome, $n(E_4) = \pi(6^2 - 4^2) \text{ in}^2$

Probability, $P(E_4) = \frac{n(E_4)}{n(S)}$

$$= \frac{\pi(6^2 - 4^2)}{\pi 8^2}$$

$$= \frac{20}{64} = \frac{5}{16}$$

(iii) Use a graph paper for this question (take 10 small divisions = 1 unit on both the axes). P and Q have coordinates (0, 5) and (–2, 4).

(a) P is invariant, when reflected in an axis. Name the axis.

(b) Find the image of Q on reflection in the axis found in (i).

(c) (0, k) on reflection in the origin in invariant. What the value of k.

(d) Write the coordinates of the image of Q, obtained by reflection it in origin followed in X -axis.

Answer:

(i) The axis is y-axis or x = 0.

(ii) Image of 'Q'

$Q' = M_{x=0}(-2, 4)$

$= (2, 4)$

(iii) $\because M_0 (a, b) = (-a, -b)$

$\therefore M_0 (0, k) = (o, -k)$

$\therefore k = -k$

$\therefore 2k = 0$

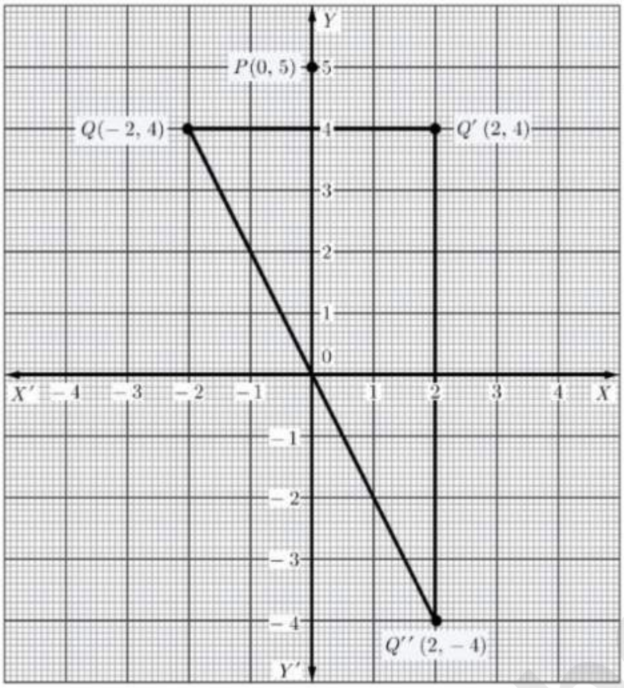
$\therefore k = 0$

(iv) Q'' = MxMoQ

$= M_x M_0 (-2, 4)$

$= Mx (2, -4)$

$= (-2, -4)$



Y
P(0, 5)
Q(− 2, 4)
Q′ (2, 4)
X′
X
Y′
Q′′ (2, − 4)