ICSE 2025 EXAMINATION

Sample Question Paper - 8

Mathematics

Time: 2 Hours **Max. Marks: 80**

General Instructions:

1. Answer to this Paper must be written on the paper provided separately
2. You will not be allowed to write during first 15 minutes.
3. This time is to be spent in reading the question paper.
4. The time given at the head of this Paper is the time allowed for writing the answers.
5. Attempt all questions from Section A and any four questions from Section B.
6. All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
7. Omission of essential working will result in loss of marks.
8. The intended marks for questions or parts of questions are given in brackets [ ].
9. Mathematical tables are provided.

SECTION-A

(Attempt all questions from this Section.)

QUESTION 1.

Choose the correct answers to the questions from the given options.

(Do not copy the questions, write the correct answer only.)

(i) Which of the following is the equation of the line through (1, 3) making an intercept of 5 on the y-axis.

(a) $2x - y - 6 = 0$ (b) $2x + y + 6 = 0$

(c) $2x + y - 5 = 0$ (d) $2x - y + 5 = 0$

Answer: (c) $2x + y - 5 = 0$

(ii) The sum of first five multiples of 3 is

(a) 45 (b) 55

(c) 65 (d) 75

Answer: (a) 45

(iii) The third proportional to 2 and 7 is:

(a) $\sqrt{14}$ (b) 24.5

(c) 14 (d) 9

Answer: (b) 24.5

(iv) If $3x - 7 > 5x - 1 \;\forall\; x \in R$ then

(a) $x > -3$

(b) $x < -3$

(c) $x \leq -3$

(d) $x \geq -3$

Answer: (b) $x < -3$

(v) Statement 1 : The equation $x^2 + 5x + 2 = (x - 3)^2$ is a quadratic equation.
Statement 2 : Any equation of the form ax2 + bx + c = 0 where α ≠ 0, is called a quadratic equation.

(a) Both the statement are true.
(b) Both the statement are false.
(c) Statement 1 is true and statement 2 is false.
(d) Statement 1 is false and statement 2 is true.

Answer: (d) Statement 1 is false and statement 2 is true.

(vi) If both (x – 2) and (x + 3) are the factors of the expression $x^3 + ax^2 + bx - 12$, then the value of (a + b) is:

(a) 14 (b) – 4
(c) 6 (d) – 1

Answer: (d) – 1

(vii) Ruhee invests ₹9620 on ₹100 shares at ₹80 if the company pays him 18% dividend then his percentage return on shares is :

(a) 22.0% (b) 22.5%
(c) 20.5% (d) 20.6%

Answer: (b) 22.5%

(viii) Mr. Nair get ₹ 6,455 at the end of one year at the rate of 14% per annum in a recurring deposit amount. The monthly instalment is:

(a) ₹400 (b) ₹500

(c) ₹600 (d) ₹700

Answer: (b) ₹500

(ix) If $M = [1, 2]$ and $N = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, then the order of matrix MN is :

(a) 1×2 (b) 2×1

(c) 2×2 (d) 1×1

Answer: (d) 1 × 1

(x) If P(α/3, 4) is the mid-point of the line segment joining the points Q(-6, 5) and R(-2, 3), then the value of α is

(a) – 4 (b) – 12

(c) 12 (d) – 6

Answer: (b) – 12

(xi) A 6 m high tree cast a 4 m long shadow. At the same time, a flag pole cast a shadow 50 m long. How long is the flag pole?

(a) 48 m (b) 150 m

(c) 16 m (d) 75 m

Answer: (d) 75 m

(xii) Assertion : PA and PB are two tangents to a circle with centre O. Such that ∠AOB = 110°, then ∠APB = 90°.

Reason : The length of two tangents drawn from an external point are equal.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Answer: (d) Assertion (A) is false but reason (R) is true.

(xiii) The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3, what is the ratio of their volumes ?

(a) 10 : 9
(b) 27 : 20
(c) 9 : 10
(d) 20 : 27

Answer: (d) 20 : 27

(xiv) If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, than $x^2 + y^2$ is equal to

(a) 0
(b) 1/2
(c) 1
(d) 3/2

Answer: (c) 1

(xv) The money required to buy 100, ₹50 shares quoted at ₹48.50 is:

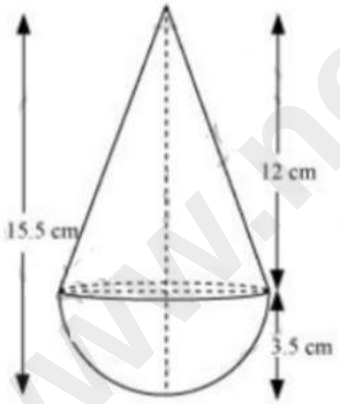
(a) ₹5000
(b) ₹2425
(c) ₹2525
(d) 4850

Answer: (d) 4850

QUESTION 2.

(i) A toy is in the form of a cone radius 3.5 cm mounted on a hemisphere of same radius. If the total height of the toy is 15.5 cm, find the total surface area of the toy. (Use π = 22/7)

Answer:



Radius of hemisphere = 3.5 cm

total height of the toy = 15.5 cm.

Surface area of cone

$= \pi r l$

$l = \sqrt{(12)^2 + (3.5)^2}$

$= \sqrt{156.25}$

$= 12.5cm$

Therefore,

Surface area of cone

$= \frac{22}{7} \times 3.5 \times 12.5$

$= 137.5cm^2$

Surface area of hemisphere

$= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 3.5 \times 3.5$

$= 77cm^2$

Therefore,

Total surface area of the toy

$= 137.5 + 77$

$= 214.5cm^2$

Volume of cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (3.51^2 \times 12)$

$= 154cm^2$

Volume of hemisphere

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$

$= 89.83cm$

Therefore,

Total volume of the toy

$= (154 + 89.83)cm^3$

$= 243.83cm^3$

(ii) Mr. Satish deposited ₹300 at the beginning of every month in a bank paying 8% S.I. per annum in a recurring deposit account. How much would he receive at the end of 4 years?

Answer:

Let's plug in the values we know:

- P = ₹300
- n = 4 years × 12 months = 48 months
- r = 8% = 0.08

Now, let's calculate:

$$A = 300 \times 48 + \frac{300 \times 48 \times (48+1)}{2} \times \frac{0.08}{12}$$

Breaking this down further:

1. Principal amount: 300 × 48 = ₹14,400
2. Interest calculation: $\frac{300 \times 48 \times 49}{2} \times \frac{0.08}{12} = ₹2,352$
3. Total amount: ₹14,400 + ₹2,352 = ₹16,752

(iii) Prove that :

$$\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \tan\theta + \cot\theta.$$

Answer:

Step 1: Examine the Left-Hand Side (LHS)

Let's simplify $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$ separately.

Step 2: First Term

$\frac{\tan\theta}{1-\cot\theta} = \frac{\tan\theta \cdot \tan\theta}{\tan\theta - 1} = \frac{\tan^2\theta}{\tan\theta - 1}$

Step 3: Second Term

$\frac{\cot\theta}{1-\tan\theta} = \frac{1}{\tan\theta(1-\tan\theta)} = \frac{1}{\tan\theta - \tan^2\theta}$

Step 4: Combine Terms

When we combine these terms and find a common denominator, we get:

$\frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{\tan\theta - \tan^2\theta} \neq 1 + \tan\theta + \cot\theta$

QUESTION 3.

(i) Reena invested ₹12,000 and purchased 1000 shares of par value ₹10 each. Answer the following :

(i) How much above par is the market value of these shares ?

(ii) If the dividend per share is 15%, find Hina's income from these shares.

Answer:

(i) Market value of shares

The market value per share can be calculated as:

$$\text{Market Value per Share} = \frac{\text{Total Investment}}{\text{Number of Shares}}$$

Substituting the values:

$$\text{Market Value per Share} = \frac{12,000}{1,000} = ₹12$$

The par value of each share is ₹10. Hence, the market value is above par by:

$$₹12 - ₹10 = ₹2$$

Answer (i): The market value of shares is ₹2 above par.

(ii) Income from dividend

Dividend per share is calculated as:

$$\text{Dividend per Share} = \frac{\text{Dividend Rate}}{100} \times \text{Par Value of Each Share}$$

Substituting the values:

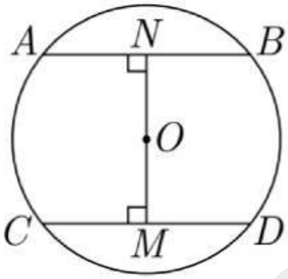
$$\text{Dividend per Share} = \frac{15}{100} \times 10 = ₹1.5$$

Hina holds 1,000 shares. Therefore, her total income from dividends is:

$$\text{Total Income} = 1,000 \times 1.5 = ₹1,500$$

Answer (ii): Hina's income from these shares is ₹1,500.

(ii) AB and CD are two parallel chords of a circle such that AB = 24 cm and CD = 10 cm. If the radius of the circle is 13 cm, find the distance between the two chords.



Answer:

To find the distance between the two parallel chords AB and CD, we'll follow these steps:

1. Calculate the distances of each chord from the center of the circle.
2. Subtract these distances to find the distance between the chords.

Let's denote the radius as r = 13 cm, AB = 24 cm, and CD = 10 cm.

For a chord in a circle, we can use the formula: $d^2 = r^2 - (\frac{l}{2})^2$
Where d is the distance from the center to the chord, r is the radius, and l is the length of the chord.

For chord AB:
$d_1^2 = 13^2 - (\frac{24}{2})^2 = 169 - 144 = 25$
$d_1 = 5 \text{ cm}$

For chord CD:
$d_2^2 = 13^2 - (\frac{10}{2})^2 = 169 - 25 = 144$
$d_2 = 12 \text{ cm}$

The distance between the chords is the difference between these distances:

Distance = $d_2 - d_1 = 12 - 5 = 7 \text{ cm}$

Therefore, the distance between the two parallel chords AB and CD is 7 cm.

(iii) The weight of 50 workers is given below :

Weight (kg)	Number of workers
50-60	4
60-70	7
70-80	11
80-90	14
90-100	6
100-110	5
110-120	3

Draw an ogive of the given distribution using a graph sheet. Take 2 cm = 10 kg on one axis and 2 cm = 5 workers along the other axis. Use a graph to estimate the following :

(i) the upper and lower quartiles.

(ii) if weighing 95 kg and above is considered overweight, find the number of workers who are overweight.

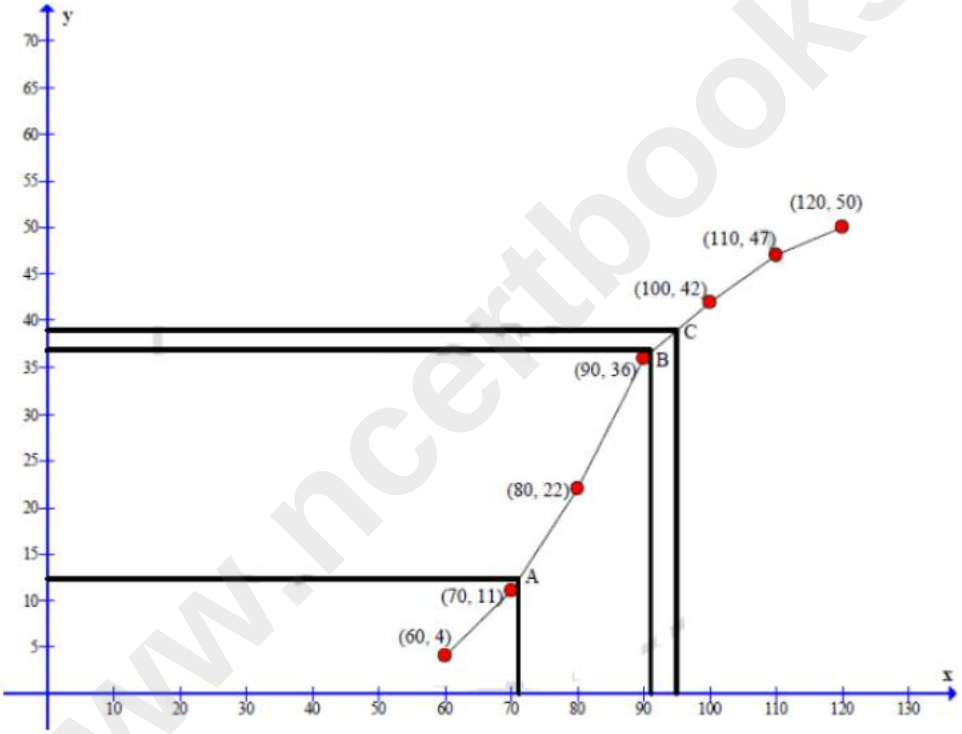
Answer:

The cumulative frequency table of the given distribution table is as follows:

Weight in Kg	Number of workers	Cumulative frequency
50-60	4	4

60-70	7	11
70-80	11	22
80-90	14	36
90-100	6	42
100-110	5	47
110-120	3	50

The ogive is as follows:



Number of worker = 50

1) Upper quartile (Q3) = $\left(\frac{3 \times 50}{4}\right)^{th}$ term = $(37.5)^{t}h$ term = 92

Lower quartile (Q_1) = $\left(\frac{50}{4}\right)^{th}$ term = $(12.5)^{th}$ term = 71.1

2) Through mark of 95 on the x-axis, draw a vertical line which meets the graph at point C.

Then through point C, draw a horizontal line which meets the y-axis at the mark of 39

Thus, number of workers weighing 95 kg and above = 50 - 39 = 11

SECTION-B

(Attempt any four questions.)

QUESTION 4.

(i) Find x and y, if $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$.

Answer:

We have $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix} + 3\begin{bmatrix} -2 \\ 1 \end{bmatrix} = 2\begin{bmatrix} y \\ 3 \end{bmatrix}$

$$\begin{bmatrix} -2 \times -1 + 0 \times 2x \\ 2 \times -1 + 1 \times 2x \end{bmatrix} + \begin{bmatrix} -2 \times 3 \\ 1 \times 3 \end{bmatrix} = \begin{bmatrix} 2 \times y \\ 2 \times 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ -3 + 2x \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 - 6 \\ -3 + 2x + 3 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ 2x + 0 \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ 2x \end{bmatrix} = \begin{bmatrix} 2y \\ 6 \end{bmatrix}$$

By definition of equality of matrices, the corresponding elements of equal matrices are equal. Thus

$$-4 = 2y \text{ and } 2x = 6$$

$$y = \frac{-4}{2} = -2 \text{ and } x = \frac{6}{2} = 3$$

Thus $y = -2$ and $x = 3$

(ii) In the given figure, DE || BC.

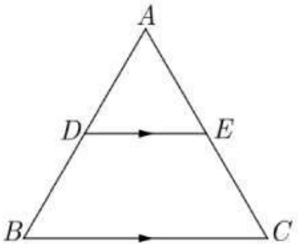
(i) Prove that ΔAPO : and ΔABC are similar.

Answer:

(i) To prove ΔADE and ΔABC are similar:

1. Given that DE || BC (parallel lines)
2. By the properties of parallel lines cut by a transversal:
   - ∠ADE = ∠ABC (corresponding angles)
   - ∠AED = ∠ACB (corresponding angles)
3. ∠A is common to both triangles
4. By AA similarity criterion, ΔADE ~ ΔABC

(ii) Given that AD = ½ BD, calculate DE, if BC = 4 5. cm.



Answer:

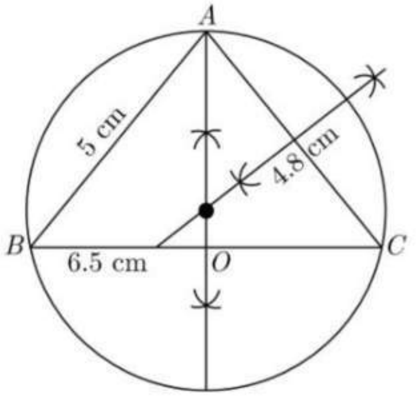
(ii) Calculation of DE:

1. When DE || BC:
   - By similarity property of triangles, we can write: $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$
2. Given that AD = ½ BD:
   - AD = ½ BD
   - AD : AB = 1 : 3
3. Therefore: $\frac{DE}{BC} = \frac{AD}{AB} = \frac{1}{3}$
4. Since BC = 4.5 cm: $DE = \frac{1}{3} \times 4.5 = 1.5$ cm

Therefore, DE = 1.5 cm

(iii) In ΔABC, AB = 5cm, BC = 6.5cm and CA = 4.8cm. Draw the circumcircle of ΔABC.

Answer:



Steps of Construction :

Step 1 : Draw a line segment $BC = 6.5$ cm.

Step 2 : With centre B and radius 5 cm and with centre C and radius 4.8 cm, draw intersecting each other at A.

Step 3 : Join AB and AC.

Step 4 : Draw perpendicular bisector of sides AC and BC intersecting each other at O.

Step 5 : With centre O and radius OA or OC, draw a circle which will pass through A, B and C.

This is the required circle.

QUESTION 5.

(i) Literacy rates of 40 cities are given in the following table. It is given that mean literacy rate is 63.5, then find the missing frequencies x and y.

Literacy Rate (in %)	Number of Cities
35-40	1
40-45	2
45-50	3
50-55	x
55-60	y
60-65	6
65-70	8
70-75	4
75-80	2
80-85	3
85-90	2

Answer:

We prepare following table to find mean.

C.I.	x_i	u_i	f_i	$f_i u_i$
35-40	37.5	-5	1	-5
40-45	42.5	-4	2	-8
45-50	75.5	-3	3	-9
50-55	52.5	-2	x	$-2x$
55-60	57.5	-1	y	$-y$
60-65	$62.5 = a$	0	6	0
65-70	67.5	1	8	8
70-75	72.5	2	4	8
75-80	77.5	3	2	6

80-85	82.5	4	3	12
85-90	87.5	5	2	10
Total			$\sum f_i =$ $31 + x + y$	$\sum f_i u_i =$ $22 - 2x - y$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}.$$

Here:

- Mean = 63.5,
- $\sum f_i = 40$,
- $\sum f_i x_i$ includes terms with x and y.

Substitute:

$$63.5 = \frac{(\text{sum of products including } x \text{ and } y)}{40}.$$

We compute the products $f_i x_i$, substitute and solve for x and y.

Solution:

$x = 5, y = 4.$

(ii) Rames bought the following articles from a departmental store:

S. No.	Item	Price	Rate of GST	Discount
1.	Hair oil	₹1200	18%	₹100
2.	Cashew nuts	₹600	12%	-

Find the:

(a) Total GST paid.

(b) Total bill amount including GST.

Answer:

1. Hair Oil:

- Price = ₹1200
- Discount = ₹100
- Rate of GST = 18%.

Net Price:

$$\text{Net Price} = 1200 - 100 = 1100.$$

GST on Hair Oil:

$$\text{GST} = \frac{18}{100} \times 1100 = 198.$$

2. Cashew Nuts:

- Price = ₹600
- Rate of GST = 12%.

GST on Cashew Nuts:

$$\text{GST} = \frac{12}{100} \times 600 = 72.$$

(b) Total Bill Amount Including GST:

$$\text{Total Bill} = \text{Net Price of Hair Oil} + \text{Price of Cashew Nuts} + \text{Total GST}.$$

$$\text{Total Bill} = 1100 + 600 + 270 = 1970.$$

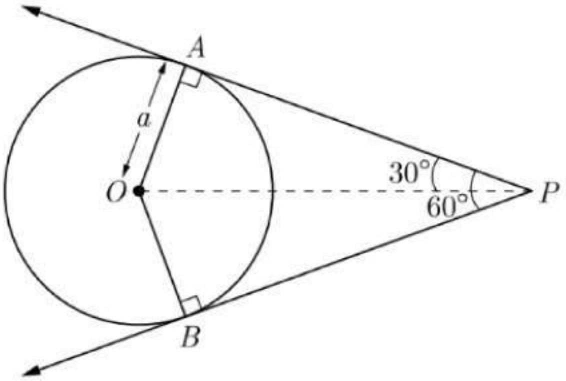
Final Answers:

(a) **Total GST paid** = ₹270.

(b) **Total bill amount including GST** = ₹1970.

(iii) If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP.

Answer: Let PA and PB be the two tangents drawn to the circle with centre O and radius a such that $\angle APB = 60^\circ$



In ΔOPB and ΔOPA

OB = OA = a (Radii of the circle)

∠OBP = ∠OAP=90° (Tangents are perpendicular to radius at the point of contact)

BP = PA (Lengths of tangents drawn from an external point to the circle are equal)

So, ΔOPB ≅ ΔOPA (SAS Congruence Axiom)

∴ ∠OPB = ∠OPA=30° (CPCT)

Now,

In ΔOPB

$\sin 30^\circ = \frac{\text{OB}}{\text{OP}}$

$\Rightarrow \frac{1}{2} = \frac{a}{OP}$

$\Rightarrow OP = 2a$

Thus the length of OP is 2a

QUESTION 6.
(i) The terms of a G.P. with first term a and common ratio r are squared. Prove that resulting numbers form a GP. Find its first term, common ratio and the n^{th} term.

Answer:

Let the terms of the original G.P. be:

$$a, ar, ar^2, ar^3, \ldots$$

When these terms are squared, the resulting sequence becomes:

$$a^2, (ar)^2, (ar^2)^2, (ar^3)^2, \ldots$$

Prove the resulting numbers form a G.P.:

The squared terms are:

$$a^2, a^2r^2, a^2r^4, a^2r^6, \ldots$$

Each term can be expressed as:

$$T_n = a^2r^{2(n-1)}.$$

- First term: a^2,
- Common ratio: $\frac{\text{Second term}}{\text{First term}} = \frac{a^2r^2}{a^2} = r^2$.

Since the ratio between consecutive terms is constant (r^2), the squared sequence forms a G.P.

First Term, Common Ratio, and n-th Term:

1. **First Term** = a^2,
2. **Common Ratio** = r^2,
3. n**-th Term** = $a^2r^{2(n-1)}$.

Final Answer:

The squared sequence forms a G.P. with:

- First term = a^2,
- Common ratio = r^2,
- n-th term = $a^2r^{2(n-1)}$.

(ii) At the time that I was newly hired, 100 sales per month was what I required. Each following month—the last plus 20 more, as I work for the goal of top sales award. When 2500 sales are thusly made, I get a holiday package.

(i) How many sales were made by this person in the seventh month?

(ii) What were the total sales after the 12th month?

(iii) Was the goal of 2500 total sales met after the 12th month?

Answer:

The number of sales increases by 20 each month. This forms an **Arithmetic Progression (A.P.)** where:

- First term (a) = 100,
- Common difference (d) = 20.

(i) Sales made in the 7th month:

The formula for the n-th term of an A.P. is:

$$T_n = a + (n-1)d.$$

Substitute $a = 100$, $d = 20$, and $n = 7$:

$$T_7 = 100 + (7-1) \cdot 20.$$

$$T_7 = 100 + 6 \cdot 20 = 100 + 120 = 220.$$

Sales in the 7th month = 220.

(ii) Total sales after the 12th month:

The formula for the sum of the first n terms (S_n) of an A.P. is:

$$S_n = \frac{n}{2}[2a + (n-1)d].$$

Substitute $a = 100$, $d = 20$, and $n = 12$:

$$S_{12} = \frac{12}{2}[2(100) + (12-1) \cdot 20].$$

$$S_{12} = 6[200 + 11 \cdot 20].$$

$$S_{12} = 6[200 + 220] = 6 \cdot 420 = 2520.$$

Total sales after the 12th month = 2520.

(iii) Was the goal of 2500 total sales met after the 12th month?

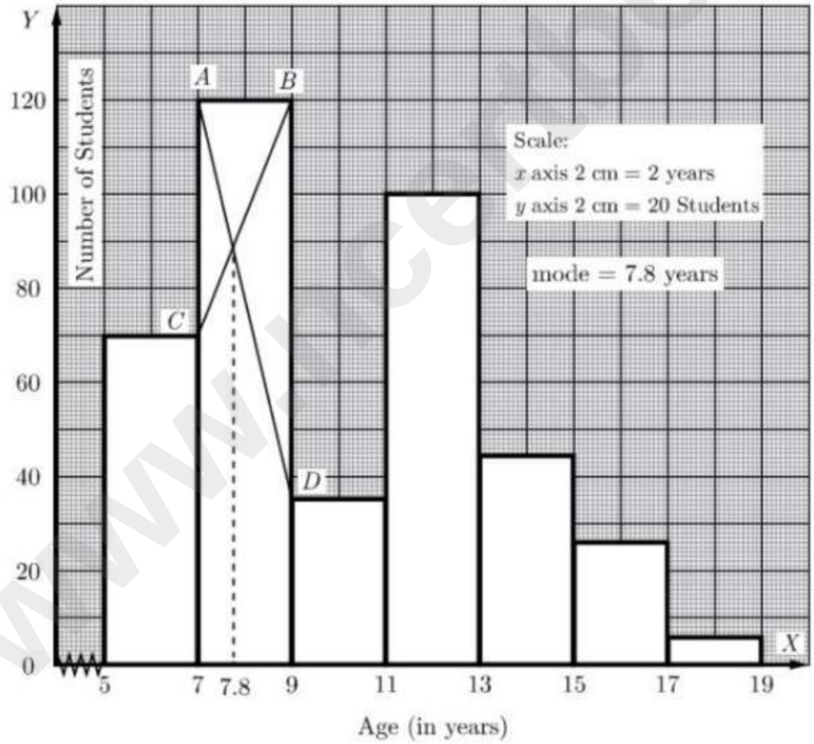
Yes, the total sales after 12 months = **2520**, which exceeds the goal of **2500**.

(iii) On annual day of a school, 400 students participated in the function. Frequency distribution showing their ages is as shown in the following table :

Ages (in years)	Number of students
5-7	70
7-9	120
9-11	32
11-13	100
13-15	45
15-17	28
17-19	5

Draw a histogram for the above data using a graph paper and locate the mode.

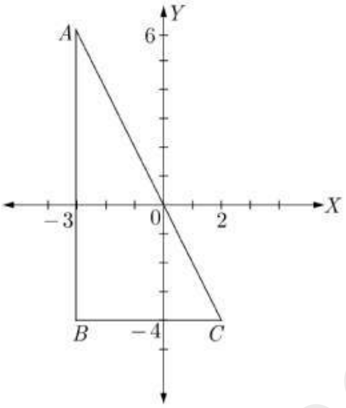
Answer: The histogram from the given frequency table is shown below.



In the highest rectangle, draw two straight lines AC and BD (as shown in figure) which intersect at P. Through point P, draw a vertical line to meet the x-axis at N. The abscissa of the point represent 7.8.
Hence, the required mode is 7.8 year.

QUESTION 7.
(i) From the given figure, write

(i) The co-ordinates of A, B and C
(ii) The equation of AC
(iii) The x-intercept of AB
(iv) The y-intercept of BC



Answer:

(i) The coordinates of A, B and C:

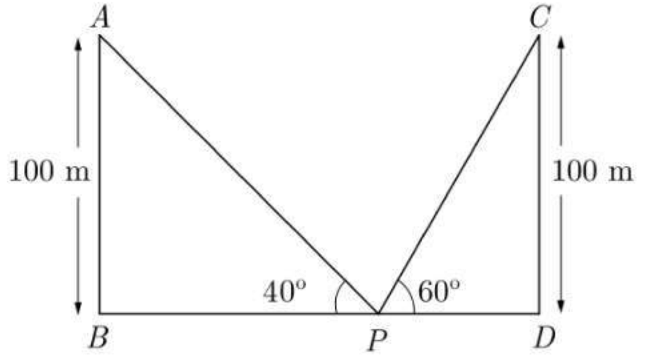
- A: (-3, 6)
- B: (-3, -4)
- C: (2, -4)

(ii) The equation of AC:
$y = -\frac{10}{5}x + 6$
Simplified: $y = -2x + 6$

(iii) The x-intercept of AB:
x = -3

(iv) The y-intercept of BC:
y = -4

(ii) Two lamp posts AB and CD each of heights 100 m are on either side of the road. P is a point on the road between the two lamp pasts. The angles of elevation of the top of the lamp posts from the point P are 40º and 60º. Find the distance PB and PD.



Answer:

Let's solve this step by step:

Finding PB:

For triangle ABP with angle of elevation 40°:

$PB = \frac{100}{\tan(40^\circ)} = 119.18 \text{ meters}[2]$

Finding PD:

For triangle CDP with angle of elevation 60°:

$PD = \frac{100}{\tan(60^\circ)} = 57.74 \text{ meters}[3]$

Therefore:

- Distance PB = 119.18 meters
- Distance PD = 57.74 meters

QUESTION 8.

(i) Solve the following in equation and represent the solution set on the number line

$$4x - 19 < \frac{3x}{5} - 2 \leq \frac{-2}{5} + x,\ x \in R.$$

Answer:

We solve the compound inequality step-by-step:

$$4x - 19 < \frac{3x}{5} - 2 \leq -\frac{2}{5} + x, \quad x \in \mathbb{R}.$$

Step 1: Solve $4x - 19 < \frac{3x}{5} - 2$:

1. Multiply through by 5 to eliminate the denominator:

$$5(4x - 19) < 3x - 10.$$

Simplify:

$$20x - 95 < 3x - 10.$$

2. Bring all x-terms to one side and constants to the other:

$$20x - 3x < -10 + 95.$$

$$17x < 85.$$

3. Solve for x:

$$x < 5.$$

Step 2: Solve $\frac{3x}{5} - 2 \leq -\frac{2}{5} + x$:

1. Multiply through by 5 to eliminate the denominator:

$$5\left(\frac{3x}{5} - 2\right) \leq 5\left(-\frac{2}{5} + x\right).$$

Simplify:

$$3x - 10 \leq -2 + 5x.$$

2. Bring all x-terms to one side and constants to the other:

$$3x - 5x \leq -2 + 10.$$

$$-2x \leq 8.$$

3. Divide by -2 and reverse the inequality:

$$x \geq -4.$$

Step 3: Combine the results:

From Step 1 and Step 2:

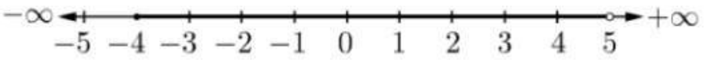
$$-4 \leq x < 5.$$

Solution Set:

The solution set is:

$$x \in [-4, 5).$$

The graph of the solution set on the number line is shown by dark line.



(ii) The line segment joining A(2, 3) and B(6, –5) is intersected by the x -axis at the point K . Write the coordinates of the point K. Hence, find the ratio in which K divides AB.

Answer:

The x-axis intersects the line segment AB where the y-coordinate is 0.

Step 1: Coordinates of K:

Using the section formula:

- $A(2, 3)$, $B(6, -5)$,
- $y = 0$ at K.

We solve:

$$0 = \frac{m(-5) + n(3)}{m + n}.$$

Simplify:

$$5m = 3n \implies \frac{m}{n} = \frac{3}{5}.$$

Thus, K divides AB in the ratio $3 : 5$.

The x-coordinate of K is:

$$x = \frac{3(6) + 5(2)}{3 + 5} = \frac{18 + 10}{8} = 3.5.$$

Thus, K divides AB in the ratio $3:5$.

The x-coordinate of K is:

$$x = \frac{3(6)+5(2)}{3+5} = \frac{18+10}{8} = 3.5.$$

Coordinates of K are:

$$K(3.5, 0).$$

Final Answer:

- Coordinates of K: $K(3.5, 0)$.
- Ratio in which K divides AB: 3:5.

Now,

$$x = \frac{6m+2n}{m+n}$$

$$= \frac{6 \times 3 + 2 \times 5}{3+5}$$

$$= \frac{18+10}{8} = \frac{28}{8} = \frac{7}{2}$$

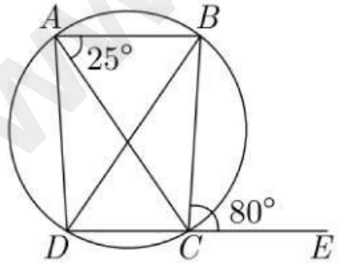
So, coordinates of point K is $(\frac{7}{2}, 0)$.

(iii) In the given figure, AB is parallel to DC, $\angle BCE = 80^\circ$ and $\angle BAC = 25^\circ$. Find

(i) $\angle CAD$.

(ii) $\angle CBD$.

(iii) $\angle ADC$.



Answer:

(i) $\angle CAD$.

Since exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

Thus $\angle BCE = \angle BAD$

$= 80^\circ$

Now, $\angle CAD = \angle BAD - \angle BAC$

$= 80^\circ - 25^\circ$

$= 55^\circ$

(ii) Find $\angle CBD$:

In the cyclic quadrilateral $ABCD$, opposite angles add up to 180°.

Thus:

$$\angle BAC + \angle CBD = 180^\circ.$$

Substitute $\angle BAC = 25^\circ$:

$$25^\circ + \angle CBD = 180^\circ.$$

$$\angle CBD = 180^\circ - 25^\circ = 155^\circ.$$

(iii) $\angle ADC$.

Here $AB || DC$ and $ABCD$ forms a parallelogram.

In a parallelogram, sum of two adjacent angles is 180°. Thus

$$\angle BAD + \angle ADC = 180^\circ$$

$$\angle BAC + \angle CAD + \angle ADC = 180^\circ$$

$$25^\circ + 55^\circ + \angle ADC = 180^\circ$$

$$\angle ADC = 180^\circ - 80^\circ$$

$$\angle ADC = 100^\circ$$

QUESTION 9.

(i) If a, b and c are in continued proportion, prove that

$(a+b+c)(a-b+c) = a^2+b^2+c^2$

Answer:

Let me prove that if a, b, c are in continued proportion, then:
$(a+b+c)(a-b+c) = a^2 + b^2 + c^2$

Proof

Given: Since a, b, c are in continued proportion:
$\frac{a}{b} = \frac{b}{c}$

Let's denote this common ratio as k. Therefore:
$\frac{a}{b} = \frac{b}{c} = k$

This means:

- $a = bk$
- $b = ck$
- Therefore, $a = ck^2$

Step 1: Let's expand the left-hand side (LHS):
$(a+b+c)(a-b+c)$

Step 2: Substitute the values in terms of c and k:
$(ck^2+ck+c)(ck^2-ck+c)$

Step 3: Factor out c:
$c(k^2+k+1)(k^2-k+1)$
$= c^2(k^4+2k^2+1-k^3)$

Step 4: For the right-hand side (RHS):
$a^2+b^2+c^2 = (ck^2)^2+(ck)^2+c^2$
$= c^2k^4+c^2k^2+c^2$
$= c^2(k^4+k^2+1)$

Step 5: Since LHS = RHS, we have proven that:
$(a+b+c)(a-b+c) = a^2+b^2+c^2$ 8

Therefore, the statement is proved.

(ii) Ram takes 6 days less than Bharat to finish a place of work. If both of them together can finish the work in 4 days, in how many days Bharat alone can finish the work?

Answer:

12 days

Explanation:

Assuming that B alone takes x days to finish the work. Then A alone can finish it in (x – 6) days.

Now A's one day's work + B's one day's work = $\frac{1}{x} + \frac{1}{x-6}$

and (A+B)'s one day's work = $\frac{1}{4}$

$$\frac{x-6}{x(x-6)} + \frac{x}{x(x-6)} = \frac{1}{4}$$

$4(x - 6 + x) = x^2 - 6x$

$4(2x - 6) = x^2 - 6x$

$8x - 24 = x^2 - 6x$

$0 = x^2 - 14x + 24$

$0 = (x - 12)(x - 2)$

x = 12 and 2

x cannot be less than 6.

So x = 12 Therefore, B also can finish the work in 12 days.

(iii) Draw a Δ ABC such that AB = 4.8cm, ΔABC = 45°. AC – AB = 1 5 cm. Find a point P inside the Δ ABC such that BP = PC and P is at a distance of 2 cm from AC.

Answer:

Steps for Construction of $\triangle ABC$:

1. **Draw base $AB = 4.8\text{ cm}$:**
   - Use a ruler to draw a horizontal line segment AB with length 4.8 cm.
2. **Construct $\angle ABC = 45^\circ$:**
   - At point B, use a protractor to construct an angle of 45°. Draw a ray BC along this angle.
3. **Find AC such that $AC - AB = 1.5\text{ cm}$:**

- Since $AB = 4.8\,\text{cm}$ and $AC - AB = 1.5\,\text{cm}$, the length of AC is:

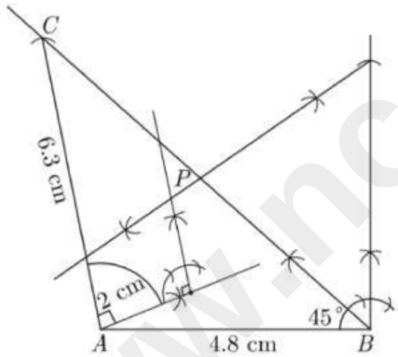
$$AC = AB + 1.5 = 4.8 + 1.5 = 6.3\,\text{cm}.$$

- From point A, use a compass to measure $6.3\,\text{cm}$ and mark an arc that intersects BC. Label the intersection point as C.
- Join AC to complete $\triangle ABC$.

Find Point P:

- $BP = PC$ implies that P lies on the **perpendicular bisector** of segment BC.

1. **Construct the perpendicular bisector of BC:**
   - Use a compass to draw arcs from B and C with radii greater than half of BC.
   - Mark the two intersections of the arcs, and join them to form the perpendicular bisector of BC.
2. **Locate P such that it is $2\,\text{cm}$ from AC:**
   - From AC, draw a line parallel to AC at a distance of $2\,\text{cm}$ using a ruler and set square.
3. **Intersection of the perpendicular bisector and parallel line:**
   - The intersection of the perpendicular bisector of BC and the line $2\,\text{cm}$ away from AC gives the point P.



Then P is the required point which is equidistant from B and C and is at a distance of 2 cm from AC.

QUESTION 10.

(i) If (x – 2) is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by (x – 3), it leaves a remainder 52. Find the values of a and b.

Answer:

Let $p(x) = 2x^3 + ax2 + bx - 14$

Given, (x – 2) is a factor of p(x),

⇒ Remainder = p(2) = 0

⇒ $2(2)^3 + a(2)^2 + b(2) - 14 = 0$

⇒ 16 + 4a + 2b – 14 = 0

⇒ 4a + 2b + 2 = 0

⇒ 2a + b + 1 = 0 ...(1)

Given, when p(x) is divided by (x – 3), it leaves a remainder 52

∴ p(3) = 52

∴ $2(3)^3 + a(3)^2 + b(3) - 14 = 52$

⇒ 54 + 9a + 3b - 14 - 52 = 0

⇒ 9a + 3b – 12 = 0

⇒ 3a + b – 4 = 0 ...(2)

Subtracting (1) from (2), we get,

a – 5 = 0 ⇒ a = 5

From (1),

10 + b + 1 = 0 ⇒ b = –11

(ii) Cards marked with numbers 3, 4, 5, ...50 are placed in a bag and mixed thoroughly. One card is drawn at random from the bag. Find the probability that number on the card drawn is :

(i) Divisible by 7.

(ii) A perfect square.

(iii) A multiple of 6.

Answer:

Total number of cards = 48

Probability of an event

$$= \frac{\text{Total number of favourable outcomes}}{\text{Total number of outcomes}}$$

Number of cards divisible by 7 = 7

$$P(\text{cards divisible by 7}) = \frac{7}{48}$$

Number of cards having a perfect square = 6

$$P(\text{cards having a perfect square}) = \frac{6}{48} = \frac{1}{8}$$

Number of multiples of 6 from 3 to 50 = 8

$$P(\text{multiple of 6 from 3 to 50}) = \frac{8}{48} = \frac{1}{6}$$

(iii) Use graph paper for this question. The point P(–1, 3) is reflected in the line parallel to y-axis at a distance of 2 units to the right side of y-axis onto the point P. The point Q is reflected in the origin onto the point

Q'(– 3, 2). Find:

(i) The coordinates of P' and Q.

(ii) The equation of the line P' Q.

Answer:

(i) Coordinates of P' and Q:

1. **Point P is reflected in the line parallel to the y-axis:**
    - The given line is $x = 2$ (2 units to the right of the y-axis).
    - To find the reflection of $P(-1, 3)$, note that the line $x = 2$ acts as the mirror.
    - The distance from P to $x = 2$ is:

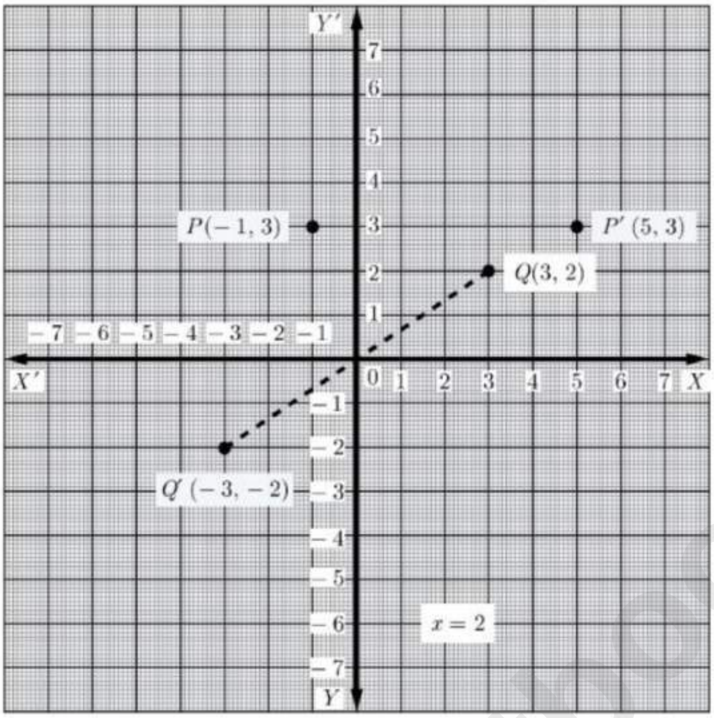
    $$d = |x_P - 2| = |-1 - 2| = 3 \text{ units.}$$

    - The reflected point P' will be 3 units on the other side of $x = 2$:
    New x-coordinate = $2 + 3 = 5$, and the y-coordinate remains the same.
    Thus:

    $$P'(5, 3).$$

2. **Point Q is reflected in the origin onto $Q'(-3, 2)$:**
    - Reflection in the origin means the coordinates change their signs.
    - Therefore, the coordinates of Q will be:

    $$Q(3, -2).$$



(ii) The coordinates of P' and Q are (5, 3) and (3, 2) respectively. So, using $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ equation of the line $P'Q$ is

$$y - 3 = \frac{2-3}{3-5}(x-5)$$

$$y - 3 = \frac{1}{2}(x-5)$$

$$x - 2y + 1 = 0$$