ICSE 2025 EXAMINATION

Sample Question Paper - 6

Mathematics

Time: 2 Hours **Max. Marks: 80**

General Instructions:

1. Answer to this Paper must be written on the paper provided separately
2. You will not be allowed to write during first 15 minutes.
3. This time is to be spent in reading the question paper.
4. The time given at the head of this Paper is the time allowed for writing the answers.
5. Attempt all questions from Section A and any four questions from Section B.
6. All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
7. Omission of essential working will result in loss of marks.
8. The intended marks for questions or parts of questions are given in brackets [ ].
9. Mathematical tables are provided.

SECTION-A

(Attempt all questions from this Section.)

QUESTION 1.
Choose the correct answers to the questions from the given options.

(Do not copy the questions, write the correct answer only.)

(i) If $A\begin{bmatrix} 2 & 0 \\ -1 & 7 \end{bmatrix}$ then A^2 is :

(a) $\begin{bmatrix} 4 & 0 \\ 1 & 49 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 0 \\ 9 & 49 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 9 \\ -9 & 49 \end{bmatrix}$

Answer:

(b) $\begin{bmatrix} 4 & 0 \\ -9 & 49 \end{bmatrix}$

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(ii) An article is marked at ₹1,000. A dealer sells it at 5% profit. If the rate of GST is 18%, then the total amount paid by a consumer to buy it, is:

(a) ₹1,324 (b) ₹1,239

(c) ₹1,120 (d) ₹1,180

Answer: (b) ₹1,239

(iii) Which of the following is the equation of the straight line whose inclination is 45 ° and whose y-intercept is –3.

(a) $x - y - 1 = 0$ (b) $-x + y + 3 = 0$

(c) $x - y - 3 = 0$ (d) $x - y + 1 = 0$

Answer: (c) $x - y - 3 = 0$

(iv) The nature of roots of the equation $5x^2 - 6x + 7 = 0$ is:

(a) two distinct real roots (b)two equal real roots

(c) no real roots (d) more than 2 real roots

Answer: (c) no real roots

(v) Naresh deposited ₹400 per month for 15 months in Federal bank's recurring deposit account. If the bank pays interest at a rate of 10% per annum, then the interest earned by Naresh during this period is:

(a) ₹350 (b) ₹250

(c) ₹400 (d) ₹300

Answer: (c) ₹400

(vi) If $x \in W$ then the solution of $x \leq 3$ is:

(a) $\{0, 1, 2\}$

(b) $\{1, 2, 3\}$

(c) $\{1, 2\}$

(d) $\{0, 1, 2, 3\}$

Answer: (d) $\{0, 1, 2, 3\}$

(vii) If $2x + 3y : 3x + 5y = 18 : 29$, then the ratio $x : y$ is:

(a) 2 : 3 (b) 3 : 5

(c) 3 : 4 (d) 5 : 29

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Answer: (c) 3 : 4

(viii) **Assertion :** $(x+2)$ and $(x-1)$ are factors of the polynomial $x^4+x^3+2x^2+4x-8$.

Reason : For a polynomial $p(x)$ of degree≥ 1, $x-a$ is a factor of the polynomial $p(x)$ if and only if $p(a)\geq 1$.

(a) Both assertion and reason are true and reason is the correct explanation of assertion.

(b) Both assertion and reason are true but reason is not the correct explanation of assertion.

(c) Assertion is true but reason is false.

(d) Assertion is false but reason is true.

Answer: (c) Assertion is true but reason is false.

(ix) The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$ is

(a) 1 (b) $\frac{1}{p}$

(c) -1 (d) $-\frac{1}{p}$

Answer: (c) –1

(x) The ratio in which the point $(2, y)$ divides the join of $(-4, 3)$ and $(6, 3)$, hence the value of y is

(a) $2:3, y=3$

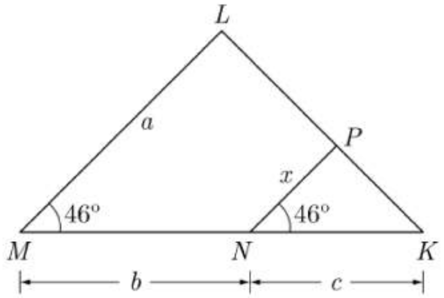
(b) $3:2, y=4$

(c) $3:2, y=3$

(d) $3:2, y=2$

Answer: (c) $3:2, y=3$

(xi) In the given figure, x is



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(a) $\frac{ab}{a+b}$

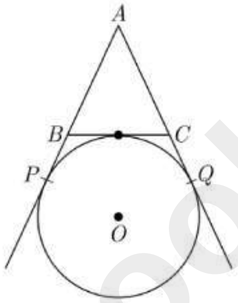
(b) $\frac{ac}{b+c}$

(c) $\frac{bc}{b+c}$

(d) $\frac{ac}{a+c}$

Answer: (b) $\frac{ac}{b+c}$

(xii) In figure, AP, AQ and BC are tangents of the circle with centre O. If $AB = 5$ cm, $AC = 6$ cm and $BC = 4$ cm, then the length of AP (in cm) is



(a) 15

(b) 10

(c) 9

(d) 7.5

Answer: (d) 7.5

(xiii) A sphere is melted and half of the melted liquid is used to form 11 identical cubes, whereas the remaining half is used to form 7 identical smaller spheres. The ratio of the side of the cube to the radius of the new small sphere is

(a) $\left(\frac{4}{3}\right)^{1/3}$

(b) $\left(\frac{8}{3}\right)^{1/3}$

(c) $(3)^{1/3}$

(d) 2

Answer: (b) $\left(\frac{8}{3}\right)^{1/3}$

(xiv) Consider the following statements

Statement 1 : The solution set of $5x - 3 < 7$, when x is an integer, is $\{..., -3, -2\ -1\}$.

Statement 2 : The solution of $5x - 3 < 7$, when x is a real number, is $(-\infty, 2)$.

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Choose the correct option.

(a) Statement 1 is true

(b) Statement 2 is true

(c) Both are true

(d) Both are false

Answer: (b) Statement 2 is true

(xv) QP is a tangent to a circle with centre O at a point P on the circle. If ΔOPQ is isosceles, then $\angle OQR$ equals to

(a) 30°

(b) 45°

(c) 60°

(d) 90°

Answer: (b) 45°

QUESTION 2.

(i) Suppose a sugar cone is 10 centimeters deep and has a diameter of 4 centimeters. A spherical scoop of ice cream with a diameter of 4 centimeters rests on the top of the cone.

(a) If all the ice cream melts into the cone, will the cone overflow? Explain.

(b) If the cone does not overflow, what percent of the cone will be filled?

Answer:

Given Data:

1. The cone has:
   - Depth (height, h) = $10\,\text{cm}$,
   - Diameter = $4\,\text{cm}$,
   - Radius (r) = $\frac{4}{2} = 2\,\text{cm}$.

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2. The ice cream scoop is a sphere with:

- Diameter = $4\,\text{cm}$,
- Radius (R) = $\frac{4}{2} = 2\,\text{cm}$.

Step 1: Calculate the Volume of the Cone

The formula for the volume of a cone is:

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

Substitute $r = 2\,\text{cm}$ and $h = 10\,\text{cm}$:

$$V_{\text{cone}} = \frac{1}{3}\pi(2)^2(10)$$

Simplify:

$$V_{\text{cone}} = \frac{1}{3}\pi(4)(10)$$

$$V_{\text{cone}} = \frac{40}{3}\pi\,\text{cm}^3$$

Step 2: Calculate the Volume of the Sphere

The formula for the volume of a sphere is:

$$V_{\text{sphere}} = \frac{4}{3}\pi R^3$$

Substitute $R = 2\,\text{cm}$:

$$V_{\text{sphere}} = \frac{4}{3}\pi(2)^3$$

Simplify:

$$V_{\text{sphere}} = \frac{4}{3}\pi(8)$$

$$V_{\text{sphere}} = \frac{32}{3}\pi\,\text{cm}^3$$

Step 3: Compare the Volume of the Sphere and Cone

The total volume of ice cream (sphere) is $\frac{32}{3}\pi\,\text{cm}^3$, and the volume of the cone is $\frac{40}{3}\pi\,\text{cm}^3$.

- Compare the two:

$$V_{\text{sphere}} = \frac{32}{3}\pi \quad \text{and} \quad V_{\text{cone}} = \frac{40}{3}\pi.$$

Since $\frac{32}{3}\pi < \frac{40}{3}\pi$, the volume of the sphere is **less than** the volume of the cone.

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Conclusion for Part (a):

The cone will **not overflow** because the volume of the ice cream (sphere) is less than the volume of the cone.

Step 4: Find the Percent of the Cone Filled

To find what percent of the cone will be filled by the melted ice cream, use the formula:

$$\text{Percent filled} = \left(\frac{\text{Volume of sphere}}{\text{Volume of cone}}\right) \times 100$$

Substitute the values:

$$\text{Percent filled} = \left(\frac{\frac{32}{3}\pi}{\frac{40}{3}\pi}\right) \times 100$$

Simplify:

$$\text{Percent filled} = \left(\frac{32}{40}\right) \times 100$$

$$\text{Percent filled} = 0.8 \times 100 = 80\%.$$

Final Answers:

(a) The cone will **not overflow** because the ice cream volume is less than the cone's capacity.

(b) The cone will be **80% filled**.

(ii) Meena deposited ₹700 per month in a R.D. account for $1\frac{1}{4}$ years at bank. If the matured value of this account is ₹11,130, find the interest received.

Answer:

Step 1: Given Data

- Monthly deposit (P) = ₹700,
- Time period = $1\frac{1}{4}$ years $= \frac{5}{4}$ years $= 15$ months,
- Maturity value (M.V.) = ₹11,130,
- Total amount deposited = $P \times n$, where n is the number of months.

Step 2: Calculate Total Amount Deposited

The total amount deposited by Meena is:

$$\text{Total deposit} = \text{Monthly deposit} \times \text{Number of months.}$$

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Substitute the values:

$$\text{Total deposit} = 700 \times 15 = ₹10,500.$$

Step 3: Find the Interest Received

The interest received is the difference between the maturity value and the total amount deposited:

$$\text{Interest} = \text{Maturity value} - \text{Total deposit}.$$

Substitute the values:

$$\text{Interest} = 11,130 - 10,500 = ₹630.$$

(iii) Prove that : $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A.$

Answer:

$$\begin{aligned}\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} &= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)} \\ &= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)} \\ &= \tan A\frac{[1 - 2(1 - \cos^2 A)]}{(2\cos^2 A - 1)} \\ &= \tan A\frac{[1 - 2 + 2\cos^2 A)]}{(2\cos^2 A - 1)} \\ &= \tan A\frac{(2\cos^2 A - 1)}{(2\cos^2 A - 1)} \\ &= \tan A \qquad \text{Hence Proved}\end{aligned}$$

QUESTION 3.

(i) How much cash is obtained by selling :

(a) fifty, ₹20 shares at ₹2.50 premium ?

(b) one hundred, ₹20 shares at ₹1.75 discount ?

(c) two hundred fifty, ₹200 shares at par ?

Answer:

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Let's calculate the cash obtained by selling shares in each scenario:

(a) Fifty ₹20 shares at ₹2.50 premium

To calculate this, we need to:

1. Find the total face value: 50 × ₹20 = ₹1000
2. Add the premium: 50 × ₹2.50 = ₹125
3. Sum these up: ₹1000 + ₹125 = ₹1125

Therefore, the cash obtained is ₹1125

(b) One hundred ₹20 shares at ₹1.75 discount

For this calculation:

1. Find the total face value: 100 × ₹20 = ₹2000
2. Subtract the discount: 100 × ₹1.75 = ₹175
3. Calculate the difference: ₹2000 - ₹175 = ₹1825

The cash obtained from this sale is ₹1825

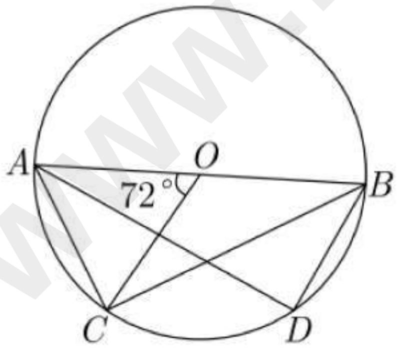
(c) Two hundred fifty ₹200 shares at par

"At par" means the shares are sold at their face value. So we simply multiply:

250 × ₹200 = ₹50,000

The cash obtained from this sale is ₹50,000

(ii) In the figure below, O is the centre of the circle and AB is diameter. If AC = BD and ∠AOC = 72°. Find



(a) ∠ABC
(b) ∠BAD
(c) ∠ABD

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Answer:

Given Information:

- O is the center of the circle
- AB is a diameter
- AC = BD
- ∠AOC = 72°

(a) Finding ∠ABC

Since AB is a diameter and O is the center:

- When an arc is viewed from the center and from a point on the circle, the angle at the center is twice the angle at the circumference
- Therefore, ∠ABC = $\frac{1}{2}$ × ∠AOC
- ∠ABC = $\frac{1}{2}$ × 72° = 36°

(b) Finding ∠BAD

- Since AB is a diameter:
- Any angle inscribed in a semicircle is 90°
- Therefore, ∠BAD = 90°

(c) Finding ∠ABD

- In triangle ABD:
- Sum of angles in a triangle = 180°
- We know ∠ABC = 36° and ∠BAD = 90°
- ∠ABD = 180° - ∠ABC - ∠BAD
- ∠ABD = 180° - 36° - 90° = 54°

Therefore:

- ∠ABC = 36°
- ∠BAD = 90°
- ∠ABD = 54°

(iii) 40 students enter for a game of shot-put competition. The distance thrown (in metres) is recorded below

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Distance in m	Number of Students
12-13	3
13-14	9
14-15	12
15-16	9
16-17	4
17-18	2
18-19	1

Use a graph paper to draw an ogive for the above distribution. Use a scale of 2 cm = 1 m on one axis and 2 cm = 5 students on the other axis.

Hence using your graph find

(a) the median

(b) Upper Quartile

(c) Number of students who cover a distance which is above 16½ m.

Answer: Cumulative frequency distribution for the given data is shown below:

Distance	Frequency f	Cumulative frequency cf
12-13	3	3
13-14	9	12
14-15	12	24
15-16	9	33
16-17	4	37
17-18	2	39
18-19	1	40

On the graph paper, we plot all points and join all these points by free drawing. The required ogive is shown on the graph paper.

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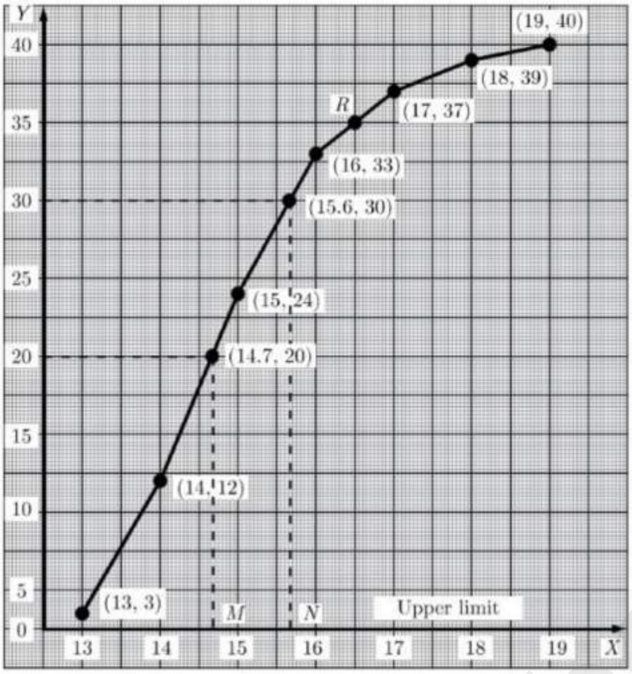

(a) Here $N = 40$ thus $\frac{N}{2} = \frac{40}{2} = 20$. Now, we locate the point on the ogive, whose y-ordinate is 20. The x co-ordinate corresponding to this ordinate is 14.7. Hence, median distance is 14.7.

The **Upper Quartile (Q3)** corresponds to the cumulative frequency:

$$Q3 = \frac{3N}{4}, \quad \text{where } N = 40.$$

Substitute $N = 40$:

$$Q3 = \frac{3 \times 40}{4} = 30.$$

Using the ogive graph:

1. Locate the cumulative frequency 30 on the y-axis.
2. Draw a horizontal line from 30 to meet the ogive curve.
3. From the point of intersection, draw a vertical line to meet the x-axis.
4. The point on the x-axis gives the **upper quartile**.

Upper Quartile (Q3) ≈ 15.5 m.

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1. Locate $16.5\,\mathrm{m}$ on the x-axis.
2. From $16.5\,\mathrm{m}$, draw a vertical line to meet the ogive curve.
3. From the point of intersection, draw a horizontal line to meet the y-axis. This gives the cumulative frequency at $16.5\,\mathrm{m}$.

Suppose the cumulative frequency at $16.5\,\mathrm{m}$ is 35 (approximate value from graph).

The number of students who cover a distance **above** $16.5\,\mathrm{m}$ is:

$$\text{Number of students} = N - \text{Cumulative frequency at } 16.5\,\mathrm{m}.$$

Substitute $N = 40$ and cumulative frequency = 35:

$$\text{Number of students} = 40 - 35 = 5.$$

SECTION-B

(Attempt any four questions.)

QUESTION 4.

(i) Given matrix $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$. Find the matrix X if, $X = B^2 - 4B$. Hence solve for a and b given $X = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

Answer:

Step 1: Given Information

- Matrix B = $\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$
- X = $B^2 - 4B$
- X = $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 50 \end{bmatrix}$

Step 2: Calculate B^2

$B^2 = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix}$

Step 3: Calculate X = B^2 - 4B

X = $B^2 - 4B = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - 4\begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

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Step 4: Solve for a and b

Comparing the matrices:

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 50 \end{bmatrix}$$

Therefore:

- a = 5
- b = 50

The values of a and b are:

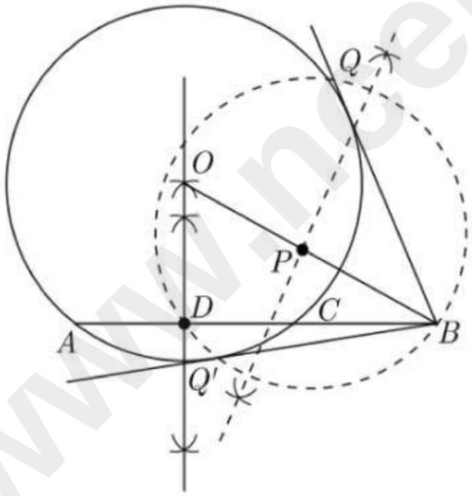
- a = 5
- b = 50

(ii) Draw a line AB = 5cm. Mark a point C on AB such that AC = 3cm. Using a ruler and a compasses only, construct

(i) a circle of radius 2.5 cm, passing through A and C .

(ii) two tangents to the circle from the external point B. Measure and record the length of the tangents.

Answer:



Steps for construction:

1. Draw AB = 5 cm using a ruler.
2. With A as the centre cut an arc of 3 cm on AB to obtain C.
3. With A as the centre and radius 2.5 cm, draw an arc above AB.

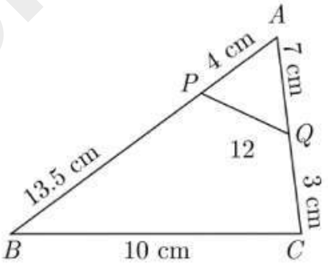
4. With same radius and C as the centre draw an arc to cut the previous arc and mark the intersection as O.
5. With O as the centre and radius 2.5 cm, draw a circle so that points A and C lie on the circle formed.
6. Join OB.
7. Draw the perpendicular bisector of OB to obtain the mid-point of OB, M.
8. With the M as the centre and radius equal to OM, draw a circle to cut the previous circle at points P and Q.
9. Join PB and QB. PB and QB are the required tangents to the given circle from exterior point B.
   QB = PB = 3 cm
   That is, length of the tangents is 3 cm.

(iii) In ΔABC, P and Q are points on AB and AC such that $AP = 4$ cm, $PB = 13.5$ cm, $AQ = 7$ cm and $QC = 3$ cm.

(i) Prove that $\Delta APQ \sim \Delta ACB$.

(ii) If $BC = 10$ cm, find the length of PQ.

(iii) Find the area of ΔAPQ : area of ΔACB.



Answer:

(i) Prove that $\triangle APQ \sim \triangle ACB$:

In $\triangle APQ$ and $\triangle ACB$:

- P divides AB such that:

$$\frac{AP}{AB} = \frac{4}{4 + 13.5} = \frac{4}{17.5}.$$

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- Q divides AC such that:

$$\frac{AQ}{AC} = \frac{7}{7+3} = \frac{7}{10}.$$

The corresponding sides of APQ and ACB are proportional.
Also, $\angle A$ is common to both triangles.
By **SAS similarity criterion**:

$$\triangle APQ \sim \triangle ACB.$$

(ii) Find the length of PQ:

Since $\triangle APQ \sim \triangle ACB$, the ratio of corresponding sides is the same:

$$\frac{PQ}{BC} = \frac{AP}{AB}.$$

Substitute the known values:

$$\frac{PQ}{10} = \frac{4}{17.5}.$$

Solve for PQ:

$$PQ = 10 \times \frac{4}{17.5}.$$

$$PQ = \frac{40}{17.5} = 2.29\,\text{cm (approx)}.$$

(iii) Find the area ratio $\triangle APQ : \triangle ACB$:

The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides:

$$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ACB} = \left(\frac{AP}{AB}\right)^2.$$

Substitute the values:

$$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ACB} = \left(\frac{4}{17.5}\right)^2.$$

Simplify:

$$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ACB} = \frac{16}{306.25}.$$

Thus, the area ratio is approximately:

$$\frac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ACB} \approx 0.052.$$

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QUESTION 5.

(i) Find the mean of the following frequency distribution :

Classes	0- 10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	3	8	10	15	7	4	3

Answer:

We prepare following table to find mean.

Classes	x_i	f_i	$f_i x_i$
0-10	5	3	15
10-20	15	8	120
20-30	25	10	250
30-40	35	15	525
40-50	45	7	315
50-60	55	4	220
60-70	65	3	195
		$\sum f_i = 50$	$\sum f_i x_i = 1640$

Mean $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{1640}{50} = 32.8$

(ii) Rate of GST for different categories are given below :

Category	**Rate of GST**
Toiletries	18%
Automobiles	28%
Dry Fruits	5%
Edible Oils	5%
Processed food	12%
Printed material	18%

Anuradha bought the following items from a departmental store :

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S. No.	Item	Price
1.	Coconut oil	₹2400
2.	Soap	₹900
3.	Cheese	₹500

Find the :

(a) Total GST paid.

(b) Total bill amount including GST.

Answer:

Step 1: Identify the GST rate for each item:

From the table of **Rate of GST**:

- **Coconut oil**: Edible oil → GST = 5%,
- **Soap**: Toiletries → GST = 18%,
- **Cheese**: Processed food → GST = 12%.

Step 2: Calculate GST for each item:

The formula for GST is:

$$\text{GST} = \text{Price} \times \frac{\text{Rate of GST}}{100}.$$

1. **Coconut oil (₹2400, 5% GST):**

$$\text{GST} = 2400 \times \frac{5}{100} = 120.$$

2. **Soap (₹900, 18% GST):**

$$\text{GST} = 900 \times \frac{18}{100} = 162.$$

3. **Cheese (₹500, 12% GST):**

$$\text{GST} = 500 \times \frac{12}{100} = 60.$$

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Total GST paid

$= \text{GST on Coconut oil}$

$+ \text{GST on Soap}$

$+ \text{GST on Cheese}$

$= 120 + 27 + 60$

$= ₹207$

Total bill amount including GST

$= \text{Price of Coconut oil}$

$+ \text{Price of Soap} + \text{Price of Cheese}$

$+ \text{Total GST paid}$

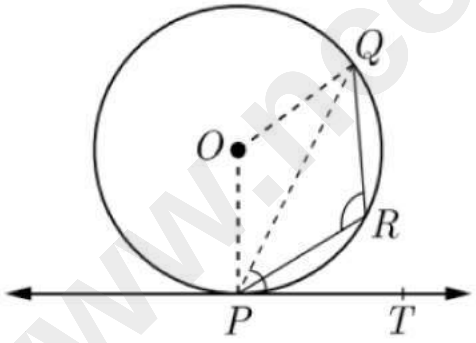
$= 2400 + 150 + 500 + 207$

$= ₹3257$

(a) Total GST Paid is ₹207
(b) Total bill amount including GST is ₹3257

(iii) In figure, PQ is a chord of a circle O and PT is a tangent. If ∠QPT = 60°, find ∠PRQ.

Answer:



m∠OPT 90° (radius is perpendicular to the tangent)

So, ∠OPQ = ∠OPT - ∠QPT

= 90° - 60°

= 30°

m∠POQ = 2m∠QPT = 2 x 60° = 120°

reflex m∠POQ = 360° - 120°= 240°

$$\angle PQR = \frac{1}{2}\text{reflex}\angle POQ$$

$$= \frac{1}{2} \times 240^\circ$$

$$= 120^\circ$$

$$m\angle PRQ = 120^\circ$$

QUESTION 6.

(i) Three numbers form an increasing G.P. If the middle term is doubled, then the new numbers are in A.P. Find the common ratio of the GP.

Answer:

Step 1: Condition when the middle term is doubled:

When the middle term ar is doubled, the new sequence becomes:

$$a,\ 2ar,\ ar^2.$$

These numbers are now in an A.P. Hence, the condition for an A.P. is that the difference between consecutive terms is the same.

$$(2ar - a) = (ar^2 - 2ar).$$

Step 2: Simplify the equation:

$$2ar - a = ar^2 - 2ar.$$

Bring all terms to one side:

$$2ar - a - ar^2 + 2ar = 0.$$

Combine like terms:

$$4ar - a - ar^2 = 0.$$

Step 3: Factorize:

Take a common:

$$a(4r - 1 - r^2) = 0.$$

Since $a \neq 0$ (as the numbers are non-zero), we get:

$$4r - 1 - r^2 = 0.$$

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Rearranging:

$$r^2 - 4r + 1 = 0.$$

Step 4: Solve for r using the quadratic formula:

The quadratic formula is:

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Here:

- $a = 1,$
- $b = -4,$
- $c = 1.$

Substitute:

$$r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}.$$

$$r = \frac{4 \pm \sqrt{16 - 4}}{2}.$$

$$r = \frac{4 \pm \sqrt{12}}{2}.$$

Simplify:

$$r = \frac{4 \pm 2\sqrt{3}}{2}.$$

$$r = 2 \pm \sqrt{3}.$$

Step 5: Select the correct root:

Since $r > 1$ (common ratio in an increasing G.P.), we take the positive root:

$$r = 2 + \sqrt{3}.$$

(ii) A spiral is made up of successive semi-circles with centres alternately A and B starting with A, of radii 1cm, 2 cm, 3 cm, ... as shown in the figure. What is the total length of spiral made up of eleven consecutive semi-circles?

(Use π = 3.14)

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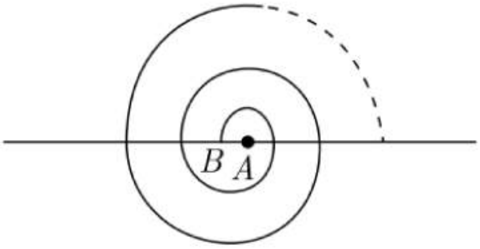

Answer:

Step 1: Total length of one semi-circle:

The circumference of a full circle of radius r is:

$$C = 2\pi r.$$

The length of a **semi-circle** is half of the circumference:

$$\text{Length of semi-circle} = \frac{1}{2} \times 2\pi r = \pi r.$$

Step 2: Total length of 11 semi-circles:

The radii of the 11 semi-circles are:

$$1, 2, 3, \dots, 11\,\text{cm}.$$

The total length of the spiral is the sum of the lengths of all 11 semi-circles:

$$\text{Total Length} = \pi \times (1 + 2 + 3 + \cdots + 11).$$

Step 3: Sum of the radii:

The sum of the first n natural numbers is given by:

$$\text{Sum} = \frac{n(n+1)}{2}.$$

Here, $n = 11$:

$$\text{Sum} = \frac{11(11+1)}{2} = \frac{11 \times 12}{2} = 66.$$

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Step 4: Calculate the total length:

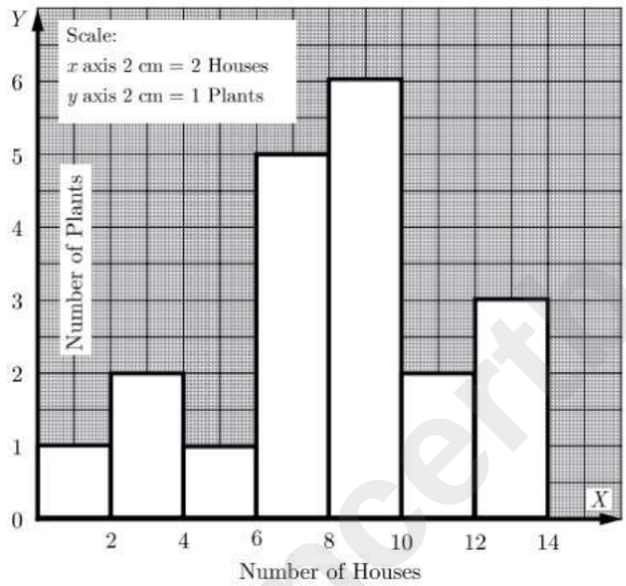
Substitute the sum into the total length formula:

$$\text{Total Length} = \pi \times 66.$$

Using $\pi = 3.14$:

$$\text{Total Length} = 3.14 \times 66 = 207.24\,\text{cm}.$$

(iii) The following histogram represents the number of plants per house in a colony.



Use the data to

(i) Frame a frequency table

(ii) To calculate mean

(iii) To determine the modal class

Answer:

(i) Frequency table

From given histogram we prepare following frequency distribution table.

Number of plants	Number of houses
0-2	1
2-4	2
4-6	1
6-8	5
8-10	6
10-12	2
12-14	3

(ii) Mean

From frequency distribution table we prepare following cumulative distribution table.

Class	x_i	f_i	$f_i x_i$
0-2	1	1	1
2-4	3	2	6
4-6	5	1	5
6-8	7	5	35
8-10	9	6	54
10-12	11	2	22
12-14	13	3	39
Total		$\Sigma f_i = 20$	$\Sigma f_i x_i = 162$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{162}{20} = 8.1$$

Mean number of plants per house is 8.1.

(iii) Modal class

We observe that the class 8-10 has maximum frequency 6. Therefore 8-10 is the modal class.

QUESTION 7.

(i) Point P divides the joining of line segment A(2, 5) and B(7, 15) in the ratio 2 : 3.

(i) Find the co-ordinates of point P

(ii) Find the equation of a line with gradient -5/3 and passing through P.

Answer:

Part (i): Find the coordinates of point P

Point P divides the line segment AB in the ratio $2:3$. The formula for a point dividing a line segment in a ratio $m_1 : m_2$ is:

$$P(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$$

Given:

- $A(2,5)$ and $B(7,15)$
- $m_1 = 2, m_2 = 3$

Substitute the values:

1. **For x-coordinate:**

$$x = \frac{2(7)+3(2)}{2+3} = \frac{14+6}{5} = \frac{20}{5} = 4$$

2. **For y-coordinate:**

$$y = \frac{2(15)+3(5)}{2+3} = \frac{30+15}{5} = \frac{45}{5} = 9$$

Coordinates of P: $(4,9)$

Part (ii): Find the equation of a line with gradient $-\frac{5}{3}$ passing through $P(4,9)$

The equation of a line in point-slope form is:

$$y - y_1 = m(x - x_1)$$

Where:

- $(x_1, y_1) = (4,9)$
- Gradient $m = -\frac{5}{3}$

Substitute the values:

$$y - 9 = -\frac{5}{3}(x-4)$$

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Simplify:

1. Expand the right-hand side:

$$y - 9 = -\frac{5}{3}x + \frac{20}{3}$$

2. Add 9 to both sides to isolate y. Express 9 with a denominator of 3:

$$y = -\frac{5}{3}x + \frac{20}{3} + \frac{27}{3}$$

3. Combine like terms:

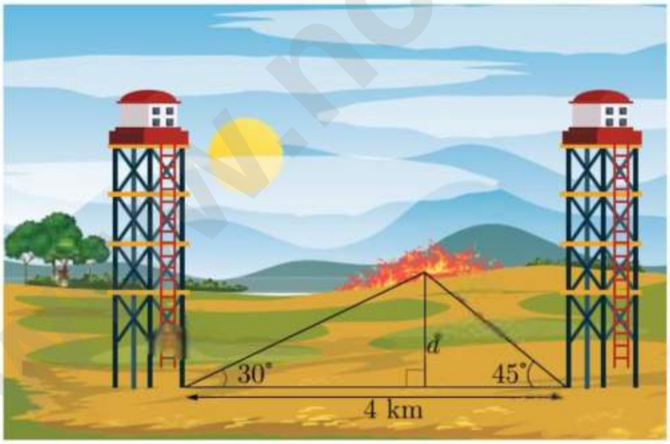
$$y = -\frac{5}{3}x + \frac{47}{3}$$

Final Answers:

1. Coordinates of P: $(4, 9)$
2. Equation of the line:

$$y = -\frac{5}{3}x + \frac{47}{3}$$

(ii) Fire towers : Two fire towers are 4 kilometers apart, where tower is due west of tower. A fire is spotted from the towers, and the angle of fire sight from tower is shown below. Find the distance of the fire from the line segment.



Answer:

Let d be the distance of the fire from the line segment. We draw a diagram of the situation as shown below.

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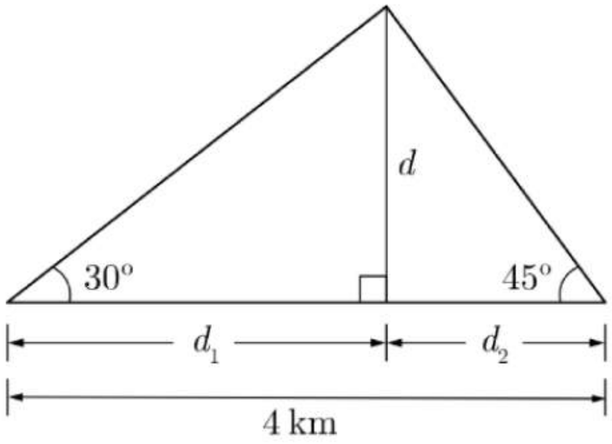

Given:

- Distance between towers = 4 km
- Angle from left tower = 30°
- Angle from right tower = 45°

Let's solve this using a different approach:

For the left triangle:
$\tan(30^\circ) = \frac{h}{x}$

For the right triangle:
$\tan(45^\circ) = \frac{h}{4-x}$

Since $\tan(45°) = 1
$h = 4 - x$

And since tan(30°) =
$h = x \cdot \frac{1}{\sqrt{3}}$

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Setting these equal:

$4 - x = x \cdot \frac{1}{\sqrt{3}}$

$4 - x = \frac{x}{\sqrt{3}}$

$4\sqrt{3} - x\sqrt{3} = x$

$4\sqrt{3} = x\sqrt{3} + x$

$4\sqrt{3} = x(\sqrt{3} + 1)$

$x = \frac{4\sqrt{3}}{\sqrt{3}+1} = 3$

Now we can find h:

$h = 4 - x = 4 - 3 = 1.732 \text{ km}$

Therefore, the correct distance of the fire from the line segment connecting the two towers is **1.732 kilometers**.

QUESTION 8.

(i) Solve the following system of inequations and graph the solution set:

$x + 5 \leq 4(x - 1)$ and $7 - 2x > -3$, $x \in R$

Answer:

We are tasked with solving the system of inequalities:

1. $x + 5 \leq 4(x - 1)$
2. $7 - 2x \geq -3$
3. $x \in \mathbb{R}$ (real numbers).

We will solve each inequality step by step and then determine the intersection of the solution sets.

Step 1: Solve $x + 5 \leq 4(x - 1)$

Expand the terms on the right-hand side:

$$x + 5 \leq 4x - 4$$

Simplify by bringing all x-terms to one side and constants to the other:

$$5 + 4 \leq 4x - x$$

$$9 \leq 3x$$

Divide by 3 (note: the inequality direction remains the same since $3 > 0$):

$$x \geq 3$$

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Step 2: Solve $7 - 2x \geq -3$

Simplify by isolating $-2x$:

$$7 + 3 \geq 2x$$

$$10 \geq 2x$$

Divide by 2 (again, inequality direction remains the same since $2 > 0$):

$$x \leq 5$$

Step 3: Combine the Solution Sets

From the two inequalities:

1. $x \geq 3$ (from the first inequality).
2. $x \leq 5$ (from the second inequality).

The solution is the intersection of these two inequalities:

$$3 \leq x \leq 5$$

Step 4: Graph the Solution Set

- Draw a number line.
- Mark $x = 3$ with a **solid dot** (since the inequality includes $\leq$ and $\geq$).
- Mark $x = 5$ with a **solid dot**.
- Shade the region **between** 3 **and** 5, including the endpoints.

Final Solution:

The solution set is:

$$x \in [3, 5]$$

(ii) In the given figure, O is the centre of the centre, ⍰BAD = 75° and chord BC = chord CD. Find

(i) ∠BOC

(ii) ∠OBD

(iii) ∠BCD.

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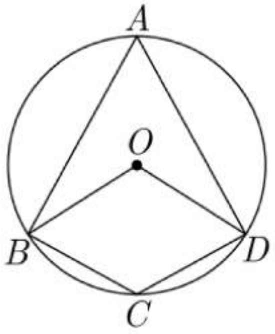

Answer:

(i) $\angle BOC$

An angle subtended by an arc of a circle at the centre is twice of the angle subtended by the same arc at any point on the remaining part of the circle. Thus

$$\angle BOD = 2\angle BAD$$
$$= 2 \times 75^\circ$$
$$= 150^\circ$$
$$\angle BOC = \tfrac{1}{2}\angle BOD$$
$$= \tfrac{1}{2} \times 150^\circ = 75^\circ$$

(ii) To find $\angle OBD$:

- Since O is the center, OB and OD are radii of the circle.
- In $\triangle OBD$, $OB = OD$, which makes $\triangle OBD$ an **isosceles triangle**.

The vertex angle $\angle BOD$ at O is part of the full central angle $\angle BOC$, where:

$$\angle BOD = \angle BOC = 150^\circ$$

The sum of the angles in $\triangle OBD$ is 180°:

$$\angle OBD + \angle ODB + \angle BOD = 180^\circ$$

Since $\triangle OBD$ is isosceles ($OB = OD$):

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Let $\angle OBD = \angle ODB = x$. Substituting into the equation:

$$x + x + 150^\circ = 180^\circ$$

$$2x = 30^\circ$$

$$x = 15^\circ$$

Thus:

$$\angle OBD = 15^\circ$$

(iii) BCD

Sum of opposite angles in a cyclic quadrilateral is 180°. Here ABCD is cyclic quadrilateral, thus

$$\angle BAD + \angle BCD = 180^\circ$$

$$75^\circ + \angle BCD = 180^\circ$$

$$\angle BCD = 180^\circ - 75^\circ$$

$$= 105^\circ$$

(iii) In what ratio does the y-axis divide the line segment joining the point P (– 4, 5) and Q(3, – 7)? Also, find the coordinates of the point of intersection.

Answer:

Step 1: General understanding

- The y-axis divides a line segment where the **x-coordinate** of the point of intersection is 0 because any point on the y-axis has $x = 0$.
- The coordinates of the point where the line segment intersects the y-axis are $(0, y)$.

Step 2: Use Section Formula

The section formula states that a point dividing a line segment joining two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio $m : n$ has coordinates:

$$(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)$$

Here:

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- $P(x_1, y_1) = (-4, 5)$
- $Q(x_2, y_2) = (3, -7)$
- The y-axis divides the segment, so the x-coordinate is 0.
  We set $x = 0$ in the section formula and solve for the ratio $m : n$.

Step 3: Solve for the ratio

From the section formula for the x-coordinate:

$$x = \frac{mx_2 + nx_1}{m + n}$$

Substitute $x = 0$, $x_1 = -4$, and $x_2 = 3$:

$$0 = \frac{m(3) + n(-4)}{m + n}$$

Simplify the numerator:

$$0 = \frac{3m - 4n}{m + n}$$

Since the denominator $m + n \neq 0$, the numerator must equal 0:

$$3m - 4n = 0$$

Rearranging gives:

$$\frac{m}{n} = \frac{4}{3}$$

Thus, the y-axis divides the line segment in the ratio **4:3**.

Step 4: Find the coordinates of the point of intersection

Now that the ratio is $m : n = 4 : 3$, substitute back into the section formula for the y-coordinate:

$$y = \frac{my_2 + ny_1}{m + n}$$

Substitute:

- $y_1 = 5$, $y_2 = -7$, $m = 4$, and $n = 3$:

$$y = \frac{4(-7) + 3(5)}{4 + 3}$$

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Simplify:

$$y = \frac{-28 + 15}{7}$$

$$y = \frac{-13}{7}$$

Final Answer:

1. The y-axis divides the line segment in the ratio $4 : 3$.
2. The coordinates of the point of intersection are:

$$\left(0, -\frac{13}{7}\right)$$

QUESTION 9.

(i) Using properties of proportion, solve for x. Given that x is positive.

$$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$

Answer:

Step 1: Cross-multiply

From the given equation:

$$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$

Cross-multiplying gives:

$$2x + \sqrt{4x^2 - 1} = 4\left(2x - \sqrt{4x^2 - 1}\right)$$

Simplify the right-hand side:

$$2x + \sqrt{4x^2 - 1} = 8x - 4\sqrt{4x^2 - 1}$$

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Step 2: Isolate the square root terms

Bring all square root terms to one side and the other terms to the opposite side:

$$\sqrt{4x^2-1}+4\sqrt{4x^2-1}=8x-2x$$

Combine like terms:

$$5\sqrt{4x^2-1}=6x$$

Step 3: Square both sides

To eliminate the square root, square both sides of the equation:

$$\left(5\sqrt{4x^2-1}\right)^2=(6x)^2$$

Simplify:

$$25(4x^2-1)=36x^2$$

Distribute on the left-hand side:

$$100x^2-25=36x^2$$

Step 4: Solve for x^2

Simplify the equation:

$$100x^2-36x^2=25$$

Combine like terms:

$$64x^2=25$$

Divide by 64:

$$x^2=\frac{25}{64}$$

Take the square root of both sides (since $x>0$):

$$x=\frac{5}{8}$$

(ii) One fourth of a herd of camels was seen in forest. Twice of square root of the herd had gone to mountain and remaining 15 camels were seen on the bank of a river, find the total number of camels.

Answer:

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Step 1: Eliminate the fraction

To remove $\frac{x}{4}$, multiply through by 4:

$$4 \cdot \frac{x}{4} + 4 \cdot 2\sqrt{x} + 4 \cdot 15 = 4x$$

Simplify:

$$x + 8\sqrt{x} + 60 = 4x$$

Step 2: Rearrange the equation

Bring all terms to one side:

$$x + 8\sqrt{x} + 60 - 4x = 0$$

Simplify:

$$-3x + 8\sqrt{x} + 60 = 0$$

Step 3: Isolate the square root term

Move the $-3x$ and 60 to the other side:

$$8\sqrt{x} = 3x - 60$$

Step 4: Square both sides

To eliminate the square root, square both sides:

$$(8\sqrt{x})^2 = (3x - 60)^2$$

Simplify:

$$64x = 9x^2 - 360x + 3600$$

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Step 5: Rearrange into standard quadratic form

Move all terms to one side:

$$0 = 9x^2 - 360x + 3600 - 64x$$

Simplify:

$$9x^2 - 424x + 3600 = 0$$

Step 6: Solve the quadratic equation

We solve $9x^2 - 424x + 3600 = 0$ using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here:

- $a = 9$,
- $b = -424$,
- $c = 3600$.

Substitute into the formula:

$$x = \frac{-(-424) \pm \sqrt{(-424)^2 - 4(9)(3600)}}{2(9)}$$

Simplify step by step:

1. $b^2 = (-424)^2 = 179776$,
2. $4ac = 4(9)(3600) = 129600$,
3. $b^2 - 4ac = 179776 - 129600 = 50176$.

Thus:

$$x = \frac{424 \pm \sqrt{50176}}{18}$$

Simplify $\sqrt{50176}$:

$$\sqrt{50176} = 224$$

Substitute back:

$$x = \frac{424 \pm 224}{18}$$

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Step 7: Solve for x

1. For the positive root:

$$x = \frac{424 + 224}{18} = \frac{648}{18} = 36$$

2. For the negative root:

$$x = \frac{424 - 224}{18} = \frac{200}{18} \approx 11.11$$

Since x represents the total number of camels, it must be a positive whole number.

Final Answer:

The total number of camels is:

$$x = 36$$

(iii) Draw a line segment AB of length 12 cm. Mark M, the mid-point of AB. Draw and describe the locus of a point which is

(i) at a distance of 3 cm from AB.

(ii) at a distance of 5 cm from the point M.
Mark the points p, Q, R, S which satisfy both the above conditions. What kind of quadrilateral is PQRS? Compute the area of the quadrilateral PQRS.

Answer:

(i) Locus of a point at a distance of 3 cm from AB

The locus of points that are 3 cm away from the line segment AB consists of two parallel lines, one above and one below AB, at a vertical distance of 3 cm. These lines are represented as $y = 3$ and $y = -3$ in the coordinate plane.

(ii) Locus of a point at a distance of 5 cm from M

The locus of points that are 5 cm away from the midpoint M is a circle centered at $M = (6, 0)$ with a radius of 5 cm. The equation for this circle is:

$$(x - 6)^2 + y^2 = 25$$

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Intersection of the loci

To find the points P, Q, R, S, we calculate the intersection points of the circle with the parallel lines $y = 3$ and $y = -3$.

1. **For $y = 3$:**
   Substituting $y = 3$ into the circle equation:

$$(x-6)^2 + 3^2 = 25$$

   Simplify:

$$(x-6)^2 = 16 \implies x - 6 = \pm 4$$

   Thus, $x = 10$ and $x = 2$. The points are:

$$P = (10, 3), \quad Q = (2, 3)$$

2. **For $y = -3$:**
   Substituting $y = -3$ into the circle equation:

$$(x-6)^2 + (-3)^2 = 25$$

   Simplify:

$$(x-6)^2 = 16 \implies x - 6 = \pm 4$$

   Thus, $x = 10$ and $x = 2$. The points are:

$$R = (10, -3), \quad S = (2, -3)$$

Quadrilateral PQRS

The points P, Q, R, S form a quadrilateral. It is a rectangle because:

- Opposite sides are parallel ($PQ || RS$, and $QR || PS$).
- All angles are right angles due to symmetry.

Area of Quadrilateral PQRS

The length of one side (PQ) is the horizontal distance between P and Q:

$$PQ = |10 - 2| = 8 \, \text{cm}$$

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The length of another side (QR) is the vertical distance between Q and R:

$$QR = |3 - (-3)| = 6\,\text{cm}$$

The area of rectangle PQRS is:

$$\text{Area} = PQ \times QR = 8 \times 6 = 48\,\text{cm}^2$$

Final Answer:

- The quadrilateral formed by points P, Q, R, S is a rectangle.
- The area of quadrilateral PQRS is **48 cm²**.

QUESTION 10.

(i) What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor?

Answer:

Let the required number be K

Let $f(x) = 16x^3 - 8x^2 + 4x + 7 - K$

$\because$ $(2x + 1)$ is a factor of $f(x)$

$\therefore \quad f\left(-\frac{1}{2}\right) = 0$

$\Rightarrow 16 \times \left(-\frac{1}{2}\right)^3 - 8 \times \left(-\frac{1}{2}\right)^2 + 4 \times \left(-\frac{1}{2}\right) + 7 - K = 0$

$\Rightarrow -16 \times \frac{1}{8} - 8 \times \frac{1}{4} - 4 \times \frac{1}{2} + 7 - K = 0$

$\Rightarrow -2 - 2 - 2 + 7 - K = 0$

$\Rightarrow -6 + 7 - K = 0$

$\Rightarrow 1 - K = 0$

$\Rightarrow K = 1$

$\therefore$ The required number = 1

Thus 1 must be subtracted from $16x^3 - 8x^2 + 4x + 7$ for getting $(2x + 1)$ as a factor.

(ii) Three digit number are made using the digits 4, 5, 9 (without repetition). If a number among them is selected at random, what is the probability that the number will :

(i) Be a multiple of 5 ?

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(ii) Be a multiple of 9 ?

(iii) Will end with 9?

Answer: Total number of three digit numbers are : 459, 495, 549, 594, 945, 954 = 6.

(i) $P(\text{multiple of } 5) = \frac{2}{6} = \frac{1}{3}$

(ii) $P(\text{multiple of } 9) = \frac{6}{6} = 1$

(iii) $P(\text{ending with } 9) = \frac{2}{6} = \frac{1}{3}$

(iii) Use graph paper for this question. The points A(2, 1), B(0, 3) and C(–2, –2) are the vertices of a triangle.

(i) Plot the points on the graph paper.

(ii) Draw the triangle formed by reflecting these points in the x-axis

(iii) Are the two triangles congruent?

Answer:

(i) Plot the points $A(2, 1)$, $B(0, 3)$ and $C(-2, -2)$ as shown in the figure given below.

(ii) Reflect the points in the x-axis:

- Reflection in the x-axis changes the sign of the y-coordinate while keeping the x-coordinate the same.

1. **Reflection of $A(2, 1)$:**
   - The reflected point is $A'(2, -1)$.
2. **Reflection of $B(0, 3)$:**
   - The reflected point is $B'(0, -3)$.
3. **Reflection of $C(-2, -2)$:**
   - The reflected point is $C'(-2, 2)$.

Steps to draw the reflected triangle $A'B'C'$:

1. Plot the reflected points $A'(2, -1)$, $B'(0, -3)$, and $C'(-2, 2)$ on the graph paper.
2. Join the points A', B', and C' to form triangle $A'B'C'$.

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(iii) The two triangles ABC and $A'B'C'$ are congruent (measure the distances and check it).